French-Ukrainian workshop
on instrumentation developments
for high enegy physics

On the classical limit of Quantum Mechanics in the Theory of Channeling Phenomenon

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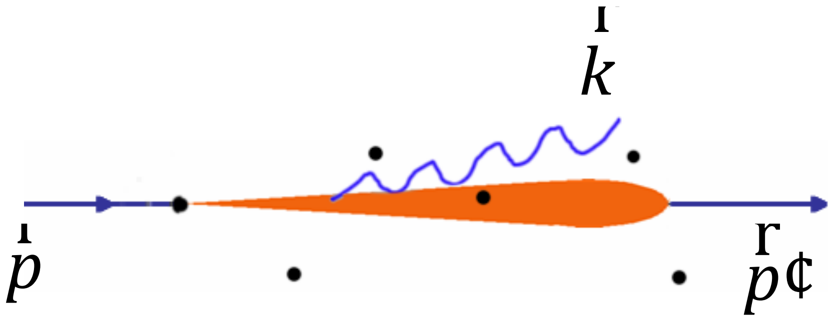
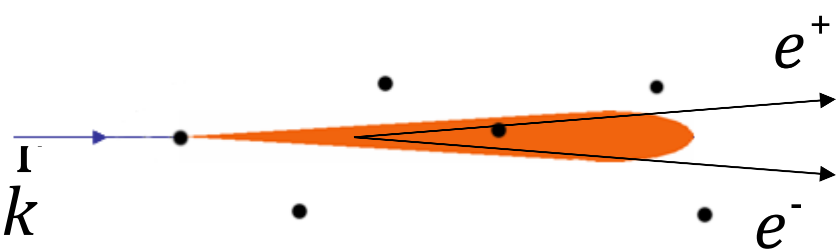
Coherent length



$$l_{coh} = \frac{1}{q_{||\min}} = \frac{2ee'}{m^2 w} >> a$$

$e = 100GeV$ $\qquad w = 500MeV$

$$l_{coh} \sim 10^{-3} cm$$



$$l_{\pm} = \frac{1}{q_{||}^{\pm}} = \frac{2e_- e_+}{m^2 w} >> a$$

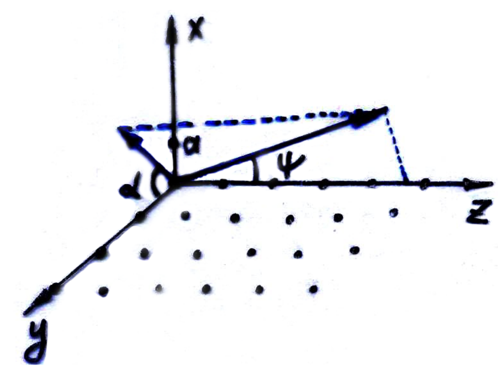
$$a_{eff} \sim N_c \frac{Ze^2}{hv} > 1$$

Coherent Bremsstrahlung in Crystal (Born Approximation)

(Ferretti 1950, Ter-Mikaelian 1952, Überall 1956)

$$\omega\frac{d\sigma}{d\omega}=\frac{2e^2\delta}{m^2\Delta}\frac{\varepsilon'}{\varepsilon}\sum_{g}\frac{g_\perp^2}{g_\parallel^2}\left[1+\frac{\omega^2}{2\varepsilon\varepsilon'}-2\frac{\delta}{g_\parallel}\left(1-\frac{\delta}{g_\parallel}\right)\right]\left|U_g\right|^2 e^{-g^2\overline{u^2}}$$

$$q_\parallel \geq \delta = \omega m^2/2\varepsilon\varepsilon', \qquad g_\parallel = g_z + \psi\left(g_y\cos\alpha + g_x\sin\alpha\right)\geq\delta$$



Discussion: E.Feinberg and M.Ter- Mikaelian with L.Landau and I.Pomeranchuk (1952)

T. - M. – Interference radiation by ultrarelativistic electrons in crystals.

Landau – That is impossible because the interference effect is possible only for

$$\lambda = \frac{h}{p} \geq a \quad , \quad \textbf{but not for } \lambda << a$$

The discussion was stopped.

Coherent length

In the theory of high energy electrons' radiation besides the length $\lambda \sim \hbar/p$ there exists another length responsible for the radiation,

$$\boxed{l_c = \frac{2\varepsilon\varepsilon'}{m^2\omega}.}$$

$$\begin{cases} \varepsilon = \varepsilon' + \omega \\ \vec{p} = \vec{p}\,' + \vec{k} + \vec{q} \\ q_{\min} = 1/l_c \end{cases}$$

Interpretations of l_c

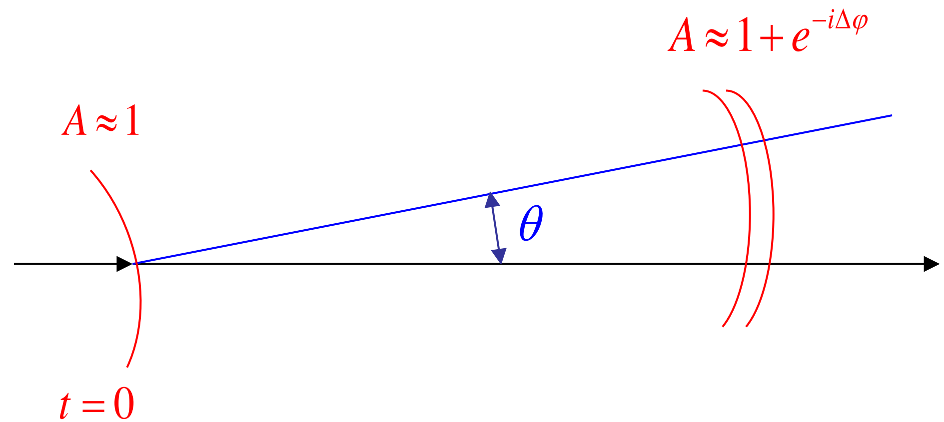
• Ter-Mikaelian (1952):	It is based on the first Born Approximation
• Landau, Pomeranchuk (1953):	It is based on classical electrodynamics
• Frish, Olsen (1959), Akhiezer, Shul'ga (1982)	It is based on the behavior of the wave packets
• Feinberg (1966) Akhiezer, Shul'ga, Fomin (1982)	Development of the process of radiation in space and time (half-bare electron)

Coherent length (Landau-Pomeranchuk, 1953)

$$\frac{dE}{d\omega\, do} = \frac{e^2}{4\pi^2}\left| \vec{k} \times \int_{-\infty}^{\infty} dt\, \vec{\upsilon}(t)\, e^{i\left(\omega t - \vec{k}\vec{r}(t)\right)} \right|^2$$

$$\vec{v}(t) \approx \vec{v}_0 \cdot \left(1 - \frac{1}{2} v_\perp^{\,2}(t)\right) + \vec{v}_\perp(t)$$

$$\Delta\varphi = \omega\Delta t - \vec{k}\vec{r}(\Delta t) <\sim 1$$

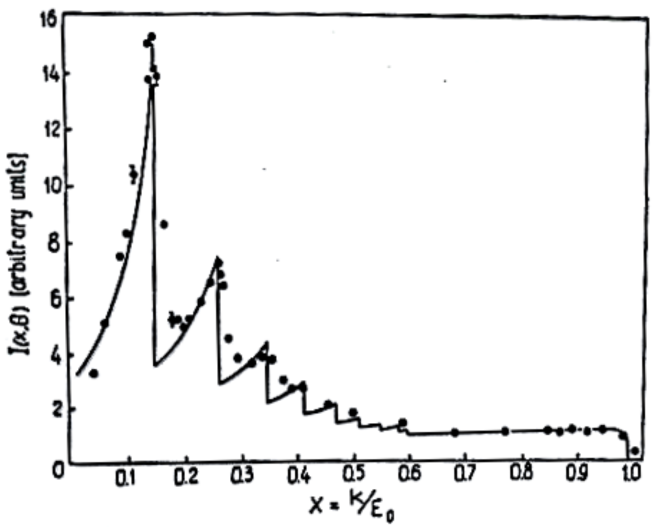


$$\Delta t \sim \frac{2\gamma^2}{\omega} \frac{1}{1 + \gamma^2 \overline{\theta^2_{\Delta t}} + \gamma^2\theta^2}$$

$$\Delta t \sim \begin{cases} 2\gamma^2/\omega & \gamma^2\overline{\theta^2_{\Delta t}} << 1 \\ << 2\gamma^2/\omega & \gamma^2\overline{\theta^2_{\Delta t}} >> 1 \end{cases}$$

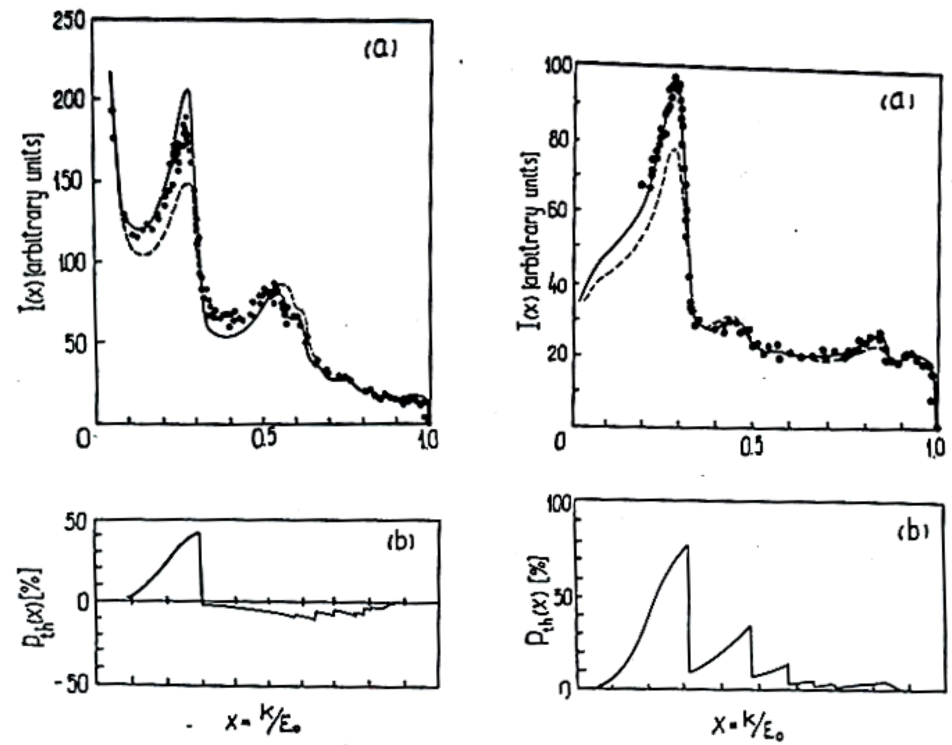
Experiment $\varepsilon \sim 1 - 5\ \mathrm{GeV}$ (1962 - 1965)

Frascati, DESY, Kharkov, Protvino, Tomsk, Yerevan, SLAC, …



Frascati

ε=1 GeV, θ =4,6 mrad



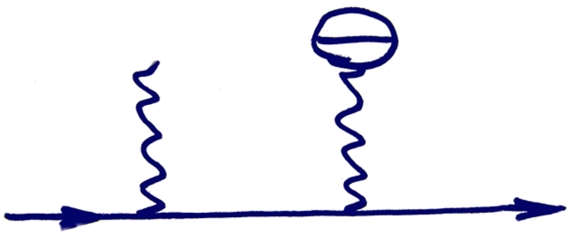
DESY

ε=4,8 GeV, θ =3,4 mrad

Generalization of CB theory

The main idea:

-For

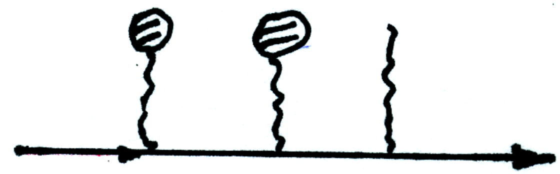


$$d\sigma_{coh} >> d\sigma_{BH}$$

-The relative contribution of higher Born approximation can be also increased (A.Akhiezer, P.Fomin, N.Shul'ga 1971)

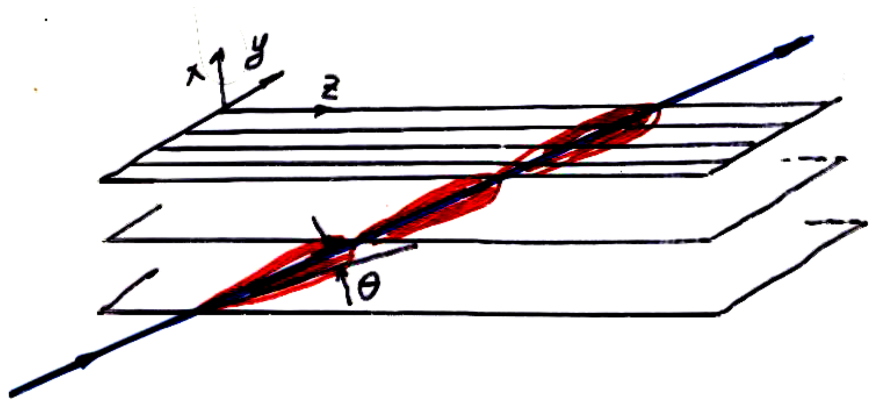
Second Born approximation in CB theory

A.Akhiezer, P.Fomin, N.Shul'ga (1970)



$$d\sigma_c = d\sigma_{coh}^{Born} \cdot \left(1 \pm \eta \frac{\theta_c^{\,2}}{\theta^2}\right), \qquad \hbar\omega << \varepsilon$$

$\eta \sim 1$ $\qquad$ $\theta_c = \sqrt{4Ze^2/\varepsilon a}$ – *critical channeling angle*



Higher Born approximation in the CB theory

A.Akhiezer, N.Shul'ga (1975)

$$N_{coh} \sim \min\left(\frac{l_{coh}}{a}, \quad \frac{R}{\psi a}\right)$$

$$l_{coh} = \frac{2\varepsilon\varepsilon'}{m^2\omega} >> a$$

$$\frac{Ze^2}{\hbar c} << 1 \quad \rightarrow \quad N_{coh}\frac{Ze^2}{\hbar c} \sim \frac{R}{\psi a}\frac{Ze^2}{\hbar c} << 1$$

Quickly destroys for $\psi \rightarrow 0$

PARADOX

This condition did not fulfill practically for experiments (1960-1970) on verification of F – T – Ü theoretical results.

But the experiments were in good agreement with this theory !!!
Why ???

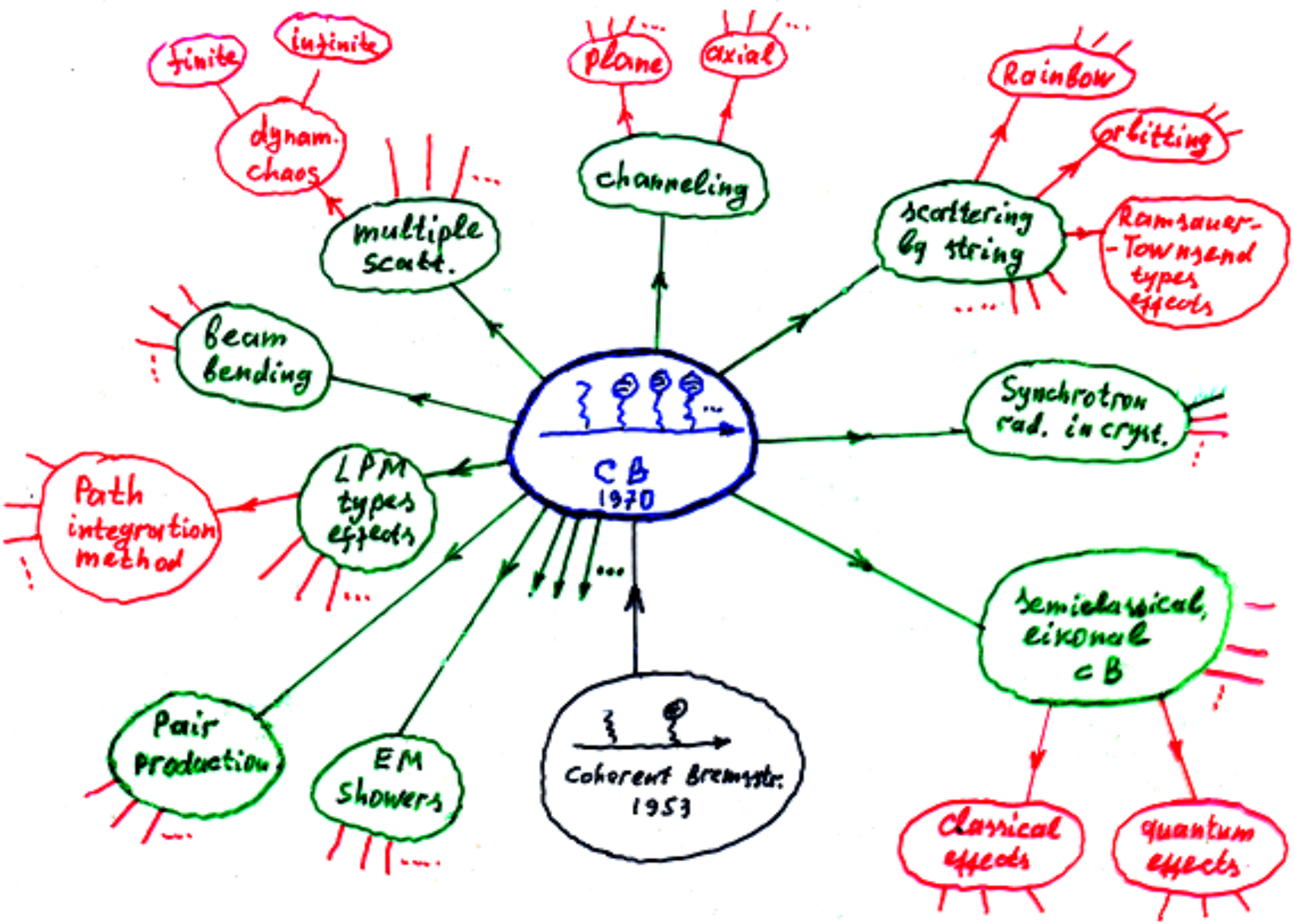
New field of research

The interaction of high-energy particles with matter in conditions of effectively strong interaction of the particle with atoms of media (semiclassical, classical approximations)

$$N_c \frac{Ze^2}{hc} >> 1$$

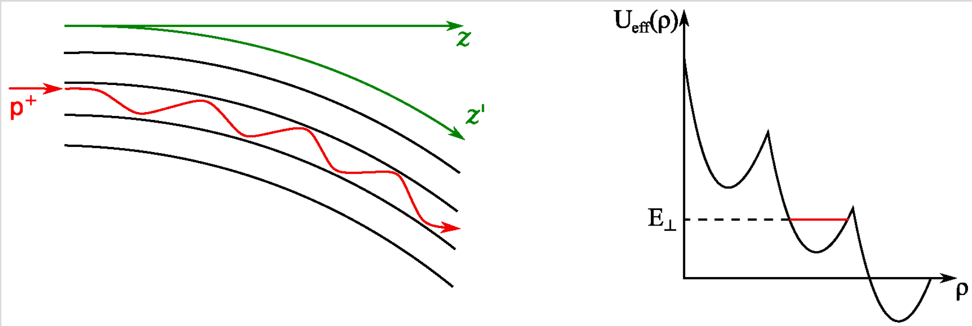
- Classical electrodynamics
- Semiclassical approximation of quantum electrodynamics
- Relation between classical and quantum effects
- Methods for description
- …

Problems generated by the theory of coherent radiation in crystals (situation up to 1995)



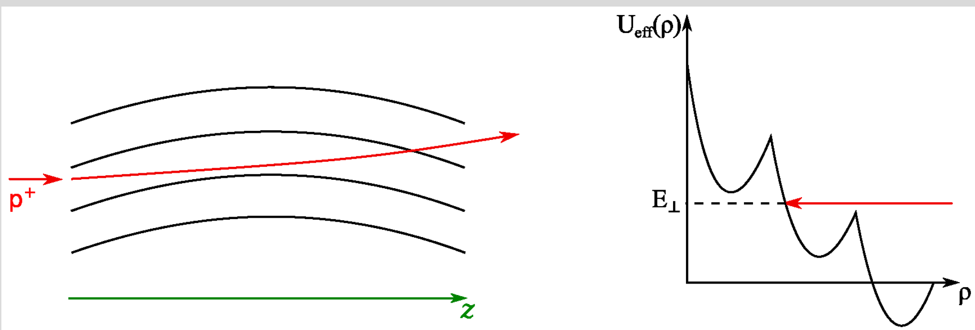
MECHANISMS OF HIGH-ENERGY CHARGED PARTICLE DEFLECTION BY BENT CRYSTALS

Planar channeling in bent crystal (*E.N. Tsyganov, 1976*)



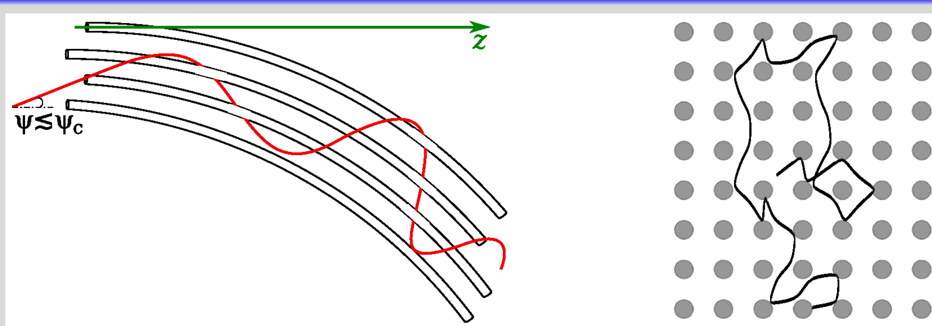
1979 — IHEP (Russia)
1980 — CERN

Volume reflection (*A.M. Taratin, S.A. Vorobiev, 1987*)



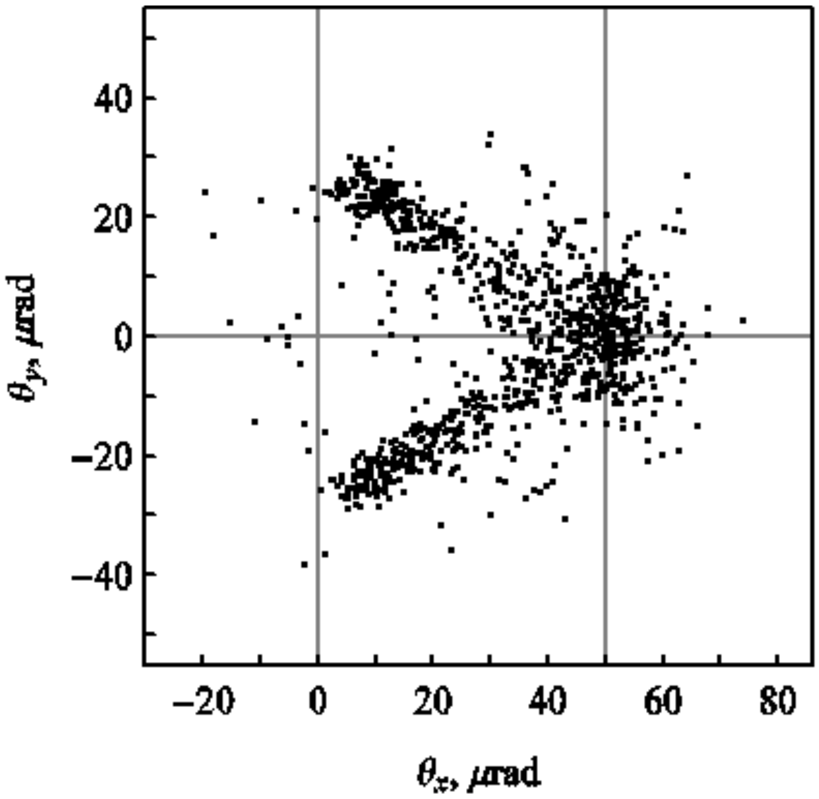
2006 — IHEP (Russia)
2006 — PNPI (Russia)
2007 — CERN

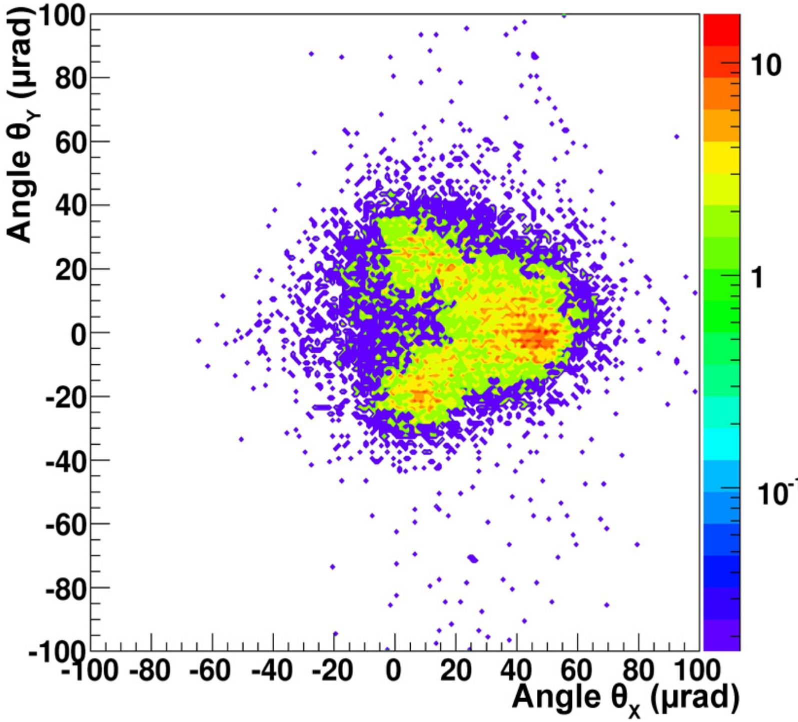
Stochastic deflection mechanism (*A.A. Greenenko, N.F. Shul'ga, 1991*)



2008 — CERN, protons
2009 — CERN, π^--mesons

Angular distribution of 400 GeV protons after passing 2 mm of bent Si crystal with R=40 m



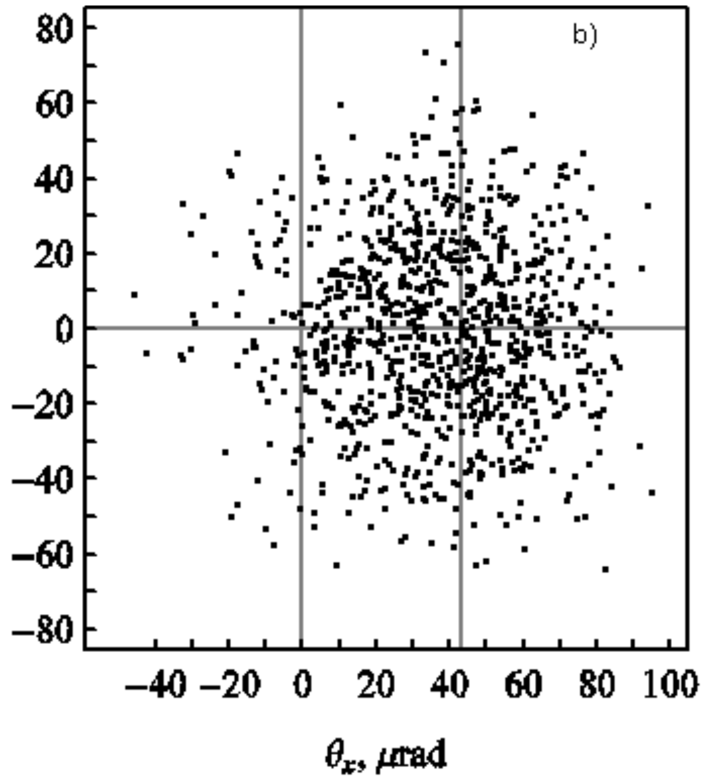


CERN experiment

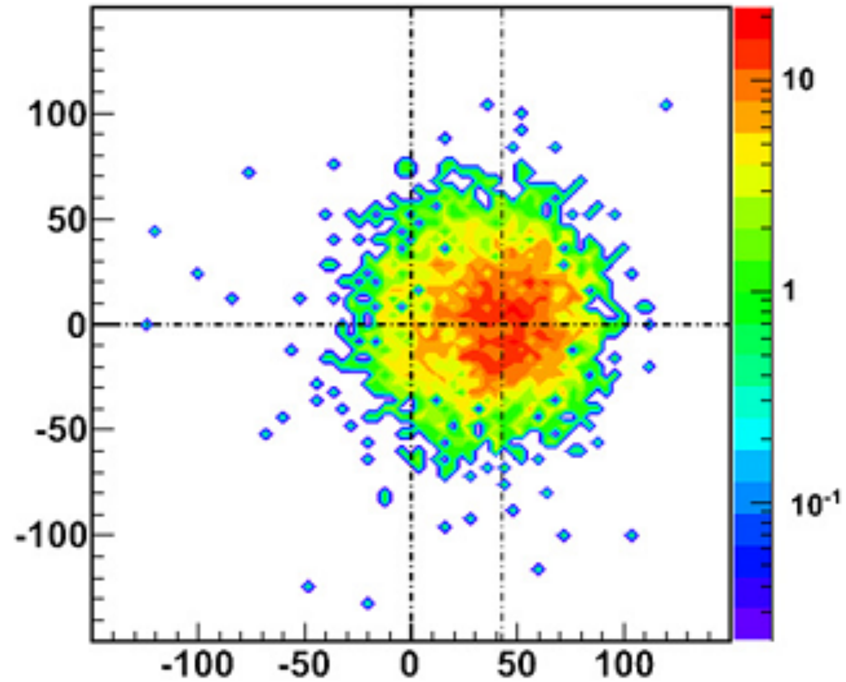
Simulation results

W. Scandale et al. Phys. Rev. Lett. 101 (2008), 164801

Angular distribution of 150 GeV π−-mesons after passing 1.172 mm of bent Si crystal with R=40 m



Simulation results



CERN experiment

W. Scandale et al. Physics Letters B 680 (2009) 301-304

Close collisions probability
(proposition for experiment at CERN)

p^+, 270 GeV, Si <110>, L = 5 mm, R = 50 m

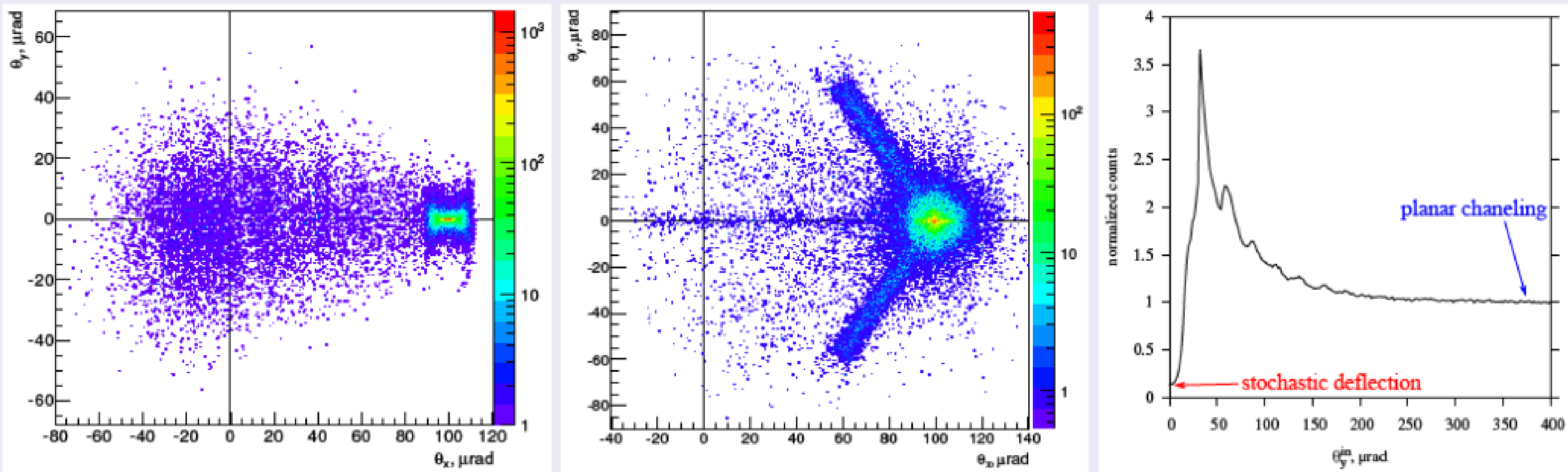
Planar channeling | Stochastic deflection | Close collisions probability

Yu.A. Chesnokov, I.V. Kirillin, W. Scandale, N.F. Shul’ga, V.I. Truten’ // Physics Letters B 731 (2014) 118

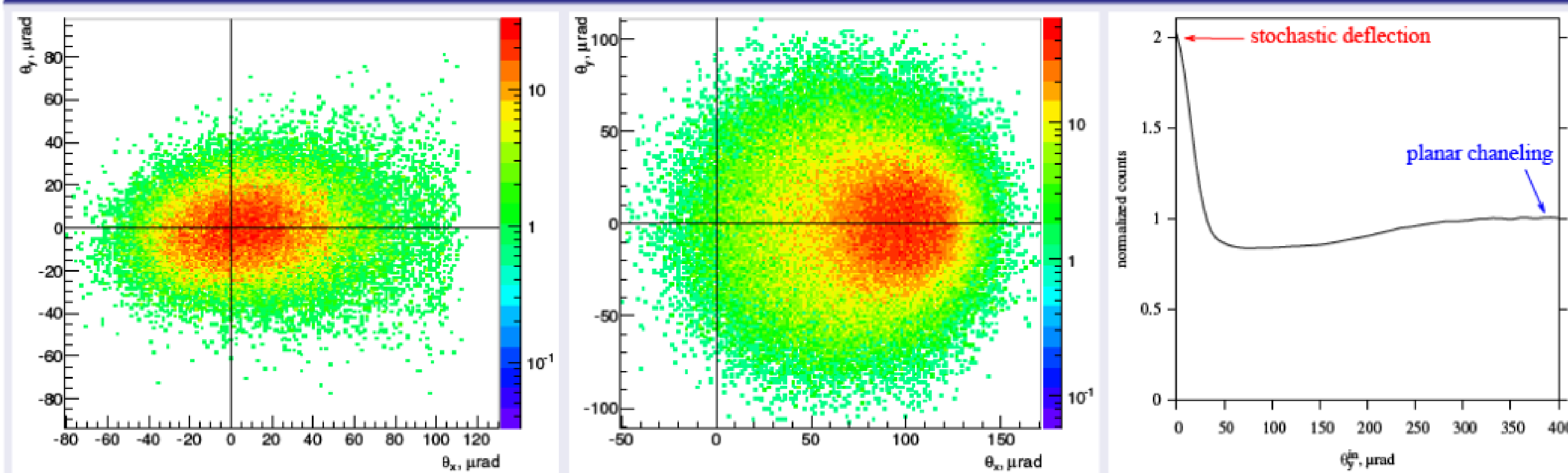
Probability of close collisions in bent crystal

planar channeling | stochastic mechanism | probability of close collisions

p^+, 270 GeV



π^-, 270 GeV



Yu.A. Chesnokov, I.V. Kirillin, W. Scandale et al. // **Phys. Lett. B.** *– 2014. – Vol. 731. – P. 118–121.*

Probability of close collisions of positively charged particles in a bent crystal (experiment)

p^+, 400 GeV, Si <110>, L = 2 mm, R = 35 m

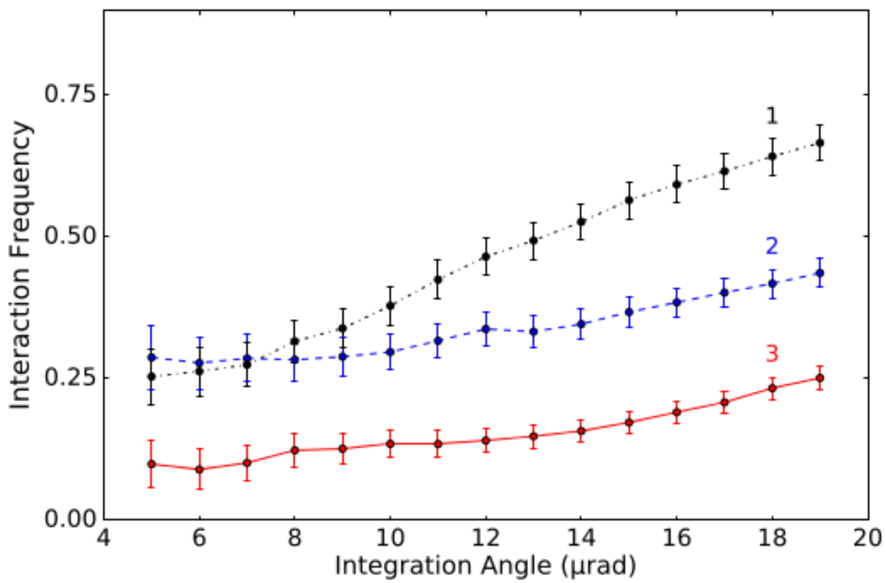


Fig. 5. Measured inelastic nuclear interaction (INI) frequency of 400 GeV/c protons interacting with the ⟨111⟩ and ⟨110⟩ crystals as a function of the angular region around the (110) planar channeling (black dash-dotted line, 1), the ⟨111⟩ axial channeling (blue dashed line, 2) and ⟨110⟩ (red continuous line, 3) orientations. The values are normalized to the INI frequencies for the amorphous crystal orientation. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

…

In summary, we compared the deflection efficiency and INI frequency under AC of ⟨111⟩ and ⟨110⟩ axes. The experiment confirms the theoretical predictions proposed in [22] and paves the way to the use of AC as an efficient manipulator of charged particle beams.

…

[22] Y. Chesnokov, I. Kirillin, W. Scandale, N. Shul'ga, V. Truten', About the probability of close collisions during stochastic deflection of positively charged particles by a bent crystal, Phys. Lett. B 731 (2014) 118–121, http://dx.doi.org/10.1016/j.physletb.2014.02.024.

W. Scandale et al. // Physics Letters B 760 (2016) 826

Feasibility of measurement of the magnetic moments of the charm baryons at the LHC using bent crystals

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[4]*V.N. Karazin Kharkiv National University, 61022 Kharkiv, Ukraine*
[5]*IPNO (Institut de Physique Nucléaire), Université Paris-Sud/IN2P3, Orsay, France*
[6]*CERN, European Organization for Nuclear Research, CH-1211 Geneva 23, Switzerland*
[7]*INFN Sezione di Roma, Piazzale Aldo Moro 2, 00185 Rome, Italy*

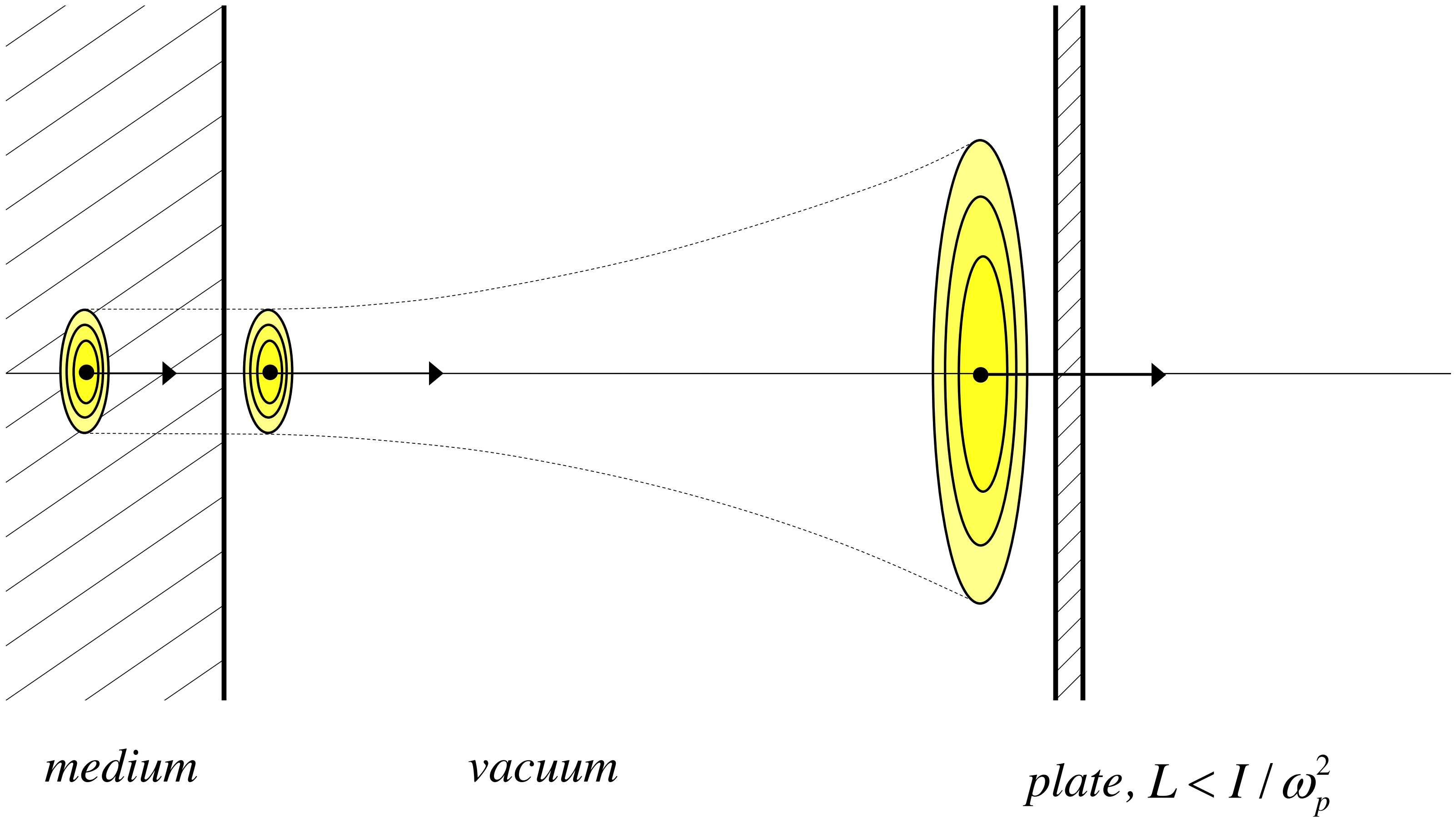
(Dated: April 28, 2017)

In this paper we revisit the idea of measuring the magnetic dipole moments of the charm baryons and in particular of Λ_c^+ by studying the spin precession induced by the strong effective magnetic field inside the channels of a bent crystal. We present a detailed sensitivity study showing the feasibility of such an experiment at the LHC in the coming years.

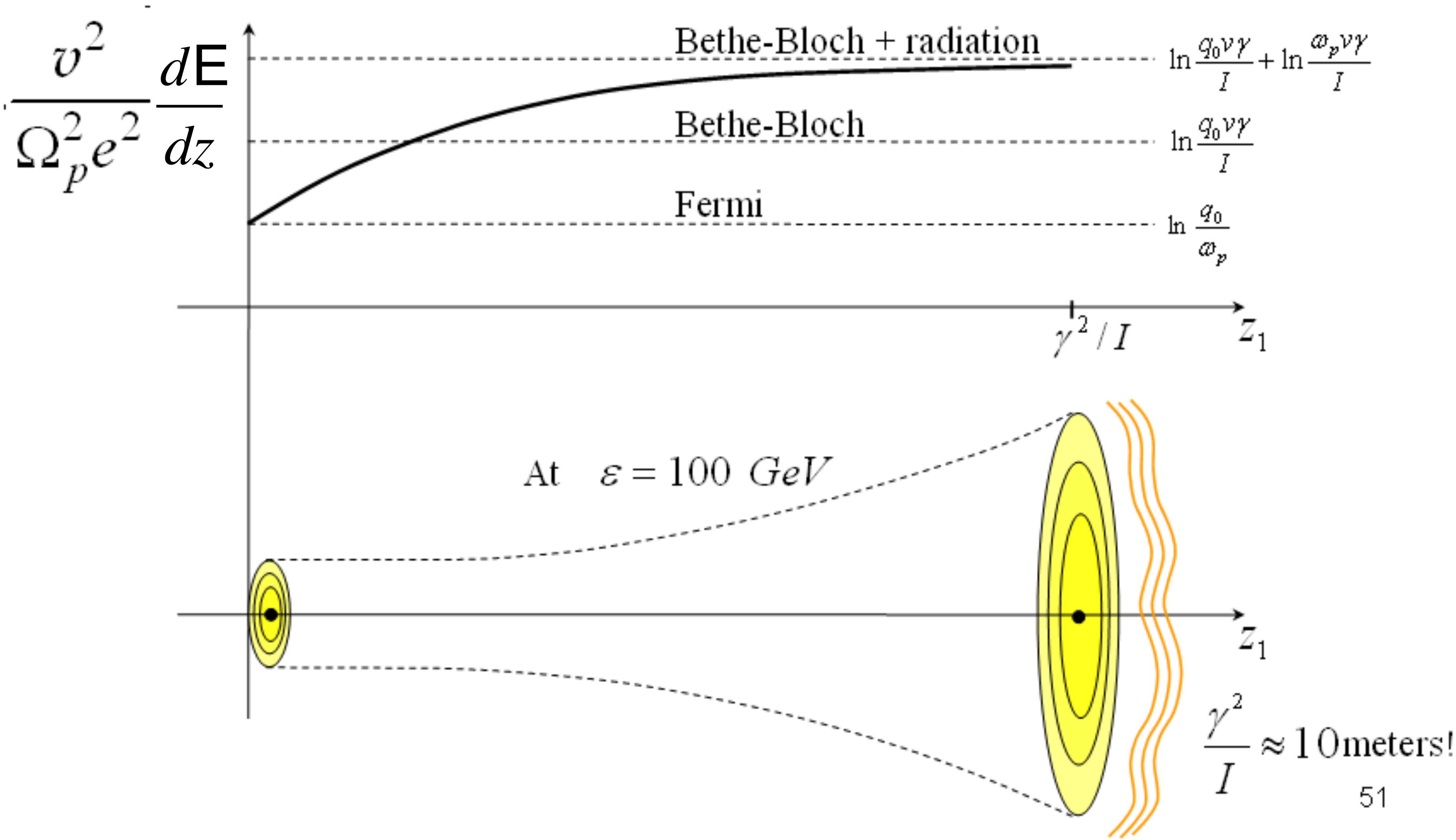
The article is published in arXiv:1705.03382 [hep-ph], "Journal of High Energy Physics" (2017).

Ionization energy losses by half-bare electron

N. Shul'ga, S. Trofymenko, Phys. Lett. A, 2012



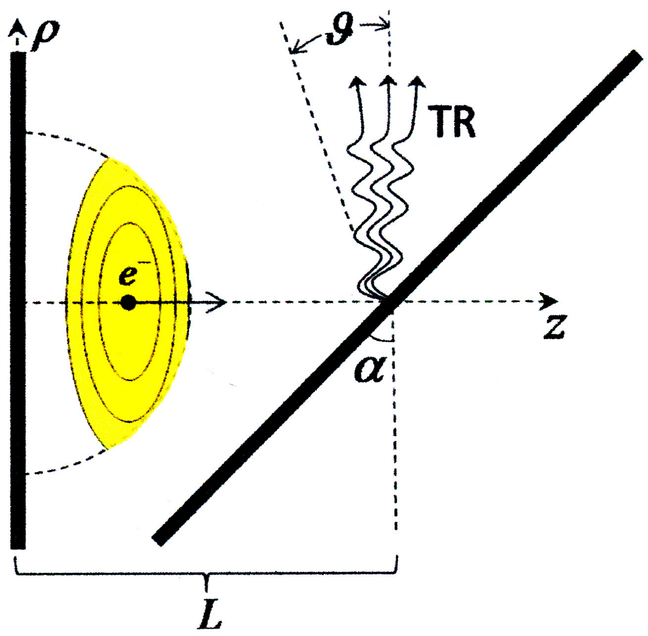
Ionization energy losses
by half-bare electron
(from Fermi to Bethe-Bloch formula)



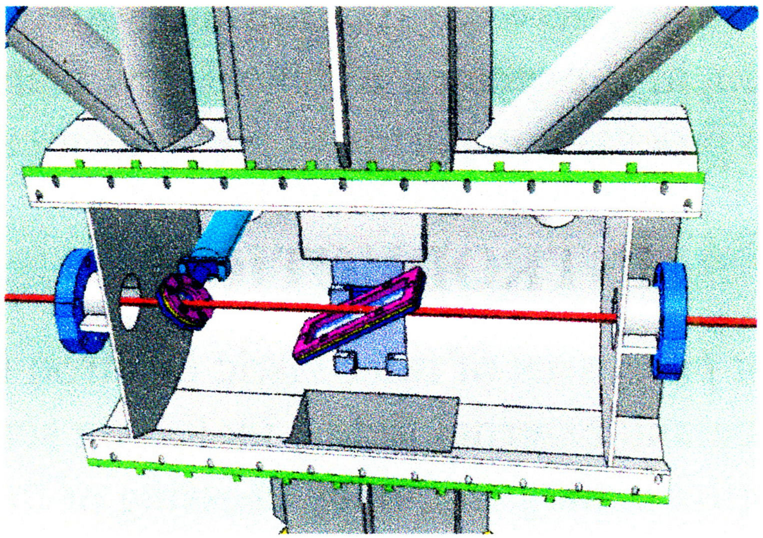
Proposal to observe the transition radiation by half-bare electrons on 45 MeV linac CLIO (2017)

S. Trofymenko, N. Shul'ga, N. Delerue et al

NSC KIPT, Karazin Kharkiv National Univ.,
LAL, Univ. Paris-Sud, CNRS/IN2P3, Univ. Paris-Saclay



Theory



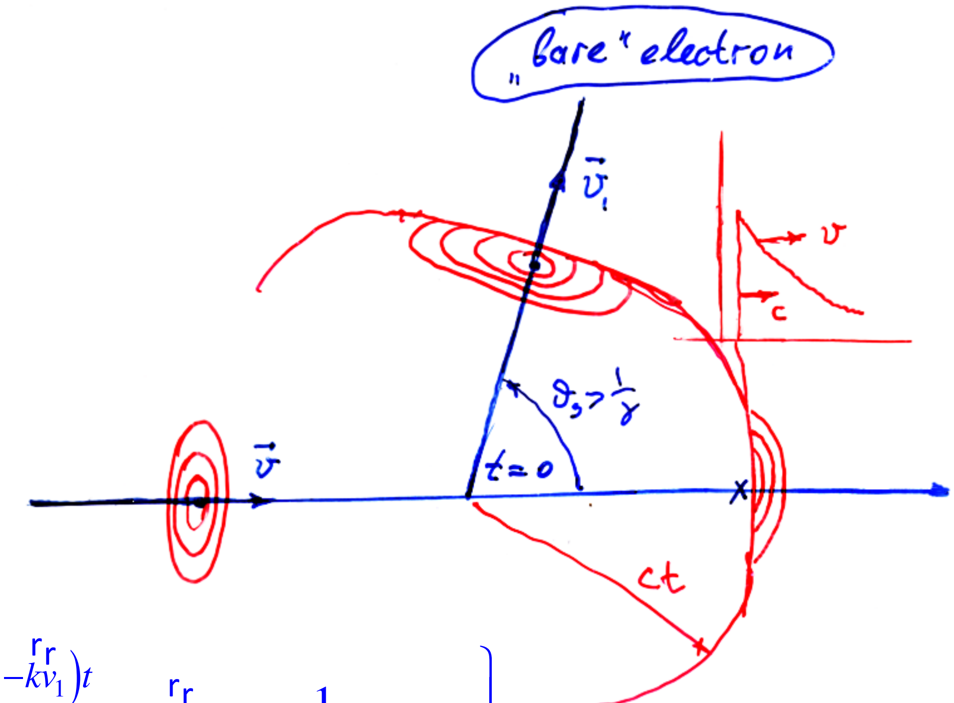
Experiment (LAL-2017)

The “half-bare” electron problem.

(electromagnetic field evolution at electron’s scattering)

$$\left(\Delta - \frac{\partial^2}{\partial t^2}\right)\varphi = 4\pi e\delta\left(\vec r - \vec r(t)\right)$$

$$\varphi_v(\vec r,t) = \frac{e}{\sqrt{(z-vt)^2+\rho/\gamma^2}}, \qquad t<0$$



$$\varphi_{ret}(\vec r,t)\Big|_{t>0} = \frac{e}{2\pi^2}\mathrm{Re}\int\frac{d^3k}{k}e^{i\vec k\vec r}\left\{\frac{1-e^{-i(k-\vec k\vec v_1)t}}{\omega-\vec k\vec v}e^{-i\vec k\vec v_1 t}+\frac{1}{k-\vec k\vec v}e^{-ikt}\right\}=$$

$$=\Theta(t-r)\varphi_{v_1}(\vec r,t)+\Theta(r-t)\varphi_v(\vec r,t)$$

$$\Delta t \ll (k-kv_1)^{-1} \approx 2\gamma^2/v = l_c$$

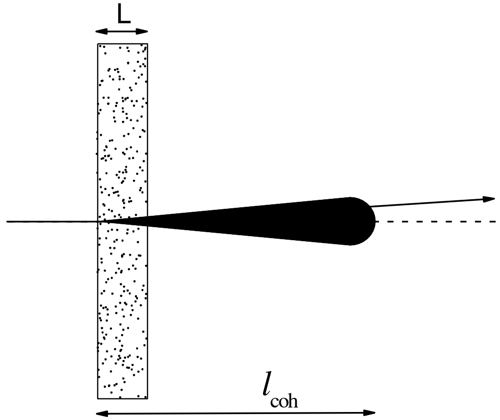
For $\varepsilon = 50\,MeV$, $\lambda = 1cm$, $l_c = 200\ m$

E.Feinberg JETP 50(1966)202,

A. Akhiezer, N.Shul’ga, S.Fomin Sov.Phys.Usp. 30(1987)197

Phys.Lett.A 114(1986)148

Radiation cross-section factorization



$$l_{coh} = \frac{2\varepsilon\varepsilon'}{m^2\omega} >> L$$

$$d\sigma_{rad}(q) = dw(q)\, d\sigma_{scatt}(q)\left\{1 + O\left(\frac{L}{l_{coh}}\right)\right\}$$

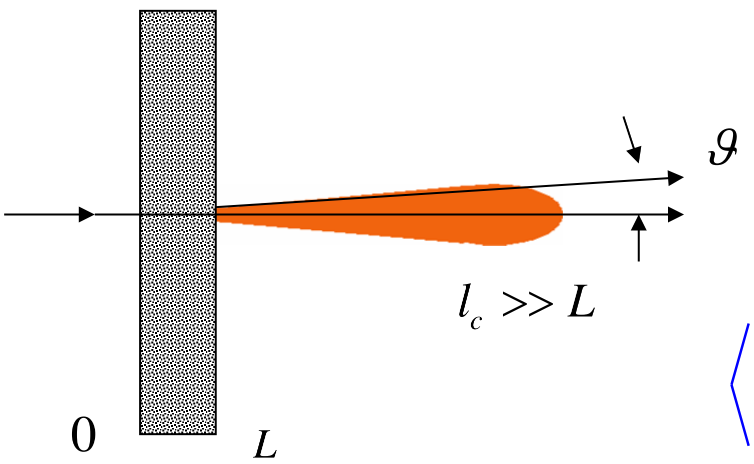
$$dw(q) = \frac{2e^2}{\pi}\frac{d\omega}{\omega}\frac{\varepsilon'}{\varepsilon}\left\{\frac{2\xi^2\left(1+\frac{\omega^2}{2\varepsilon\varepsilon'}\right)+1}{\xi\sqrt{\xi^2+1}}\ln\left(\xi+\sqrt{\xi^2+1}\right)-1\right\}, \qquad \xi = \frac{q}{2m}$$

$$d\sigma_{scatt} = |a(q)|^2\, do \qquad\qquad q \approx p\vartheta$$

$$a(q) = -\frac{1}{4\pi\hbar^2}\int d^2\rho\, dz\, e^{-i\vec{p}'\vec{r}/\hbar}\,\bar{u}'\gamma_0\psi(\vec{r})U(\vec{r})$$

Radiation in thin target (TSF-effect)

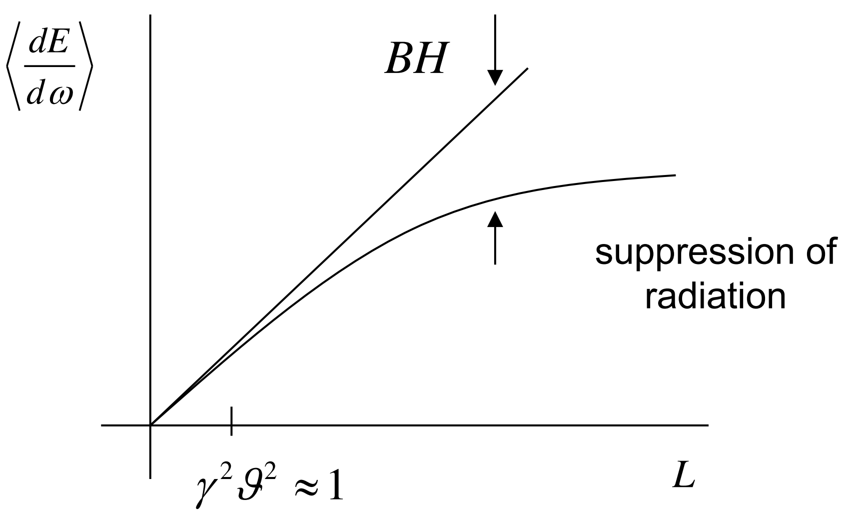
F. **T**ernovskii, JETF 1960, N. **S**hul'ga, S. **F**omin JETP Lett. 1978, 1996



$$l_c = \frac{2\gamma^2}{\omega} >> L$$

$$\left\langle \frac{dE}{d\omega} \right\rangle = \frac{2e^2}{\pi} \left\langle \left[\frac{2\xi^2+1}{\xi\sqrt{\xi^2+1}} \ln\left(\xi + \sqrt{\xi^2+1}\right) - 1 \right] \right\rangle \approx$$

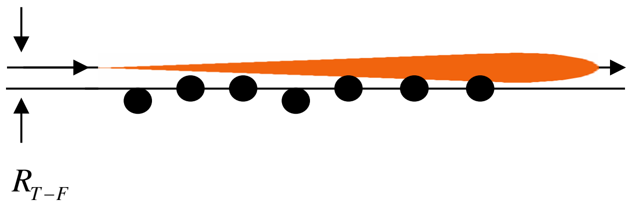
$$\approx \frac{2e^2}{3\pi} \begin{cases} \gamma^2 \overline{\vartheta^2} \\ 3\ln \gamma^2 \overline{\vartheta^2} \end{cases} \approx \begin{cases} E'_{BH}, & \gamma^2\overline{\vartheta}^2 << 1 \\ < E'_{BH} & \gamma^2\overline{\vartheta}^2 >> 1 \end{cases}$$



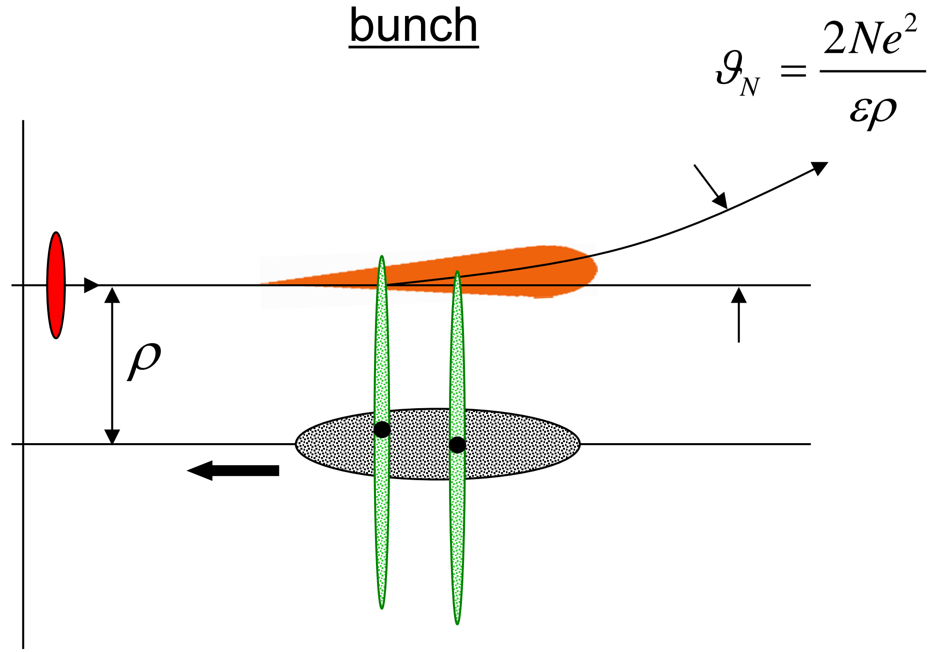
$$\overline{\vartheta}^2 = \frac{\varepsilon_s^2}{\varepsilon^2}\frac{L}{L_{rad}}, \qquad \xi = \frac{\gamma\vartheta}{2}$$

Coherent radiation in crystal and at electron collision with a short bunch

crystal atomic string



bunch



N. Shul'ga, D. Tyutyunnik

JEPT Lett. 78(2003)700., Proc. of SPIE, v. 5974(2005)60, NiM B227 (2005) 152

New field of research

The interaction of high-energy particles with matter in conditions of effectively strong interaction of the particle with atoms of media (semiclassical, classical approximations)

$$N_c \frac{Ze^2}{\hbar c} >> 1$$

- Classical electrodynamics
- Semiclassical approximation of quantum electrodynamics
- Relation between classical and quantum effects
- Methods for description
- …

Classical S-matrix in molecular collisions

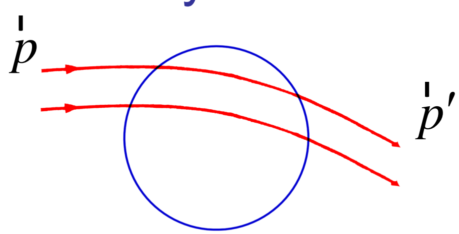
W.H. Miller (Adv. in Chemical Physics v.30 (1975) 77-136)

$$A+(BC) \to A+(BC)^{*}, \qquad A+BC \to AB+C, \quad .....$$

- Complex character of interaction (potential)
- numerical methods of solution of the motion equations
- classical trajectories method
- semiclassical approximation

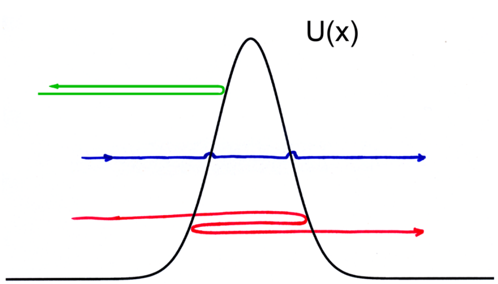
$$p_{1,2} = \left|S_{1,2}\right|^2 \qquad S_{1,2}^{cl} = \sqrt{p\left(\vec{r}(t)\right)}\; e^{i\Phi\left(\vec{r}(t)\right)}$$

- boundary conditions problem (rainbow scattering, …)



$$p_{1,2}^{cl} = p_1 + p_2 \qquad S_{1,2}^{semiclas.} = \sqrt{p_I}\; e^{i\Phi_1} + \sqrt{p_{II}}\; e^{i\Phi_2}$$

- tunnel effects (analytical continuation of classical mechanics),



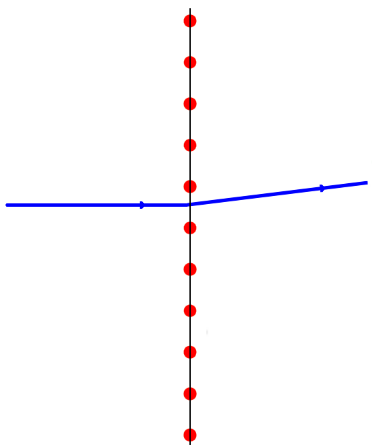
imaginary time method

$$m\ddot{x} = -\frac{\partial}{\partial x} U(x)$$

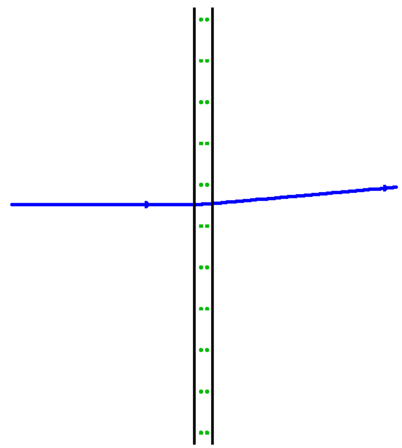
- relation between quantum and classical effects
-

Graphene, ultrathin and thin crystals

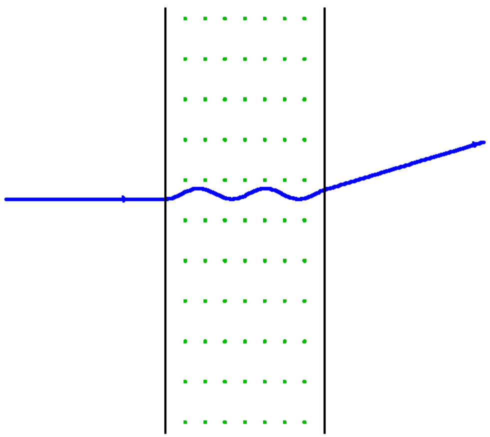
N.F. Shul'ga, S.N. Shul'ga, Phys. Lett. B 769 (2017) 141-145.
S.N. Shul'ga, N.F. Shul'ga S. Barsuk, I. Chaikovska, R. Chehab, NIM B 402 (2017) 16-20.



Graphene



Ultrathin crystal
coherent effects



Channeling

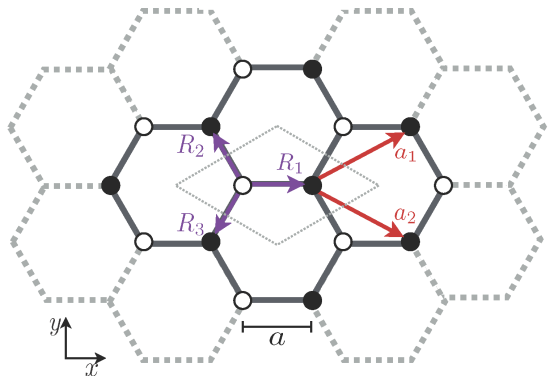
Experiments:

J.S. Rosner, Golovchenko et al. Phys. Rev. B18 (1978) 1066.
M. Mothapothula et al. NIM B283 (2012) 29
V. Guidi et al. Phys. Rev. Lett. (2012)
Y. Hochberg, Y. Kahn et al. hep-ph:1606.08849 (2016)

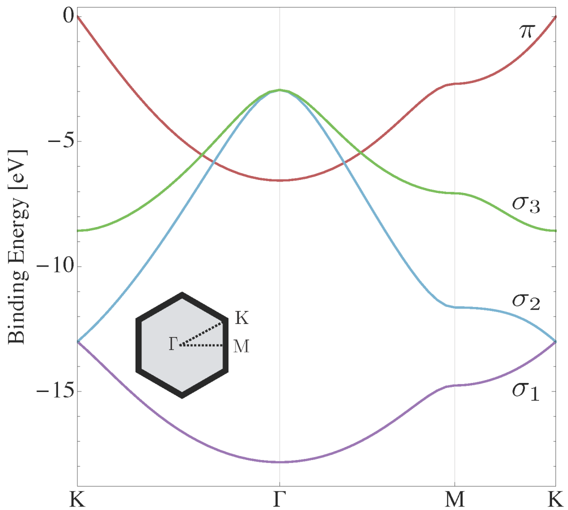
Directional Detection of Dark Matter with 2D Targets

Y. Hochberg, Y. Kahn et al. arXiv:1606.08849 [hep-ph] (2016)

University of California, Berkeley, CA 94720
LEPP, Cornell University, Ithaca, NY 14853
Princeton University, Princeton, NJ 08544



Graphene



"We propose two-dimensional materials as targets for direct detection of dark matter. Using graphene as an example, we focus on the case where dark matter scattering deposits sufficient energy on a valence-band electron to eject it from the target. We show that the sensitivity of graphene to dark matter of MeV to GeV mass can be comparable, for similar exposure and background levels, to that of semiconductor targets such as silicon and germanium…"

Experiment: 2MeV protons scattering in Si (L=55nm)

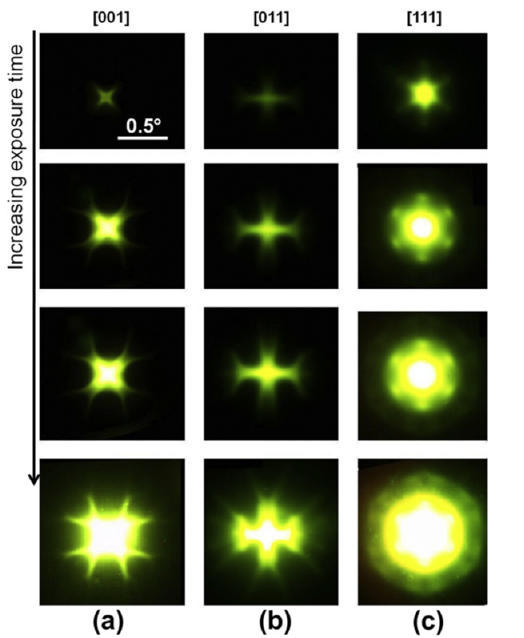
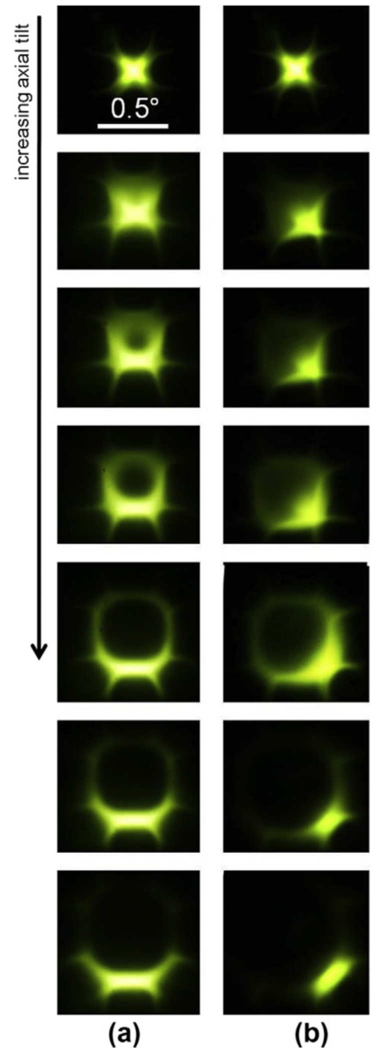


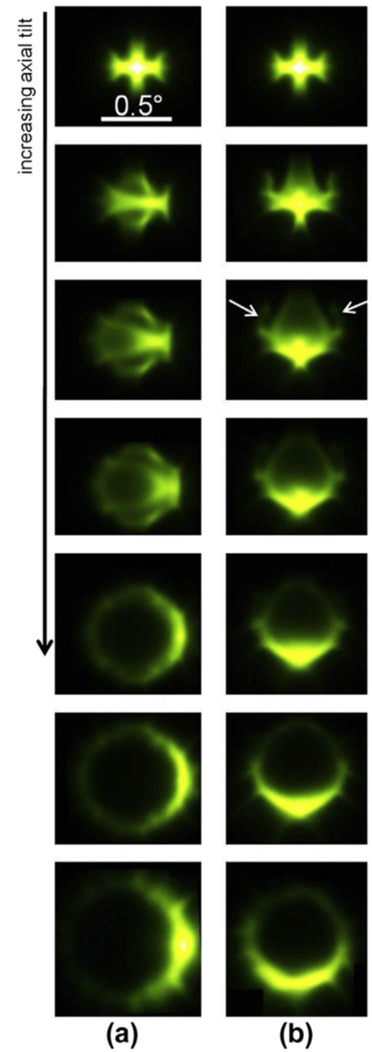
Fig. 2. Experimental channeling patterns for 2 MeV protons from a 55 nm [001] Si membrane at alignment with the (a) [001], (b) [011] and (c) [111] axes. Downwards direction shows the effect of increasing camera exposure.

$$\psi = 0$$

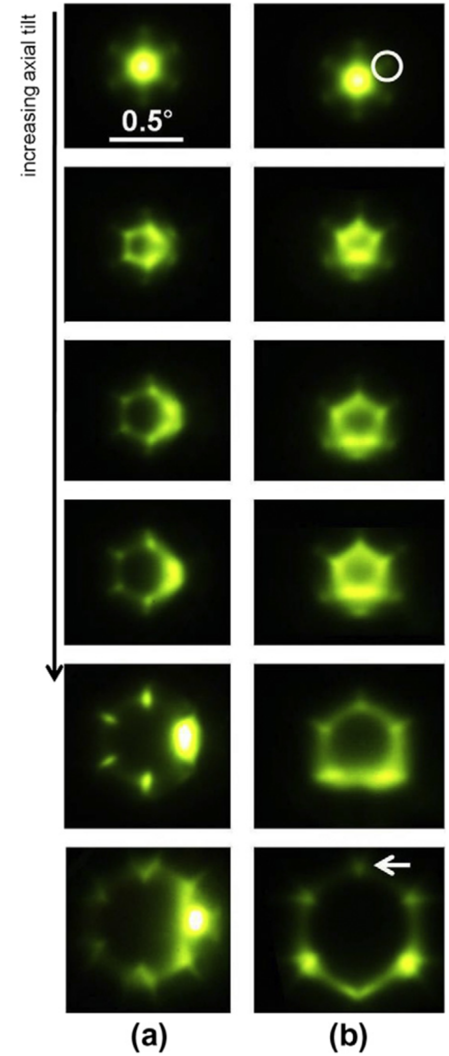
different expositions



$\langle 100 \rangle$



$\langle 110 \rangle$



$\langle 111 \rangle$

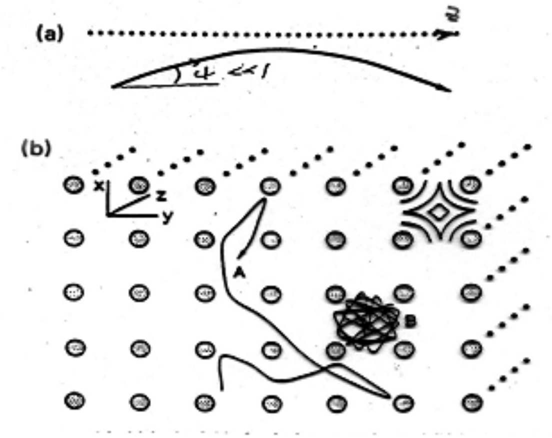
$$\psi_c > \psi > 0$$

M. Mothapothula et al. NIM B283 (2012) 29

Classical scattering in crystal (continuous strings potential)

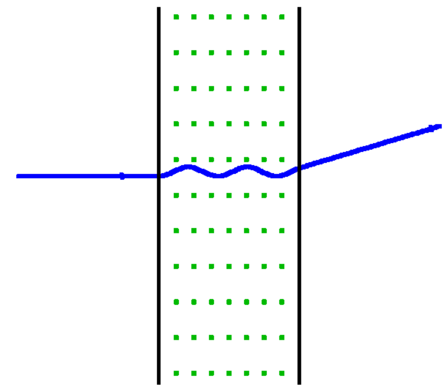
$$\frac{d\vec{p}}{dt} = -\nabla U(\vec{r})$$

$$U(\vec{r}) \to U(x,y) = \frac{1}{L}\int_0^L dz \sum_n u(\vec{r}-\vec{r}_n)$$



$$p_z = const >> p_\perp$$

$$\dot{\vec{\rho}} = -\frac{1}{\varepsilon}\nabla U(x,y)$$



Axial case

dynamical chaos

Planar channeling

$$d\sigma_{cl}(\vec{\vartheta}) = \sum_n d^2 b_n(\vec{\vartheta}) = \sum_n \frac{1}{\left|\partial\vec{\vartheta}/d\vec{b}\right|_n}\Bigg|_{\vec{b}=\vec{b}_n(\vec{\vartheta})}$$

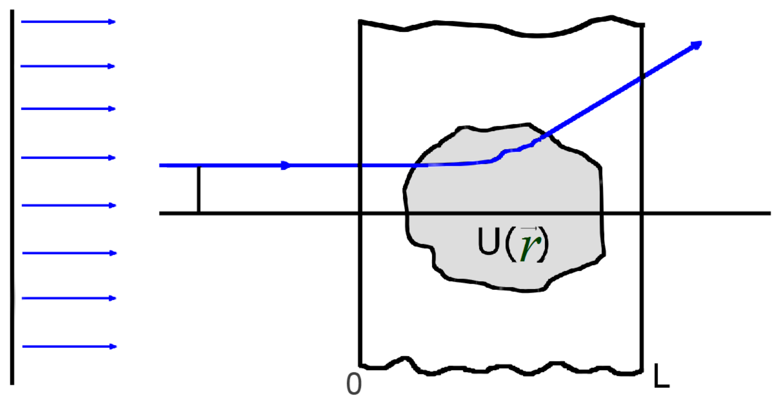
$$d^2\vec{\vartheta} = \int d^2 b\, \delta\left(\vec{\vartheta} - \vec{\vartheta}(\vec{b})\right)$$

$$dW(\vec{\vartheta}) = \frac{dN(\vec{\vartheta})}{N}$$

$$dN(\vec{\vartheta}) = \frac{N}{S} d\sigma_{cl}(\vec{\vartheta})$$

Gauss Theorem in Quantum Scattering Theory

N. Bondarenco, N. Shul'ga Phys. Lett. B 427 (1998) 114



$$\psi = \varphi(\vec r)\, e^{i\vec p\vec r} u_p$$

$$a(\vartheta) = -\frac{1}{4\pi}\int_V d^3r\, e^{-i\vec p'\vec r}\,\bar u'\gamma_0 U(\vec r)\,\psi(\vec r) = -\frac{1}{4\pi}\int_V d^3r\ \mathrm{div}\left[\bar u'\vec\gamma\,\psi(\vec r)\,e^{-i\vec p'\vec r}\right] =$$

$$= -\frac{i}{4\pi}\oint d\vec S\,\bar u'\vec\gamma\,\psi(\vec r)\,e^{-i\vec p'\vec r} =$$

$$= -\frac{ip}{2\pi}\oint d^2\rho\, e^{i\vec q\vec r}\left(\varphi(\vec r)-1\right)\Big|_{z=-L/2}^{z=L/2}$$

$$\frac{d\sigma_q}{do} = \left|a(\vartheta)\right|^2$$

$$\vec q = \vec p - \vec p'$$

Semiclassical approximation for wave function

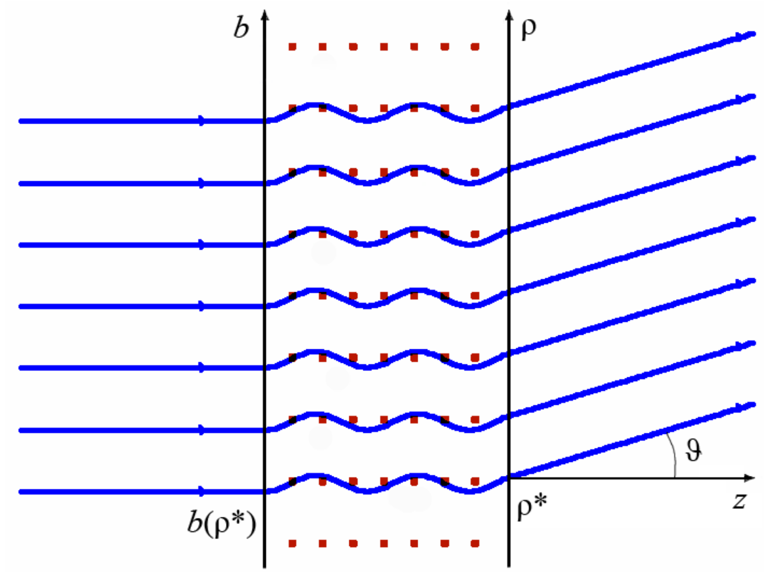
$$\left[\left(\varepsilon-U\right)^2-\left(i\hbar\nabla\right)^2-m^2+i\hbar\gamma_0\boldsymbol{\gamma}\boldsymbol{\nabla}U\right]\psi=0$$

$$\psi^{semicl.}\left(\vec{r}\right)=f\left(\vec{\rho},z\right)e^{\frac{i}{\hbar}\left(pz+\chi\left(\rho,z\right)\right)}$$

$$-v\,\partial_z\chi=U_c\left(\vec{\rho}\right)+\frac{1}{2\varepsilon}\left(\nabla_\perp\chi\left(\vec{\rho},z\right)\right)^2$$

$$\begin{cases}\chi\left(\vec{\rho}\left(z\right),z\right)=-\dfrac{1}{v}\displaystyle\int_0^z dz'\left[2U\left(\vec{\rho}\left(z'\right)\right)-\varepsilon_\perp\right], \qquad \varepsilon_\perp=U\left(\vec{b}\right)\\ f\left(\vec{\rho},z\right)=\sqrt{\displaystyle\int d^2b\,\delta\left(\vec{\rho}-\vec{\rho}\left(\vec{b},z\right)\right)}\\ \dfrac{d^2\vec{\rho}\left(z\right)}{dz^2}=-\dfrac{1}{\varepsilon}\dfrac{\partial}{\partial\vec{\rho}}U\left(\vec{\rho}\left(z\right)\right)\end{cases}$$

Semiclassical scattering in thin crystal



$$a(q_\perp) = -\frac{ip}{2\pi\hbar}\int d^2\rho\, e^{\frac{i}{\hbar}\left[\vec{q}\vec{\rho}+\chi(\rho,L)\right]} f(\rho,L) =$$

$$\boxed{\vec{q} = -\nabla\chi\left(\rho^*\right) \quad \to \quad \vec{\rho} = \vec{\rho}_n^{\,*}\left(\vec{q}\right)}$$

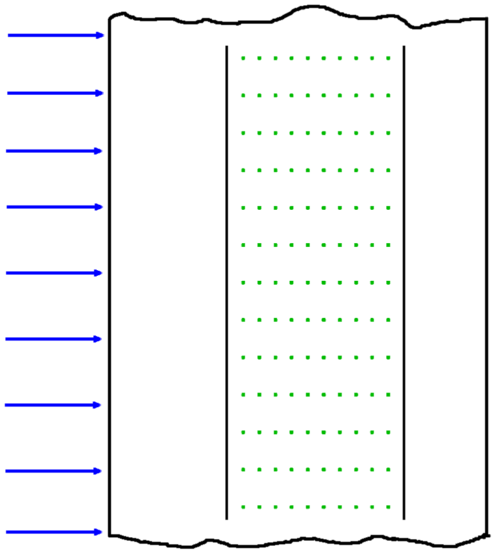
$$= \quad -\frac{p}{\left|\partial\vec{q}\left(\vec{\rho}^{\,*}\right)/\partial\vec{\rho}^{\,*}\right|^{1/2}} f\left(\rho^*,L\right) e^{\frac{i}{\hbar}\Phi\left(\rho^*,L\right)} =$$

$$= \quad -p\sum_n \frac{1}{\left|\partial\vec{q}^{\,*}/\partial\vec{b}\right|_n^{1/2}} e^{\frac{i}{\hbar}\Phi\left(\vec{q},\vec{\rho}^{\,*}\left(\vec{b}_n\right)\right)}$$

$$a(q) = \sum_n \sqrt{p_n}\, e^{\frac{i}{\hbar}\Phi_n}$$

A. Akhiezer, N. Shul'ga, Phys. Rep. 234 (1993) 297

Operator method



$\psi_{in} = e^{i\vec{p}\vec{r}}$ $\quad$ $\psi = e^{i\vec{p}\vec{r}}\varphi(\vec{r})$

$$\psi = e^{i(pz-\varepsilon t)}\varphi(\vec{\rho},z)$$

$$i\hbar v\partial_z\varphi(\vec{\rho},z) = \left(\frac{\vec{p}_\perp^{\,2}}{2\varepsilon} + U(\vec{\rho})\right)\varphi(\vec{\rho},z) =$$

$$= \left(\hat{H}_0 + U(\vec{\rho})\right)\varphi$$

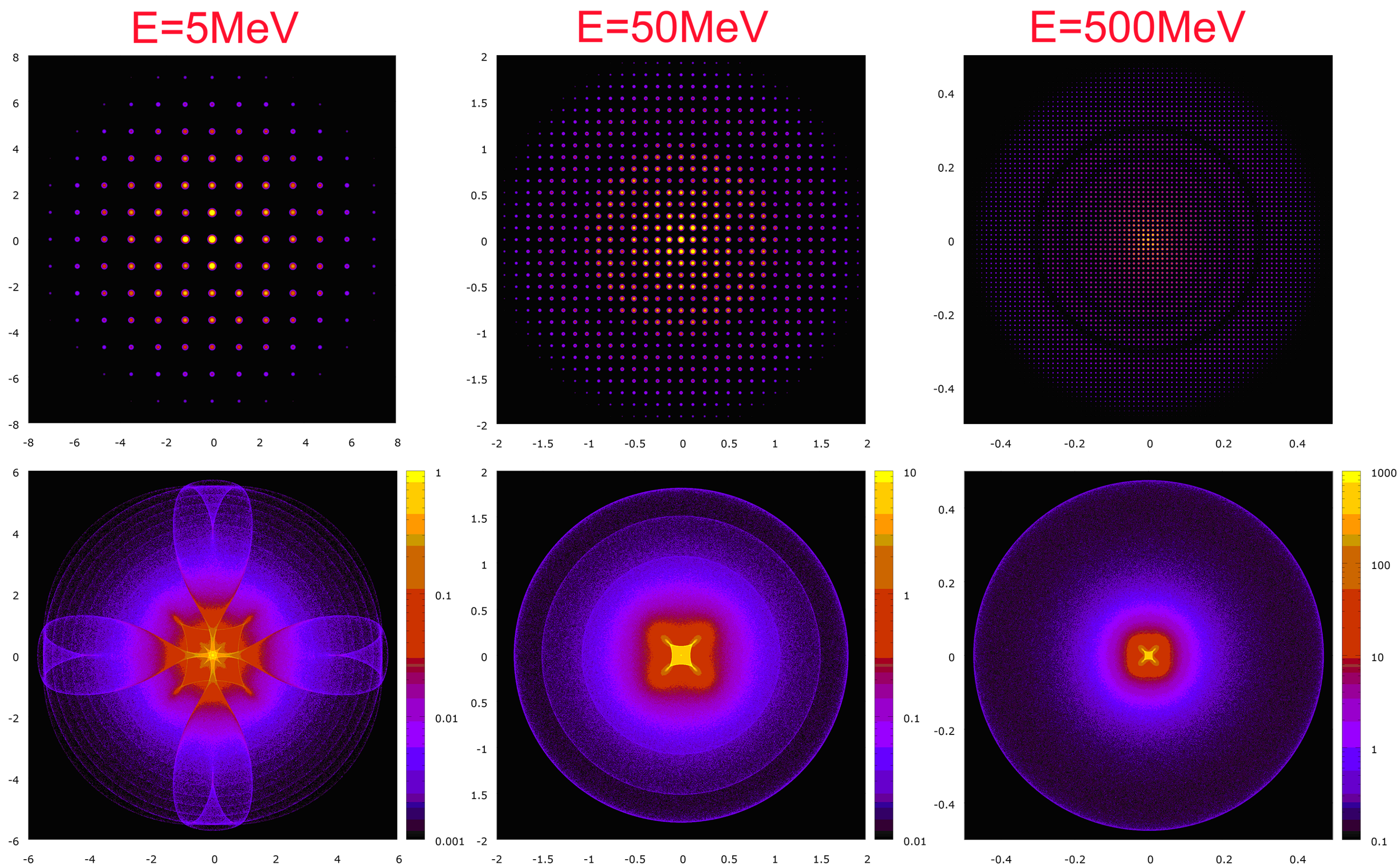
wave function $\quad$ $\varphi(\vec{\rho},z+\Delta z) = e^{-\frac{i}{\hbar}\left(\hat{H}_0+U(\vec{\rho})\right)\Delta z}\varphi(\vec{\rho},z)$

+ iteration procedure

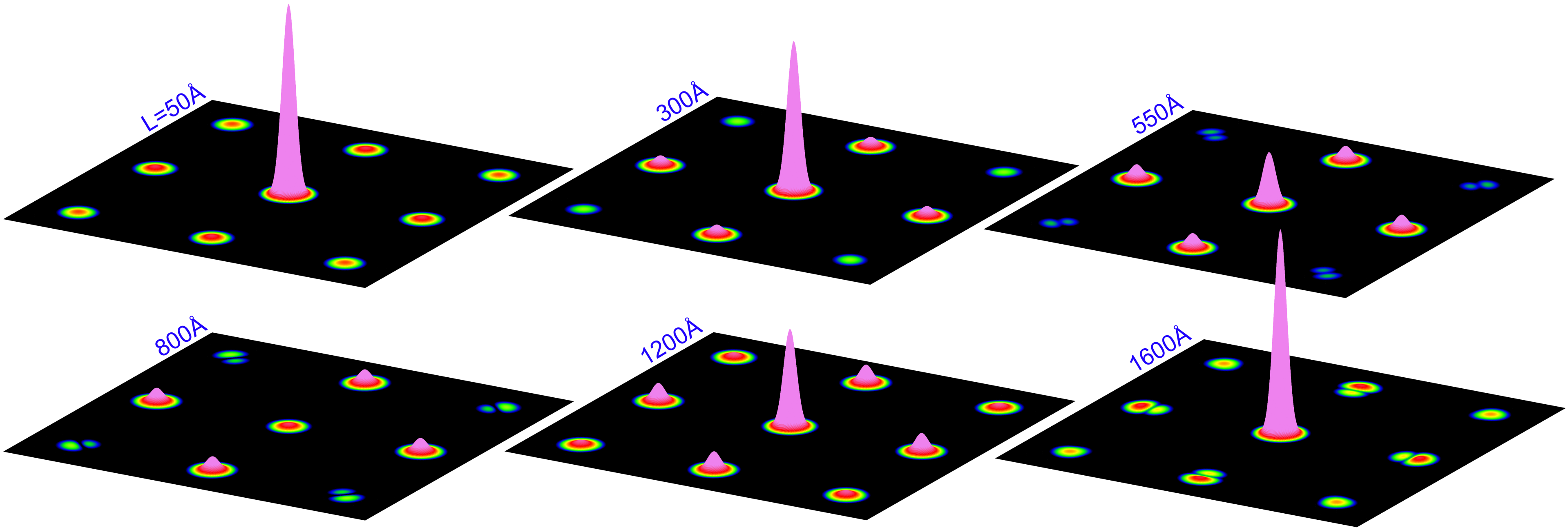
M. Feit, J. Fleck et al., J. Comput. Phys. 47 (1982) 412
S. Dabagov, L. Ognev, NIM B 30 (1988) 185
N. Shul'ga, S. Shul'ga Phys. Lett. B769 (2017) 141

Quantum and classical angular distributions of electrons in 1000Å Si <100>

N. Shul'ga, S. Shul'ga Phys. Lett. B769 (2017) 141



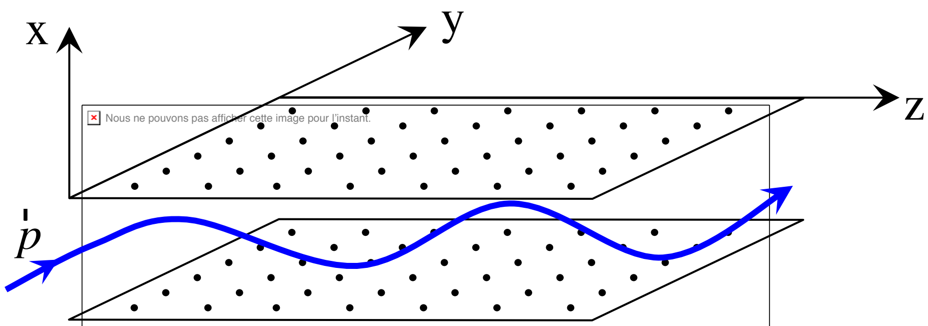
Quantum angular distributions of electrons in ultrathin Si <100> crystal



electrons 5MeV Si <100> 50-1600Å

Phenomenon of Planar Channeling

J.Lindhard (1965)



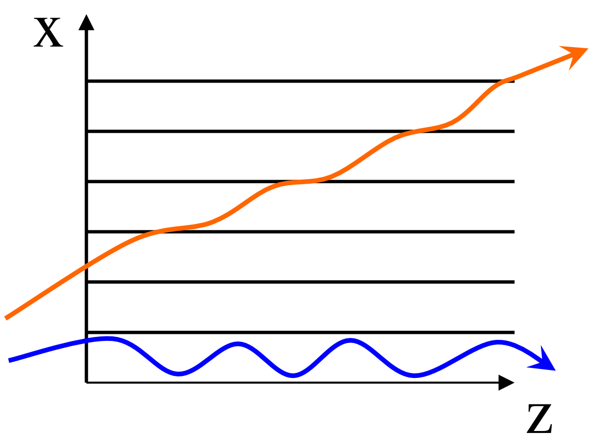
$$U_p(x) = \frac{1}{L_y L_z}\int dydz \sum_n u(\vec{r} - \vec{r}_n)$$

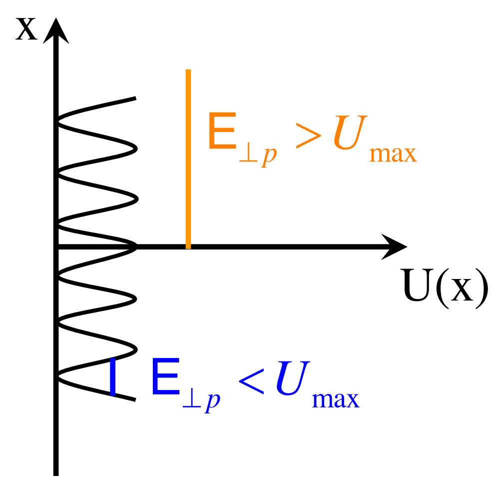
$$p_z = const \approx p$$

$$p_y = const \approx 0$$

$$\ddot{x} = -\frac{1}{E}\frac{\partial}{\partial x}U_p(x)$$

$$E_{\perp p} = \frac{E\dot{x}^2}{2} + U(x)$$



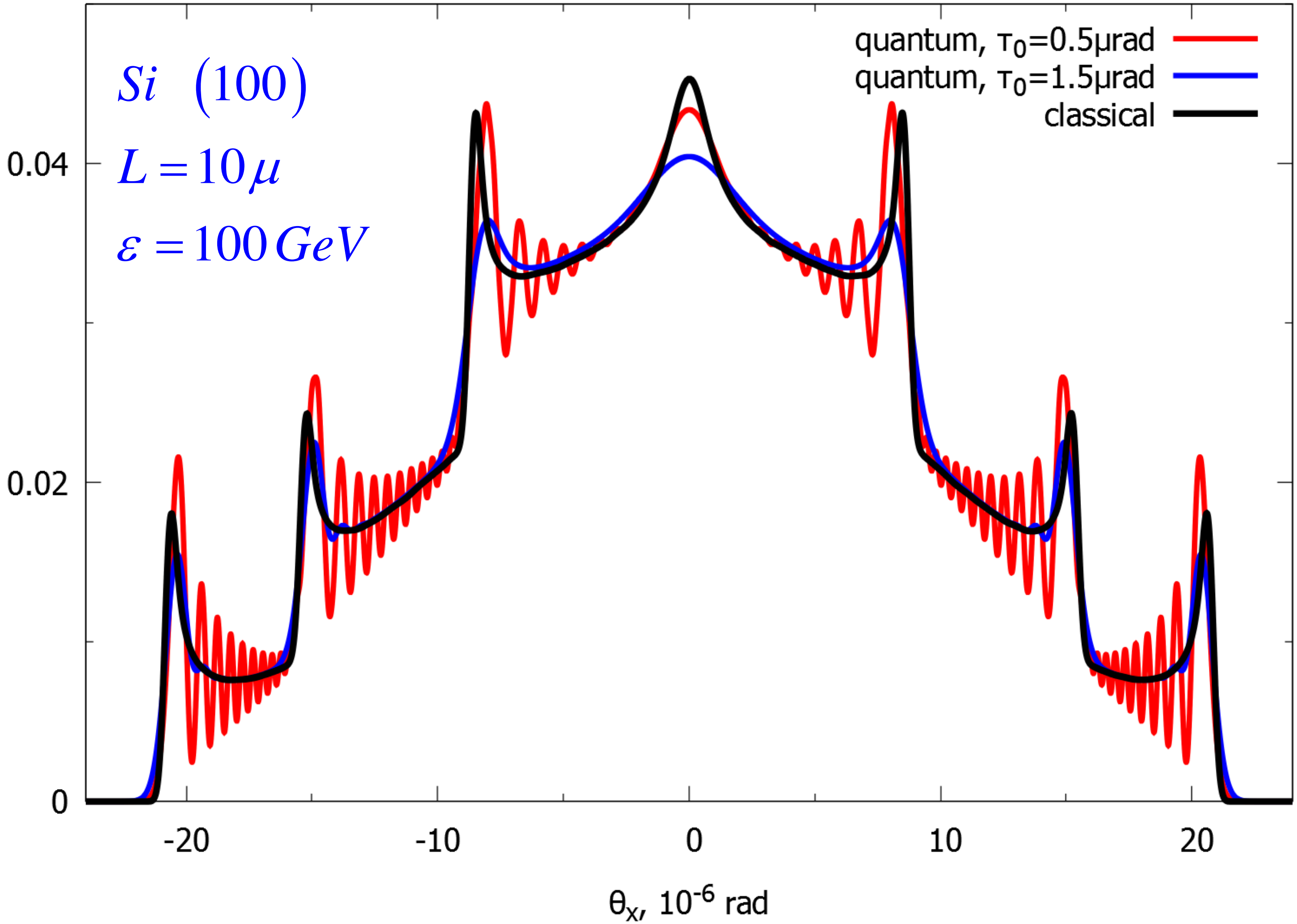


$$E_{\perp p} = U_{max} = \frac{E\psi_c^2}{2}$$

$$n_{levels} \sim \sqrt{\varepsilon_{MeV}}$$

Phenomenon of Above Barrier Motion: A.Akhiezer, N.Shul'ga (1978)

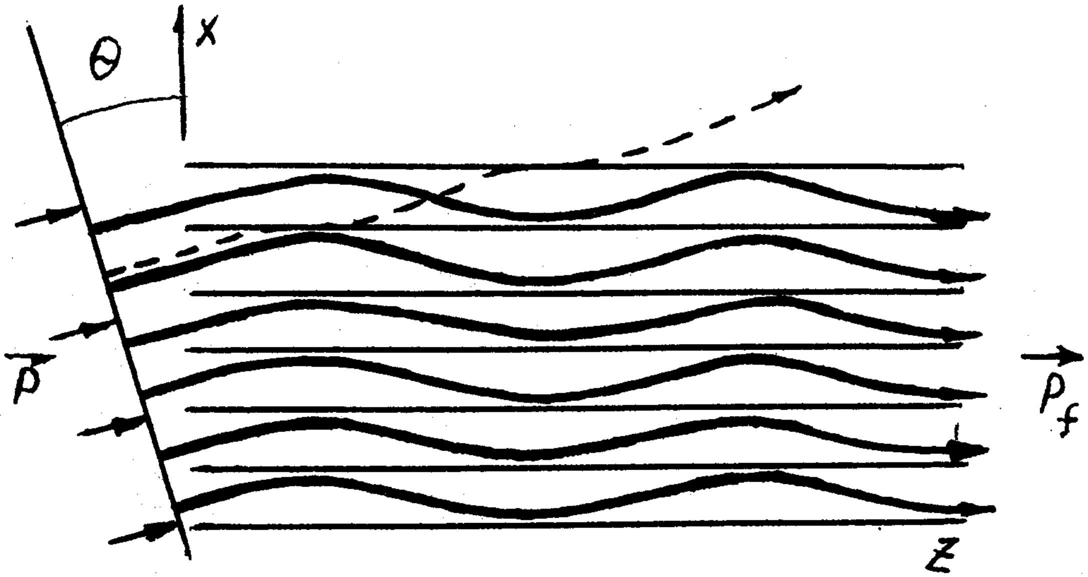
Rainbow scattering in the field of ultrathin Si (110) crystal planes



S. Shulga, N. Shul'ga et al. NIM B 402 (2017) 16

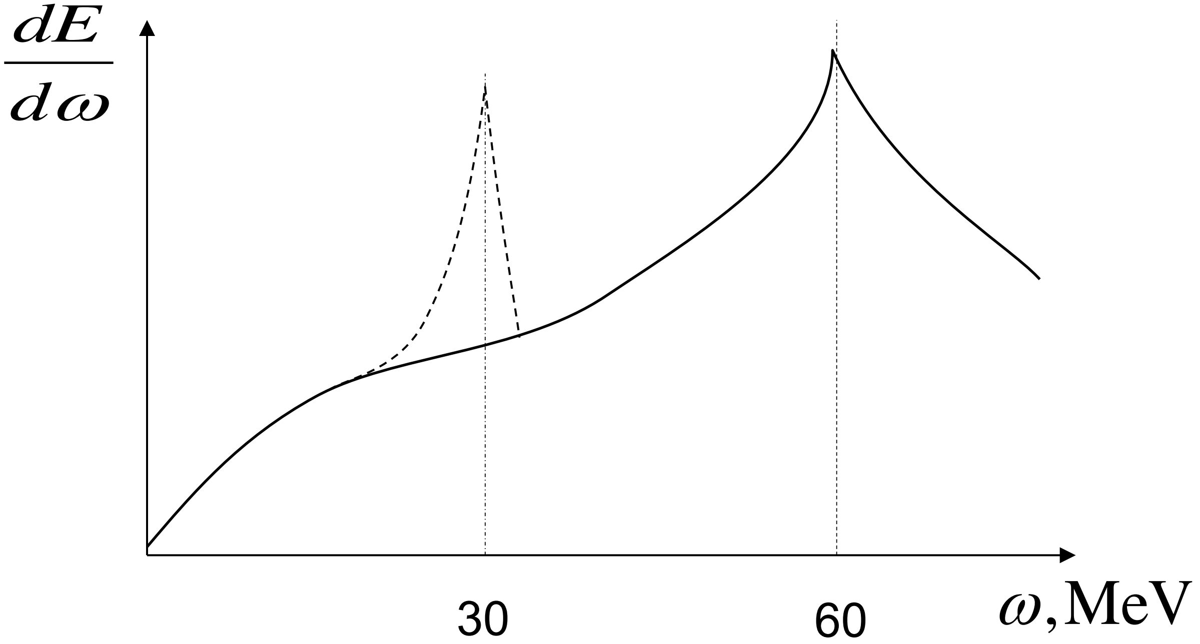
New Interference Effect in Radiation at Channeling

N. Shul'ga. Dokl. Acad. Nauk of USSR v.310 (1990) 348



A. Akhiezer, N. Shul'ga. Physics Reports v.234 (1993) 297

New Interference Effect in Radiation at Channeling



e^+

$\varepsilon = 1 GeV$

$Si, (110)$

$$\omega_{new} = \frac{\omega_d}{1 + \frac{3}{2}\alpha^2} \qquad \omega_{chan.} = \frac{\omega_d}{1 + \frac{1}{2}\alpha^2}$$

$$\omega_d = 2\gamma^2 \theta_c / a, \quad \alpha = \gamma \theta_c$$

THANK YOU FOR YOUR ATTENTION!