



Classification with Quantum Annealing on the D-Wave System

Seminar at the Laboratoire de
l'Accélérateur Lineaire - LAL
Jan 26, 2018

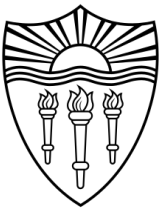
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California Institute of Technology



Outline



- Overview
 - Quantum Annealing
 - QA Machine Learning
 - A Higgs dataset
- Experiments
- Outlooks



Overview



MENU ▾

nature
International journal of science

Letter

Solving a Higgs optimization problem with quantum annealing for machine learning

Alex Mott, Joshua Job, Jean-Roch Vlimant, Daniel Lidar & Maria Spiropulu 

Nature **550**, 375–379 (19 October 2017)

doi:10.1038/nature24047

[Download Citation](#)

Computational science

Experimental particle physics Qubits

Theoretical particle physics

Received: 04 April 2017

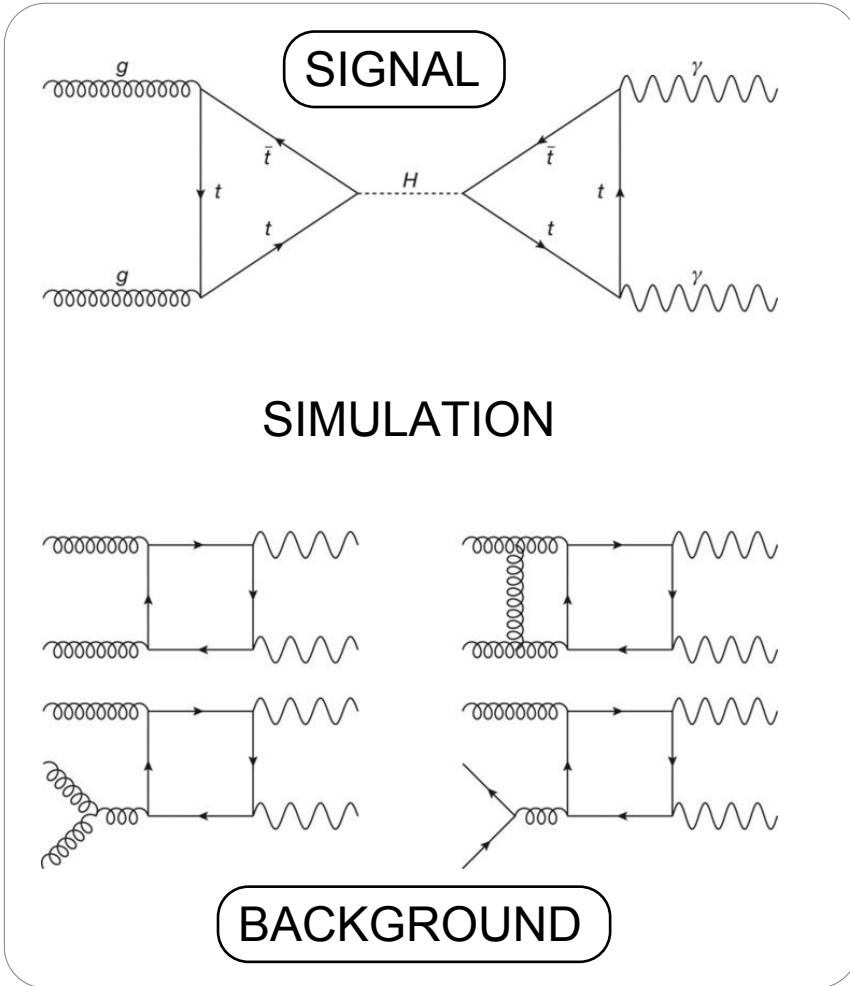
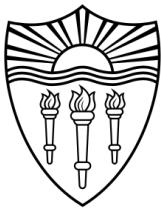
Accepted: 28 July 2017

Published online: 18 October 2017

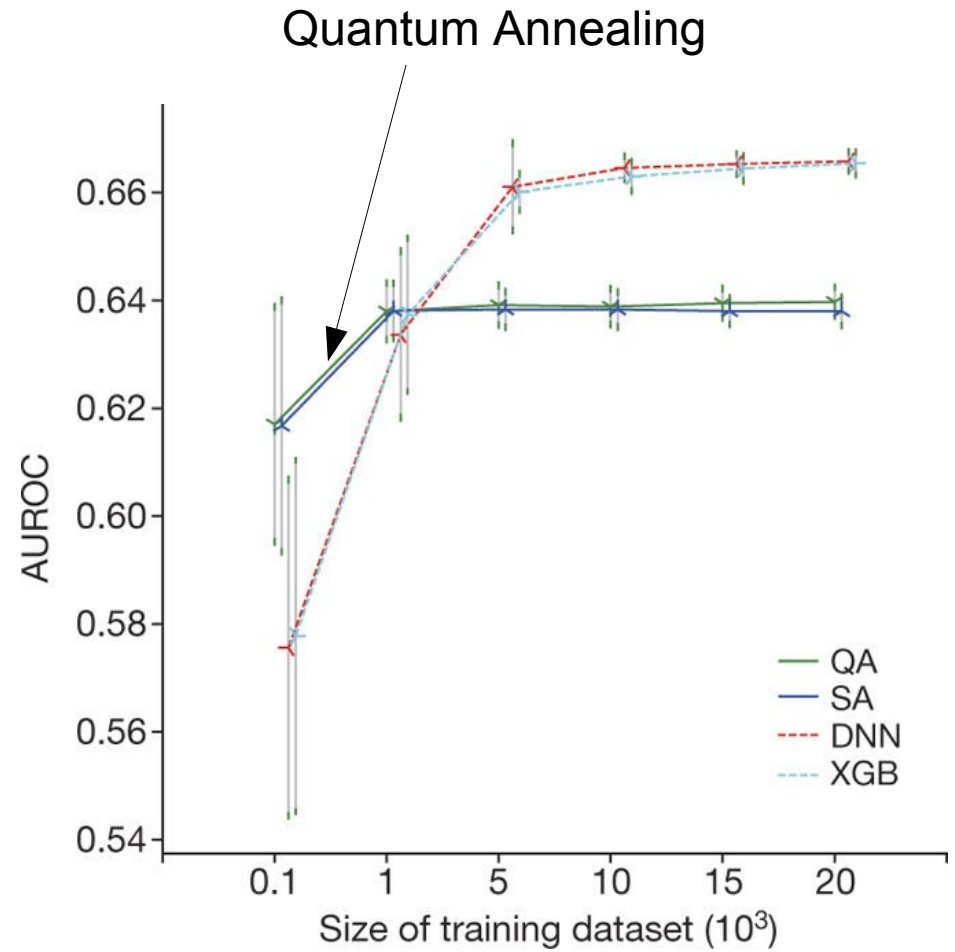
<https://www.nature.com/articles/nature24047>

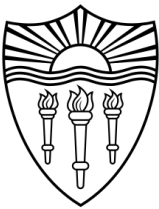


Experiment



Classification

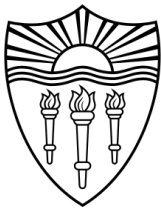




The D-Wave Computing System



The D-Wave Company



D:wave
The Quantum Computing Company™

COMPANY ▾

TECHNOLOGY ▾

COMPUTING ▾

RESOURCES ▾

NEWS ▾

Welcome to the Future

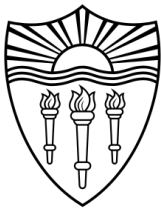
Quantum Computing for the Real World Today

<https://www.dwavesys.com/>

- 1999** Founded
- 2011** D-Wave One : 128 qubits
- 2013** D-Wave Two : 512 qubits
- 2015** D-Wave 2X : 1000 qubits
- 2017** D-Wave 2000Q : 2000 qubits
- 2019?** 5000 qubits ?

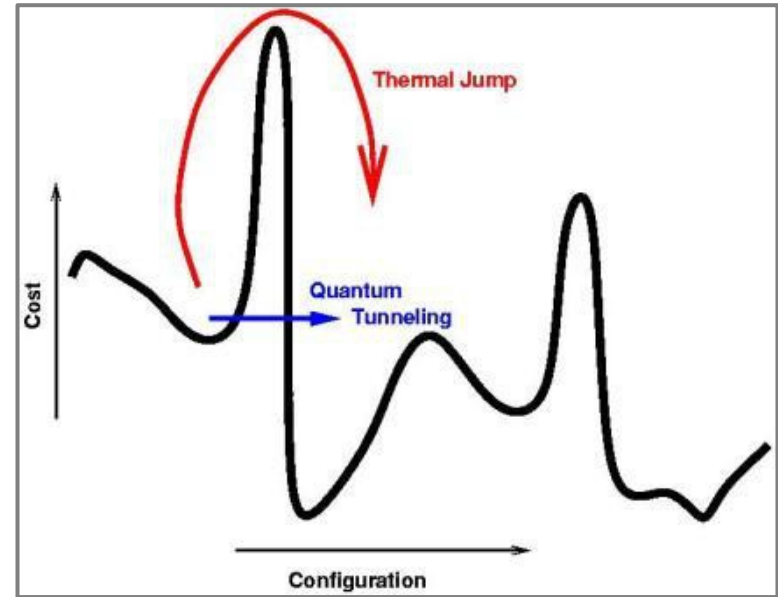
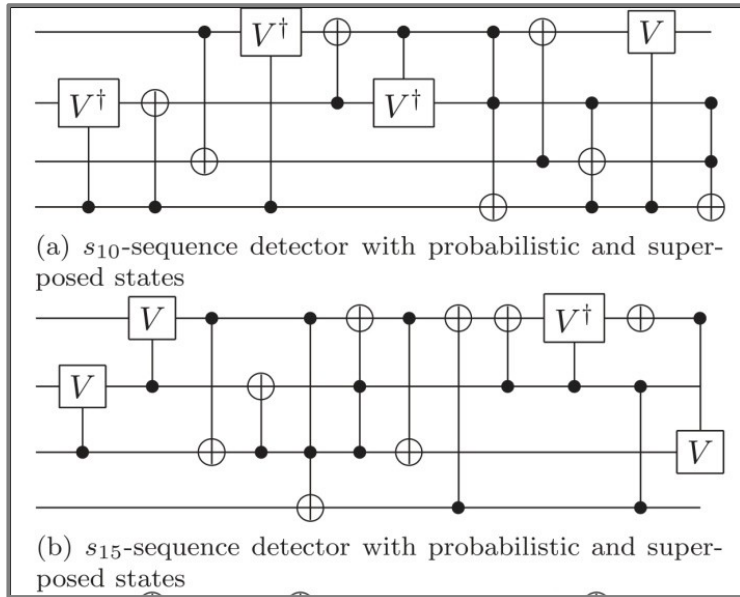
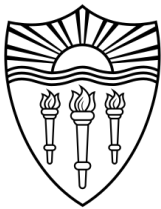


D-Wave 2X™





qubit and qubit

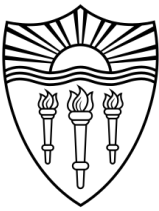


Quantum Circuits

Series of quantum gates operating on a set of quantum states.

Quantum Annealing

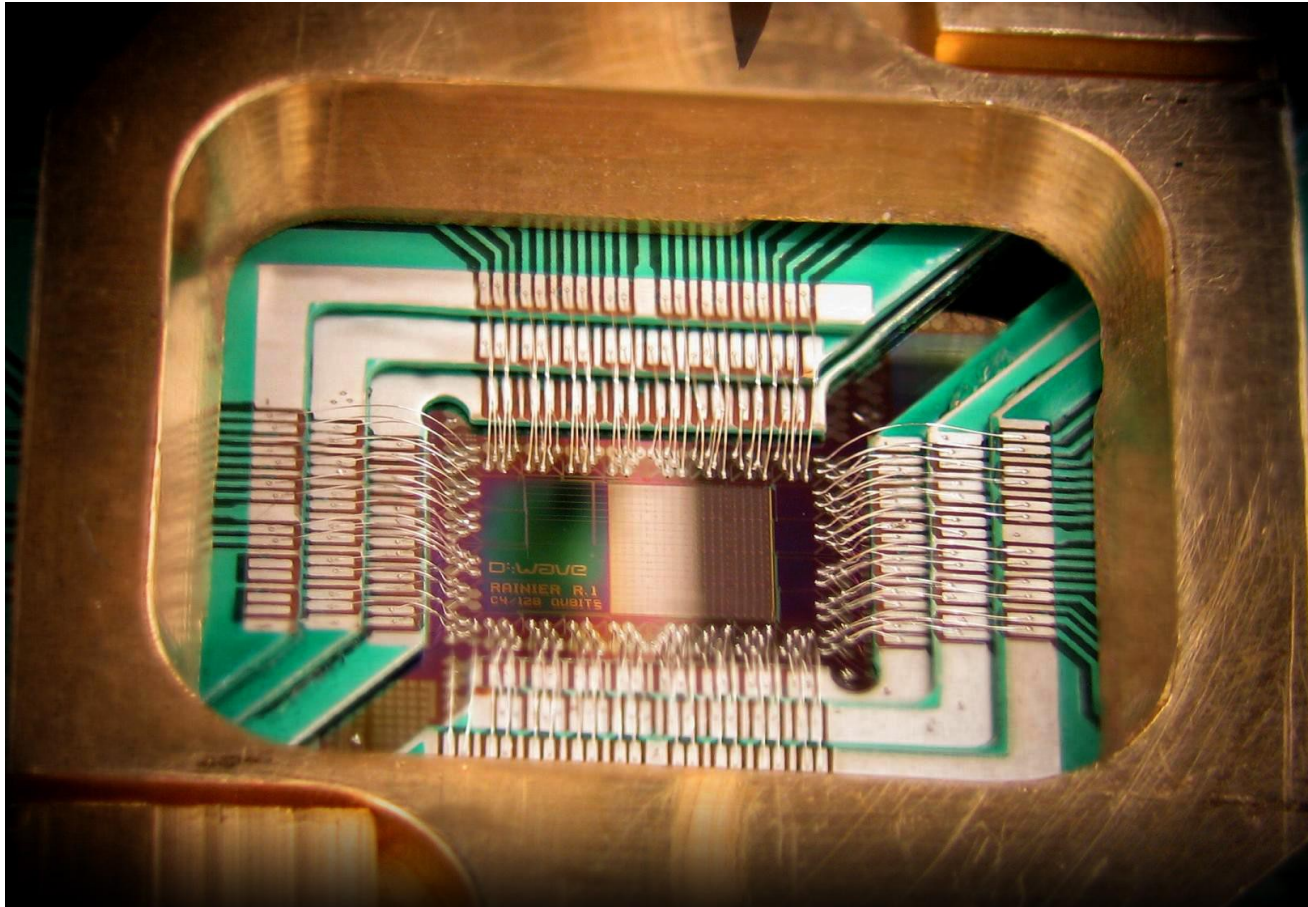
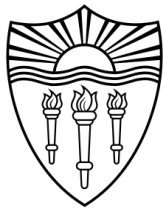
Evolution of a quantum system to a low T Gibbs state
That's D-Wave !



D-Wave Chip



D-Wave's quBit

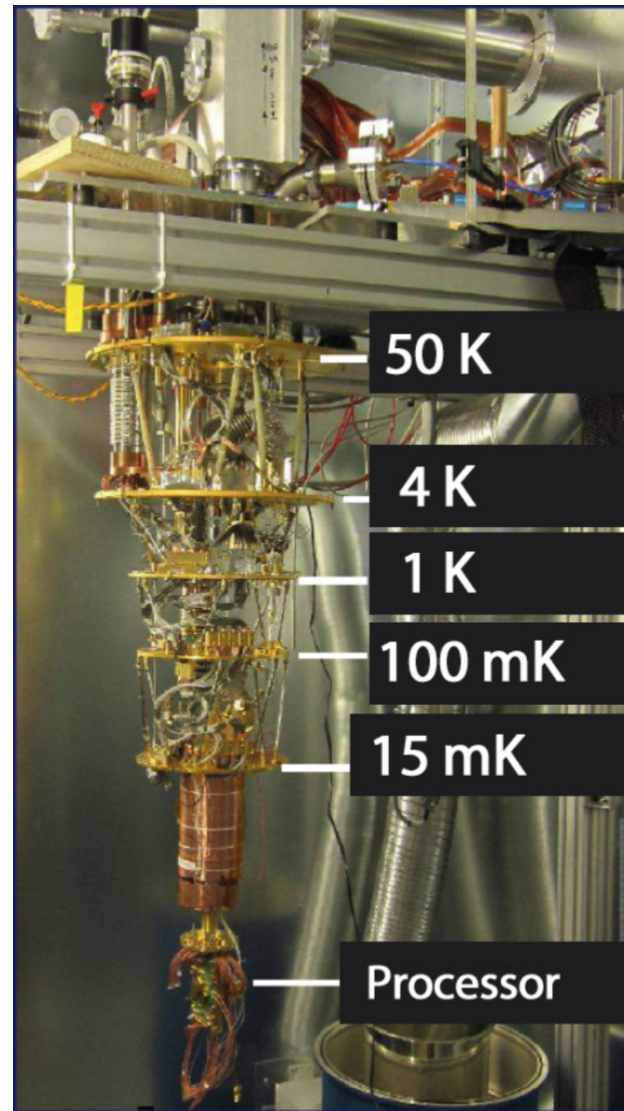
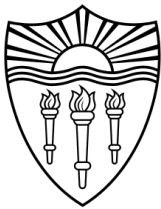


- Each qubit is a pair of Josephson junction (JJ)
- Able to apply local magnetic fields with programmable digital-to-analog flux converters (DAC)
- Operates at 15 mK to remove noise

<https://doi.org/10.1109/TASC.2014.2318294>



Thermal Noise Isolation





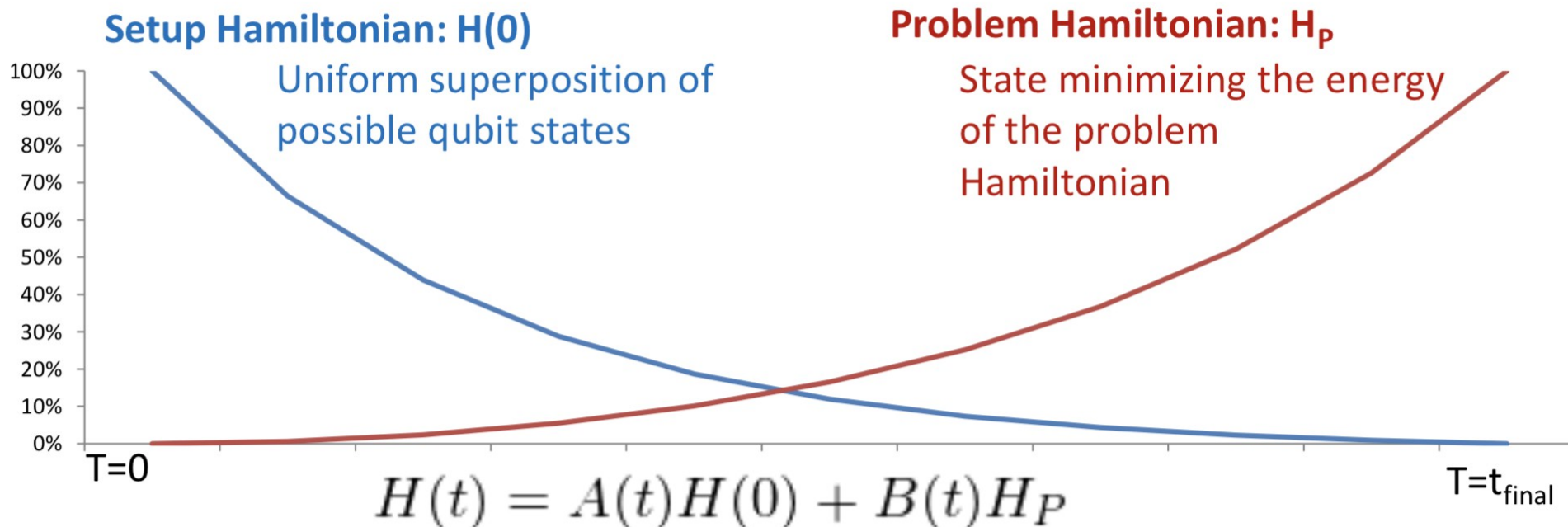
Quantum Annealing



Adiabatic Quantum Annealing

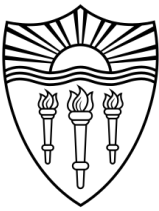


- System setup with trivial hamiltonian $H(0)$ and ground state
- Evolve adiabatically the hamiltonian towards the desired Hamiltonian H_p
- **Adiabatic theorem** : with a slow evolution of the system, the state stays in the ground state.



<https://arxiv.org/abs/quant-ph/0001106>

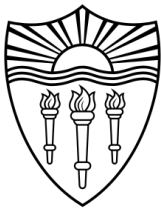
<https://arxiv.org/abs/quant-ph/0104129>



D-Wave Hamiltonian And Chimera Graph



D-Wave Hamiltonian



$$H_{\text{Ising}} = \sum_i h_i \sigma_i^z + \sum_{ij} J_{ij} \sigma_i^z \sigma_j^z$$

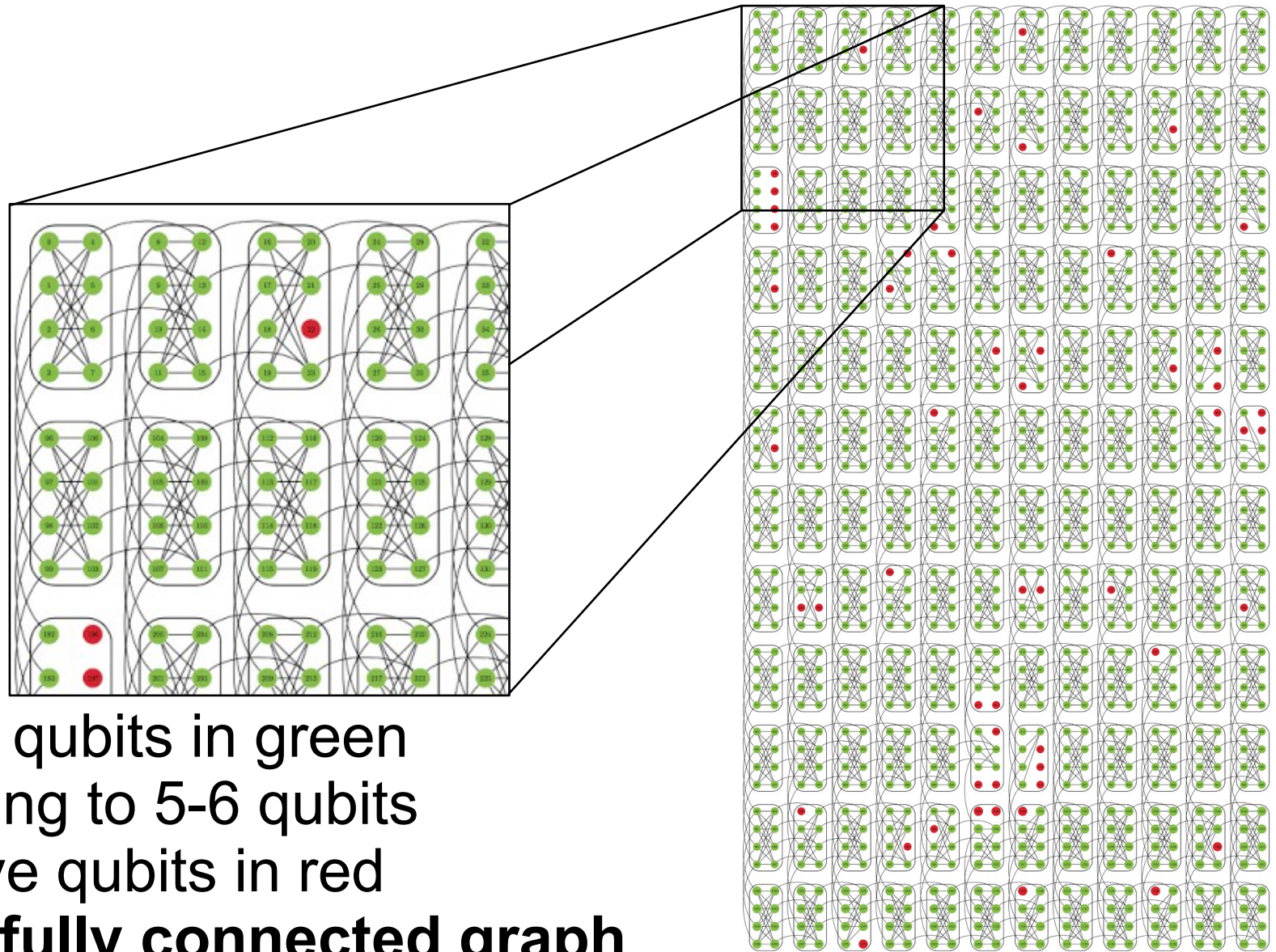
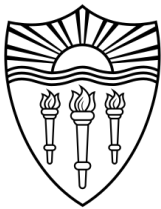
Runs over adjacent quBits

External magnetic field

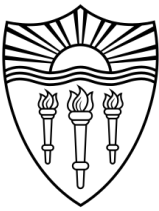
Interactions



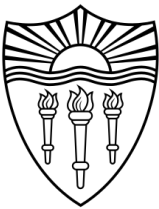
D-Wave qubit Adjacency



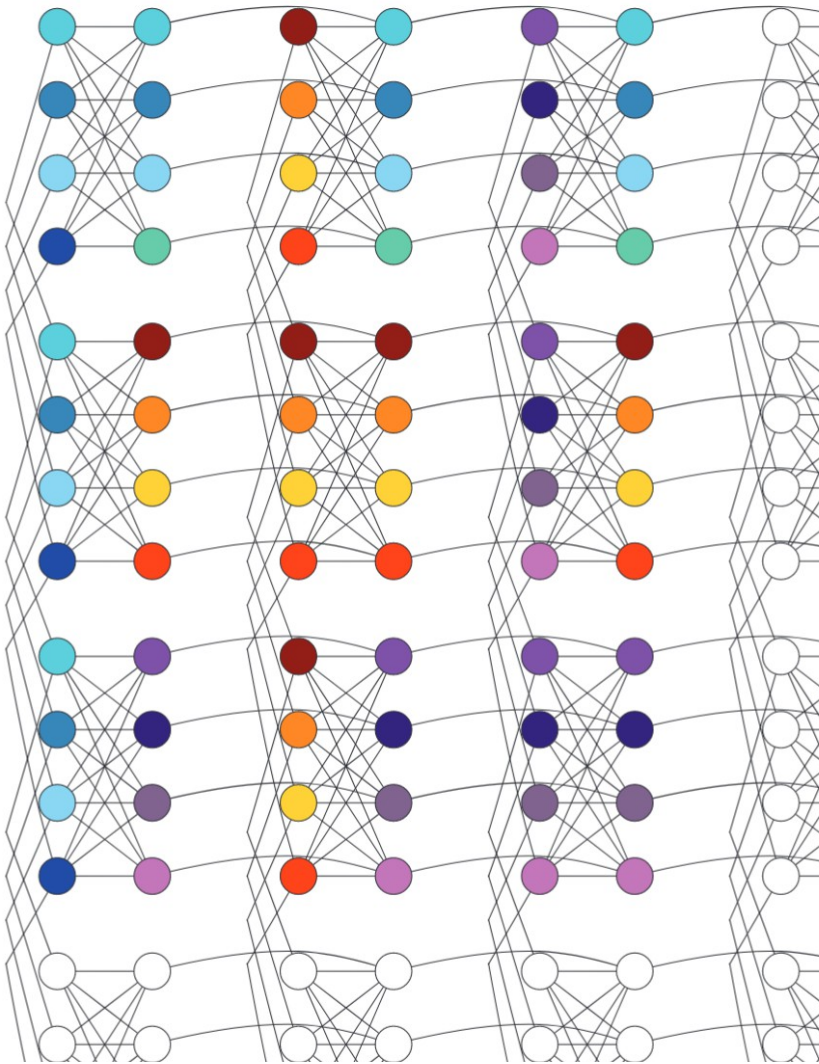
Active qubits in green
Coupling to 5-6 qubits
Inactive qubits in red
Not a fully connected graph



Model Embedding



Full Ising Model

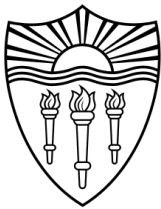


- Create chains of spins through the chimera graph
 - Split local fields across all qubits in the chain
 - Tightly couple ($J_F=6$)
 - Non-unique embedding. Heuristic approach.
 - Suppressing spin flip within chain as error correction.
 - Use majority vote
- Approximately full Ising Model with $\sim <40$ spins

<https://arxiv.org/abs/1210.8395>



Ising Hamiltonian



$$H_{\text{Ising}} = \sum_i h_i \sigma_i^z + \sum_{ij} J_{ij} \sigma_i^z \sigma_j^z$$

Runs over **all** quBit pairs

External magnetic field

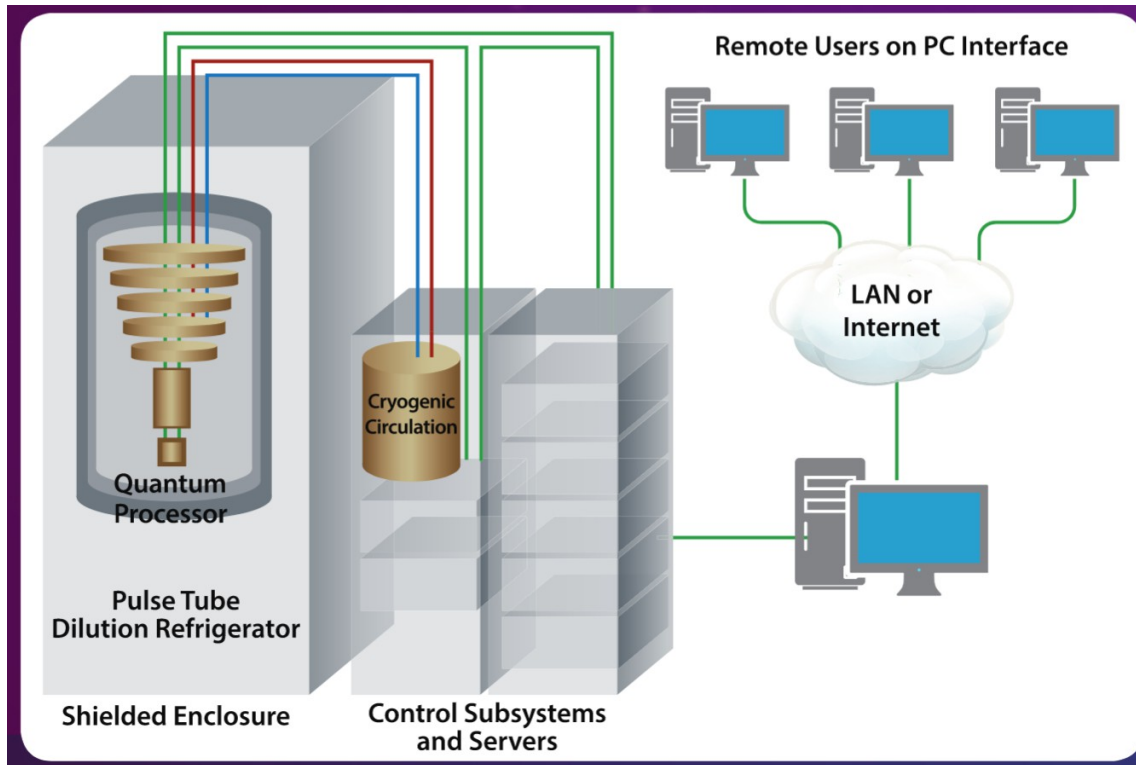
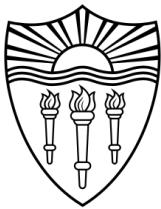
Interactions



D-Wave Computing Interface



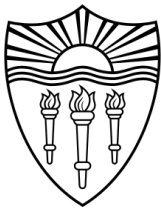
Working on a D-Wave



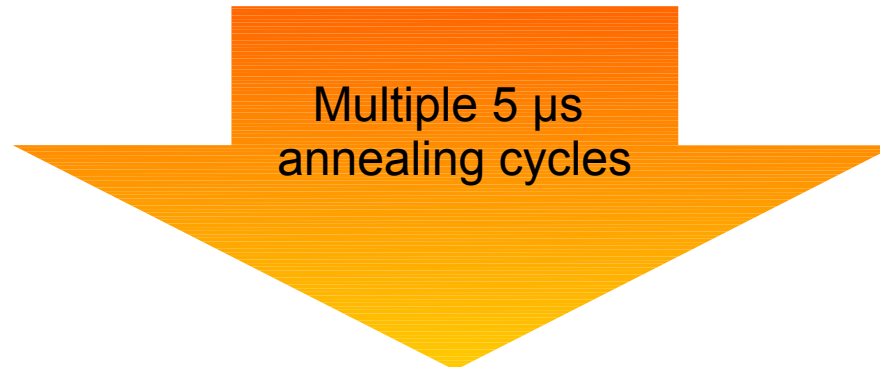
- Web Interface to post the problem settings (Hp).
- Asynchronous processing.
- Solution is made available for download.
- Distributed library for performing embedding
 - Retain full intellectual property.
- Equivalent restapi to submit and retrieve solutions
 - D-Wave processor as a service



D-Wave Sampling Solutions



User provides
(\mathbf{h}_i , \mathbf{J}_{ij})



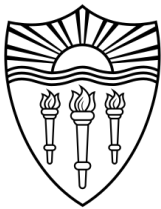
D-Wave provides sampling
from lowest energy levels
(approx. Gibbs)
($\{ \sigma_i \}_k$, $N(\{ \sigma_i \}_k)$)



Classification with Quantum Annealing



QA Machine Learning



Formalism to transform a binary
classification into an Ising model
hamiltonian optimization

K.L. Pudenz, D.A. Lidar
<https://arxiv.org/abs/1109.0325>



QAML Weak/Strong Classifier

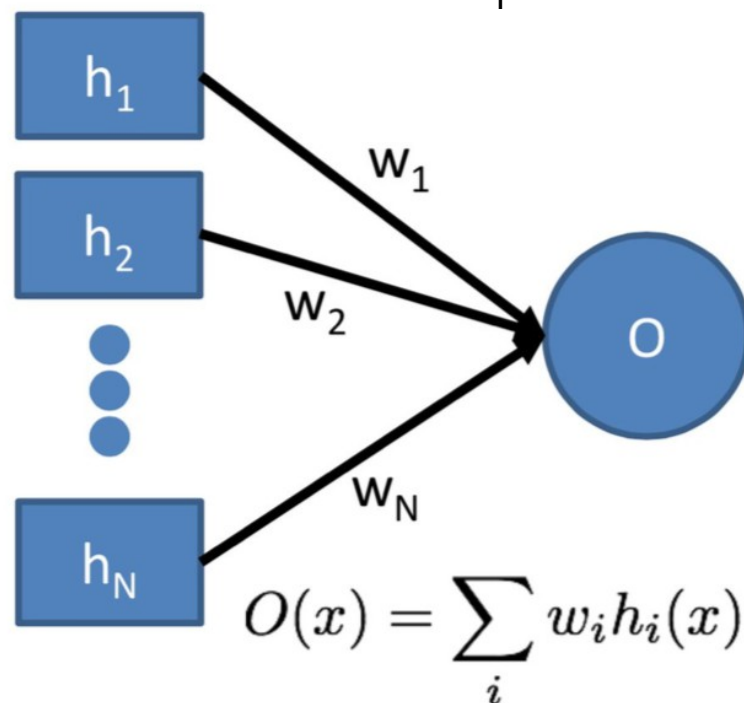


Define functions h_i of the input variables into $[-1, 1]$ such that

- $P(\text{signal}|h>0) > P(\text{bkg}|h>0)$
- $P(\text{bkg}|h<0) > P(\text{signal}|h<0)$

i.e. Most signal on $h>0$, most bkg on $h<0$

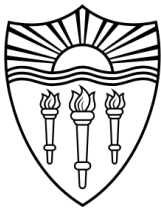
Define w_i as binary linear combination of h_i



<https://arxiv.org/abs/1109.0325>



QAML Target/Objective



Define as a “target” function

$$y(x) = \begin{cases} +1, & \text{if } x \in S, \\ -1, & \text{if } x \in B \end{cases}$$

Per event error

$$E(x) = y(x) - \sum_{i=1}^N w_i h_i(x)$$

Full error

$$\delta(\vec{w}) \propto \sum_{i,j} C_{ij} w_i w_j + \sum_i (\lambda - 2C_{iy}) w_i$$

- C_{ij} and C_{iy} are summations over the values of h_i over the training set
- λ is a parameter penalizing the number of non-zero w_i

<https://arxiv.org/abs/1109.0325>



QUBO

Quadratic Unconstrained Binary Optimization

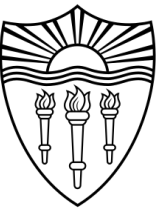
$$\delta(\vec{w}) \propto \sum_{i,j} C_{ij} w_i w_j + \sum_i (\lambda - 2C_{iy}) w_i$$

Simple conversion
of binary
weights to ± 1

$$H_{\text{Ising}} = \sum_i h_i \sigma_i^z + \sum_{ij} J_{ij} \sigma_i^z \sigma_j^z$$

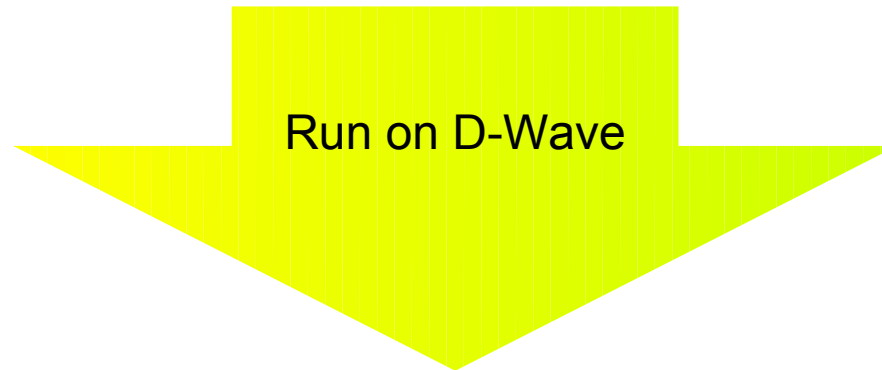


QAML End-to-End



QAML Discriminant

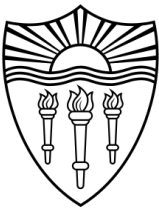
Signal and Background samples
Less than 40 features



Mask (w_i) of features contributing to

$$\sum_{i=1}^N w_i h_i(x)$$

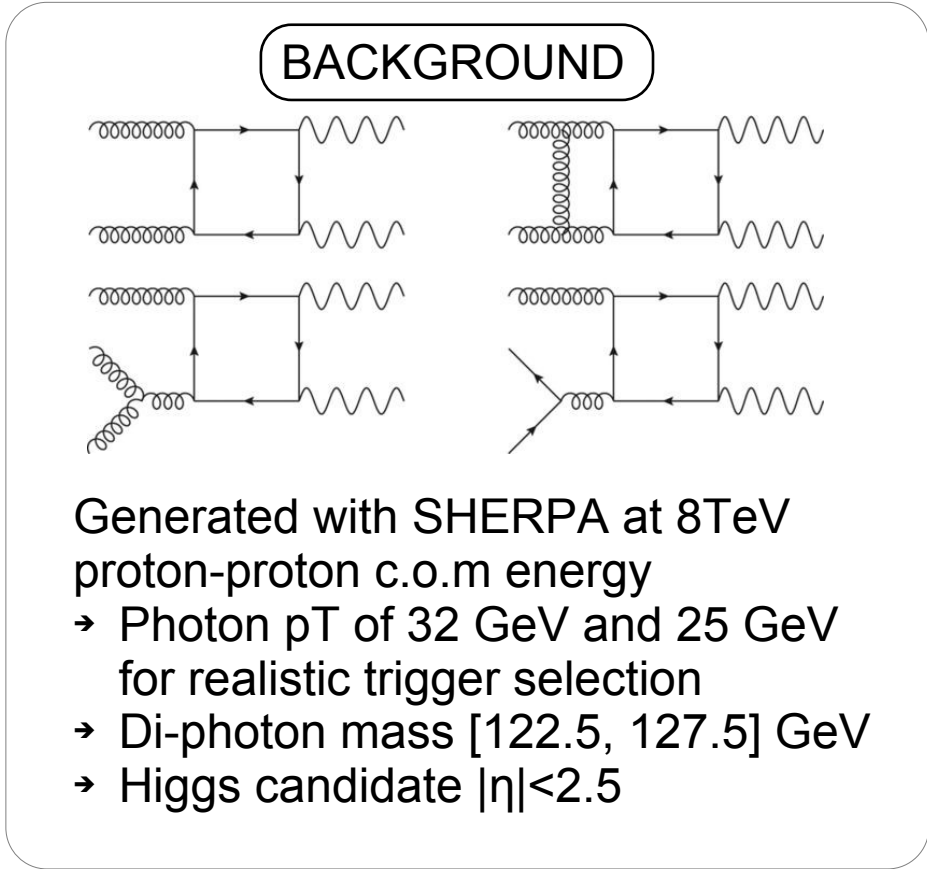
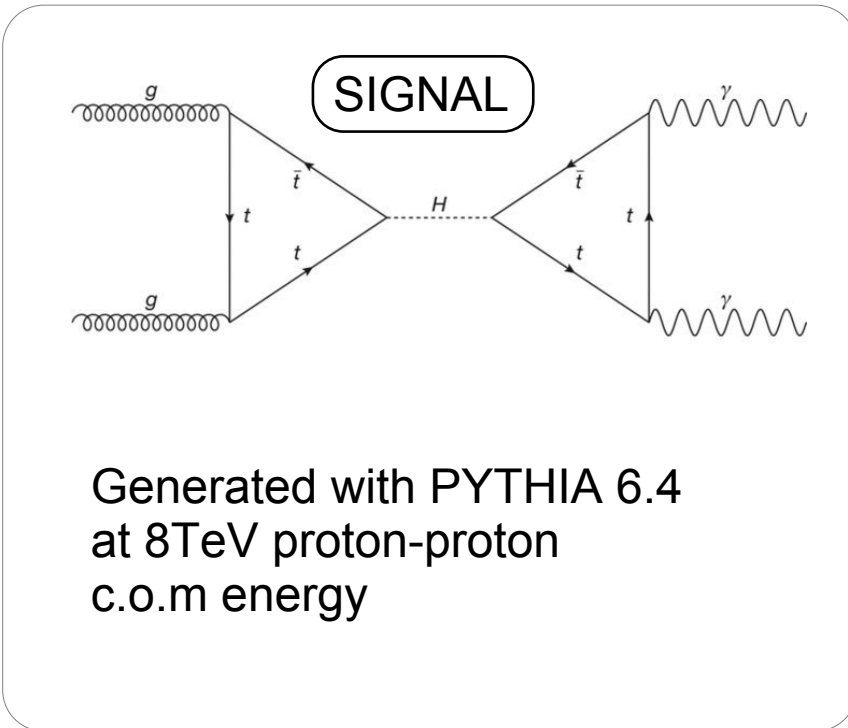
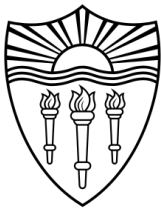
continuous discriminant function.
One mask per energy level.



A Higgs- background dataset

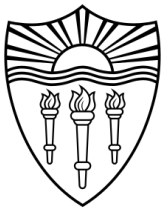


Generated Samples





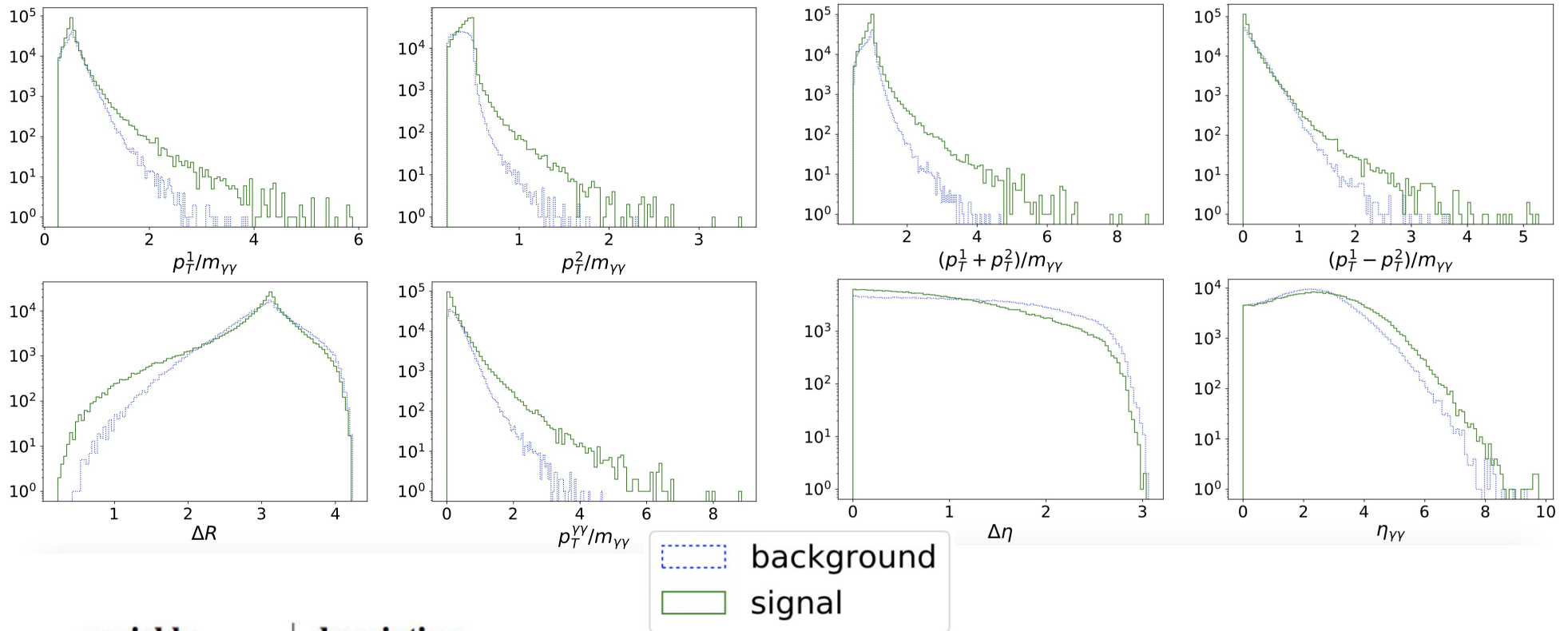
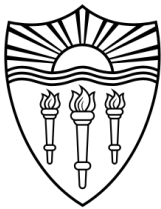
Sample Size and Folding



- 300k signal + 300k background total sample
 - **Training set**
 - 20 stratified, independent splits of sizes 100, 1000, 5000, 10k, 15k, 20k events
 - Spread of classifier performance over the folds reported as the **uncertainty due to the choice of training sample, and initialization.**
 - **Testing set**
 - Remaining 100k+100k independent sample
 - **Statistical error** on the classifier performance estimated using bootstrapping



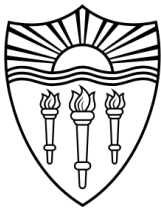
Characterizing Variables



variable	description
$p_T^1/m_{\gamma\gamma}$	transverse momentum of the highest p_T photon divided by the invariant mass of the diphoton pair
$p_T^2/m_{\gamma\gamma}$	transverse momentum of the second-highest p_T photon divided by the invariant mass of the diphoton pair
$(p_T^1 + p_T^2)/m_{\gamma\gamma}$	sum of the transverse momentum of the two photons divided by their invariant mass
$(p_T^1 - p_T^2)/m_{\gamma\gamma}$	difference of the transverse momentum of the two photons divided by their invariant mass
$p_T^{\gamma\gamma}/m_{\gamma\gamma}$	transverse momentum of the diphoton system divided by its invariant mass
$\Delta\eta$	difference in $\eta = -\log \tan\left(\frac{\theta}{2}\right)$, where θ is the angle with the beam axis
ΔR	sum in quadrature of the separation of and ϕ , the azimuthal angle of the two photons ($\sqrt{\Delta\eta^2 + \Delta\phi^2}$)
$ \eta^{\gamma\gamma} $	the η value of the diphoton system



Weak Classifier Function



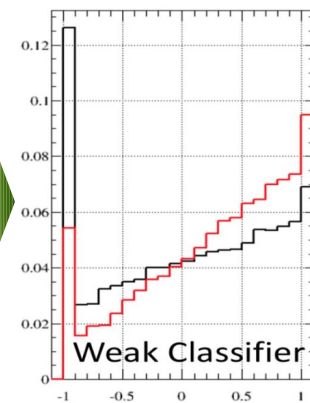
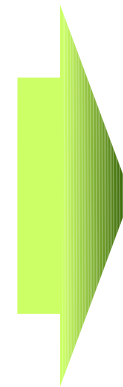
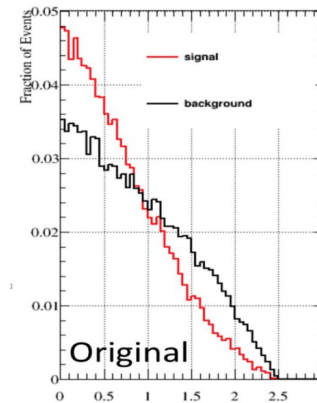
Define v_{shift}

- Based on 70th and 30th percentile of the signal distribution (s_{70} , s_{30})
- If the percentile of background at s_{70} is less than 70%, then translate to s_{70} and invert the variable
- Else, check the percentile of background at s_{30} , and if more than 30%, then translate to s_{30} .
- Else, the two distributions are “too overlapping” and we discard the variable.

Define h

- v_{+1} and v_{-1} are the 10th and 90th percentile of v_{shift}

$$h(v) = \begin{cases} +1 & \text{if } v_{+1} < v^{\text{shift}}(v) \\ \frac{v^{\text{shift}}(v)}{v_{+1}} & \text{if } 0 < v^{\text{shift}}(v) \leq v_{+1} \\ \frac{v^{\text{shift}}(v)}{|v_{-1}|} & \text{if } v_{-1} < v^{\text{shift}}(v) \leq 0 \\ -1 & \text{if } v^{\text{shift}}(v) < v_{-1} \end{cases}$$



Applied to all variables and their product (inverse if flipped)



Weak Classifiers Naming



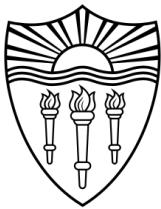
1	2	3	4	5	6	7	8	9
p_T^1	p_T^2	ΔR	$p_T^{\gamma\gamma}$	$p_T^1 + p_T^2$	$p_T^1 - p_T^2$	$\Delta\eta$	$\eta_{\gamma\gamma}$	$(p_T^1 + p_T^2)\eta_{\gamma\gamma}$
10	11	12	13	14	15	16	17	18
$\frac{p_T^2}{p_T^1 - p_T^2}$	$\frac{p_T^2}{\Delta\eta}$	$p_T^2 \eta_{\gamma\gamma}$	$\frac{1}{\Delta R p_T^{\gamma\gamma}}$	$\frac{p_T^1 + p_T^2}{\Delta R}$	$\frac{1}{\Delta R (p_T^1 - p_T^2)}$	$\frac{1}{\Delta R \Delta\eta}$	$\frac{\eta_{\gamma\gamma}}{\Delta R}$	$\frac{1}{(p_T^1 - p_T^2) \Delta\eta}$
19	20	21	22	23	24	25	26	27
$p_T^1 p_T^2$	$\frac{p_T^1}{\Delta R}$	$\frac{p_T^1}{p_T^{\gamma\gamma}}$	$p_T^1 (p_T^1 + p_T^2)$	$\frac{p_T^1}{p_T^1 - p_T^2}$	$\frac{p_T^1}{\Delta\eta}$	$\frac{p_T^1}{\eta_{\gamma\gamma}}$	$\frac{p_T^2}{\Delta R}$	$\frac{\eta_{\gamma\gamma}}{p_T^1 - p_T^2}$
28	29	30	31	32	33	34	35	36
$\frac{p_T^2}{p_T^{\gamma\gamma}}$	$p_T^2 (p_T^1 + p_T^2)$	$\frac{p_T^1 + p_T^2}{p_T^{\gamma\gamma}}$	$\frac{\eta_{\gamma\gamma}}{p_T^{\gamma\gamma}}$	$\frac{1}{p_T^{\gamma\gamma} \Delta\eta}$	$\frac{1}{p_T^{\gamma\gamma} (p_T^1 - p_T^2)}$	$\frac{p_T^1 + p_T^2}{p_T^1 - p_T^2}$	$\frac{p_T^1 + p_T^2}{\Delta\eta}$	$\frac{\eta_{\gamma\gamma}}{\Delta\eta}$



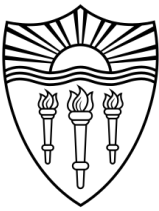
Experiments



Outline



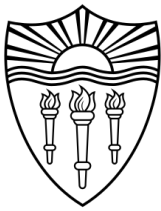
- Train classical classifiers as a baseline measurement of performance.
- Evaluate the exact solution of the problem using simulating annealing of the Ising model.
- Scan for λ , penalty on number of weak classifiers.
- Classification performance depending on the size of the training set.
- Scan on the fraction of exited states included in the classifier.



Baseline Classifiers



Classical Baseline

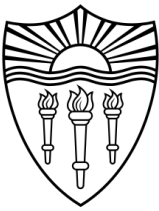


→ XGBoost (XGB)

- Extremely efficient library for training decision trees
- <http://xgboost.readthedocs.io>
- Discovered during the higgs-ml challenge
<https://www.kaggle.com/c/higgs-boson>
- Moderately optimize the hyper-parameters

→ Deep Neural Network (DNN)

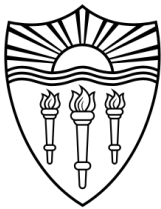
- Simple fully connected model 2 layers 1000 nodes
- <https://keras.io/>
<http://deeplearning.net/software/theano/>
- Moderately optimize the hyper-parameters



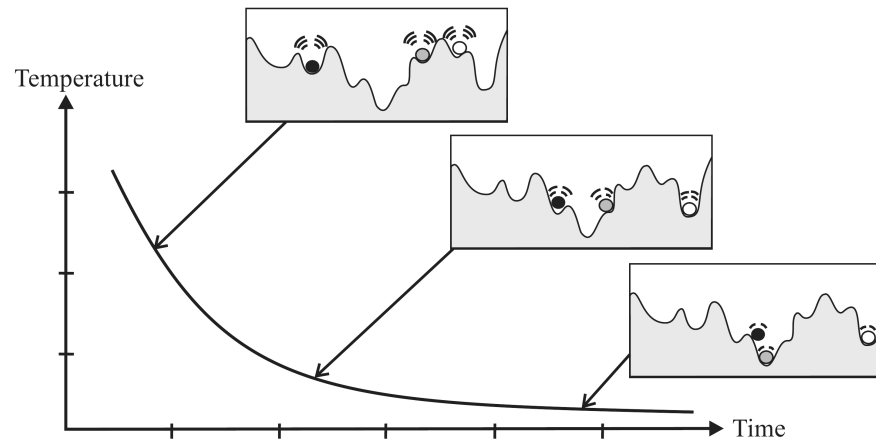
Simulated Annealing



Ising Model Heuristic Solution



- Monte-Carlo based method to find ground state of energy functions
- Random walk across phase space
 - accepting descent
 - accepting ascent with probability $e^{-\Delta E/kT}$
- Decrease T with time



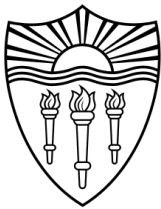
Applied to the QUBO problem, and finds the **ground state** reasonably well. SA in the legends.



Variable Importance



Weak Classifier Penalty

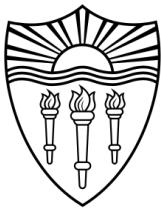


$$\delta(\vec{w}) \propto \sum_{i,j} C_{ij} w_i w_j + \sum_i (\lambda - 2C_{iy}) w_i$$

Penalize for using many
weak classifiers



Surviving Weak Classifiers

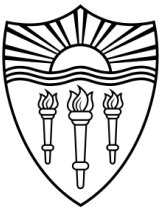


λ	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
0	0	20	20	20	19	20	20	0	5	20	20	20	20	19	20	17	20	20
0.01	0	20	20	20	19	20	20	0	4	20	20	20	20	19	20	17	20	20
0.02	0	20	20	20	19	20	20	0	4	20	20	20	20	19	20	16	20	20
0.05	0	20	20	20	19	20	20	0	1	20	20	20	20	19	20	10	20	17
0.1	0	20	20	20	19	20	20	0	0	20	20	20	20	19	20	6	14	2
0.2	0	20	20	20	19	20	20	0	0	20	14	20	20	12	20	4	1	0
0.4	0	20	0	2	19	20	20	0	0	20	17	20	20	0	20	1	0	0
0.8	0	20	0	0	0	0	9	0	0	18	0	0	20	0	2	0	0	0

λ	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
0	20	0	0	19	0	20	0	3	0	20	19	7	0	15	0	19	20	20
0.01	20	0	0	19	0	20	0	2	0	20	19	6	0	15	0	19	20	20
0.02	20	0	0	19	0	20	0	1	0	20	19	4	0	15	0	19	20	20
0.05	20	0	0	19	0	20	0	0	0	20	16	1	0	11	0	19	20	20
0.1	20	0	0	1	0	20	0	0	0	20	1	0	0	5	0	16	20	20
0.2	18	0	0	0	0	20	0	0	0	20	0	0	0	0	0	0	20	20
0.4	0	0	0	0	0	7	0	0	0	20	0	0	0	0	0	0	20	3
0.8	0	0	0	0	0	0	0	0	0	20	0	0	0	0	0	0	19	0

Table : Number of times the weak classifier of a given variable is used in the ground state solution, as a function of the penalty

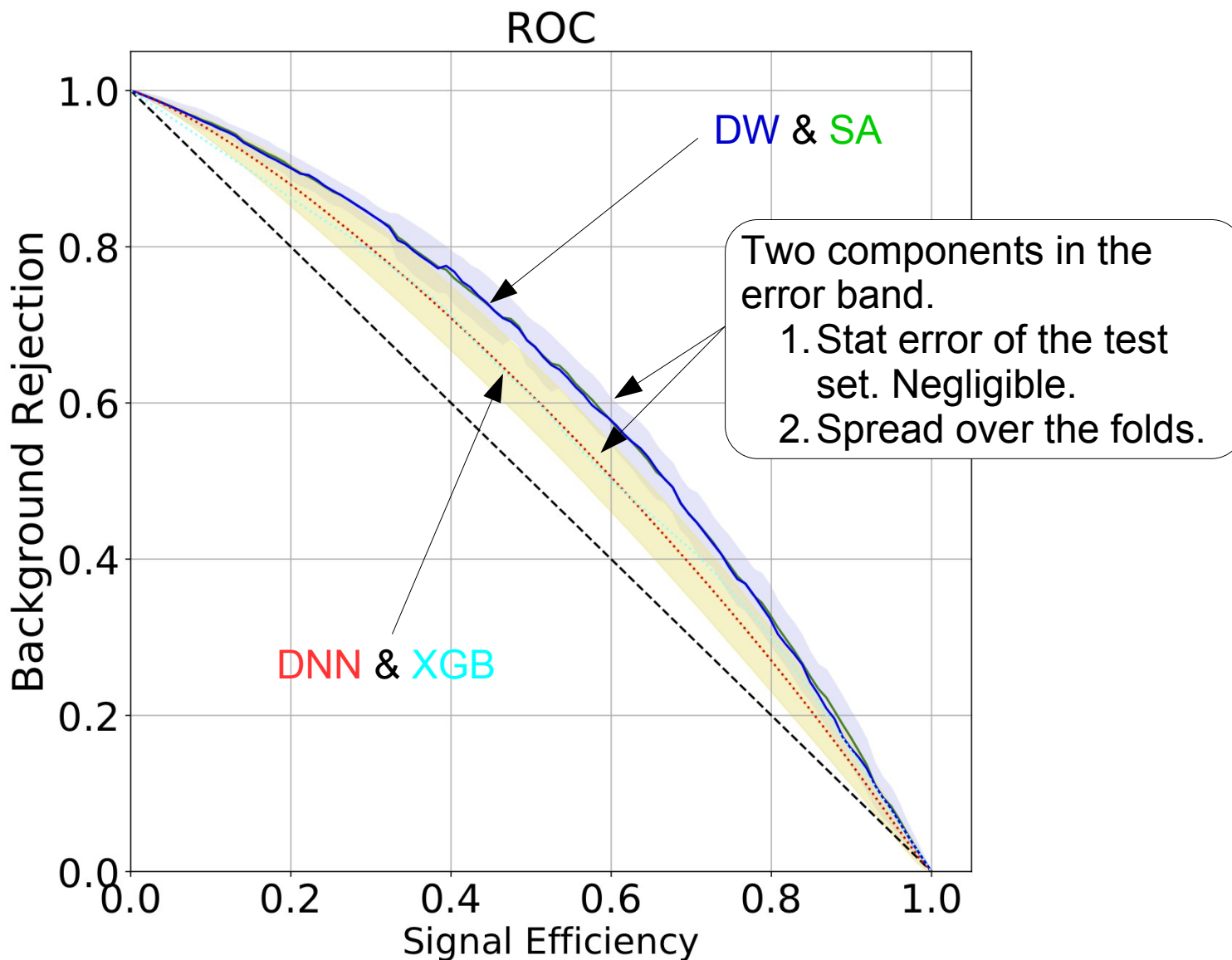
Three major variables (2, 13, 28) : p_T^2 , $(\Delta R p_T^{\gamma\gamma})^{-1}$, and $\frac{p_T^2}{p_T^{\gamma\gamma}}$
 Relates to the creation of a heavy particle (Higgs) with less transverse energy than typical QCD in the same mass range.



Classification Performance

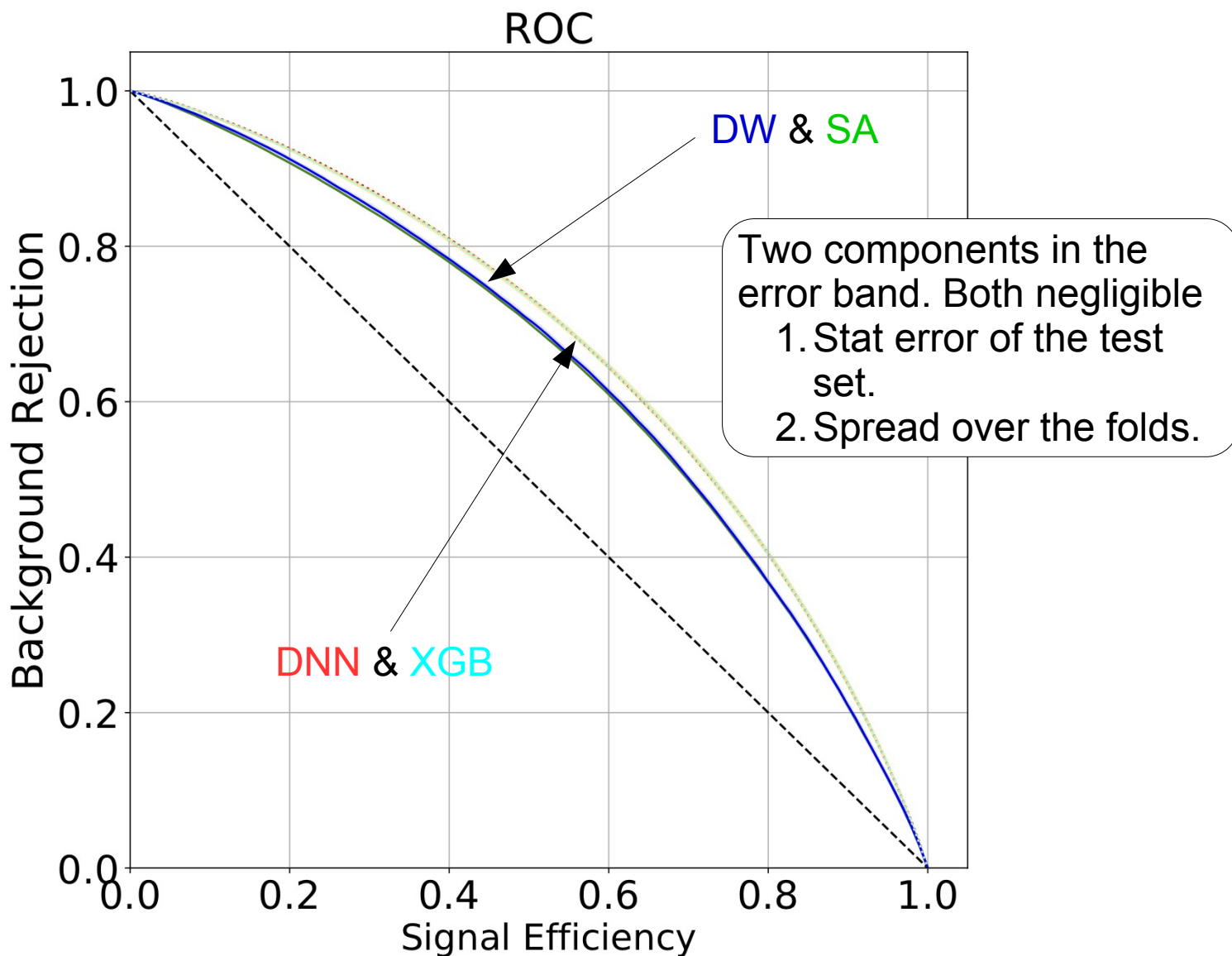


Sample Size of 100 Events



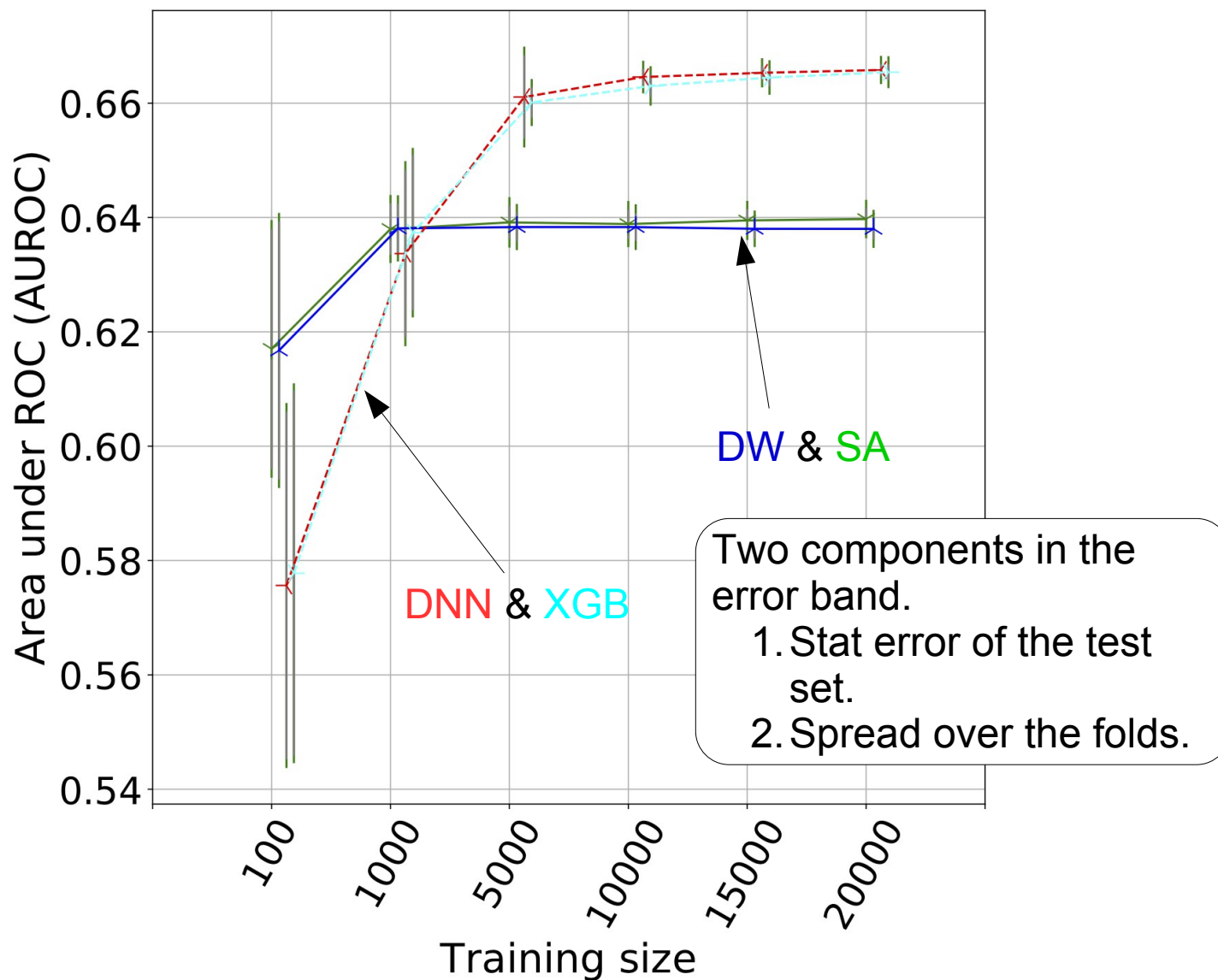


Sample Size of 20k Events

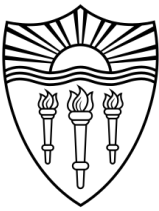




Evolution With Training Size



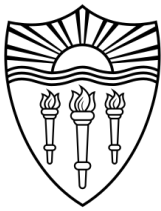
Two components in the error band.
1. Stat error of the test set.
2. Spread over the folds.



Hybrid Classifier



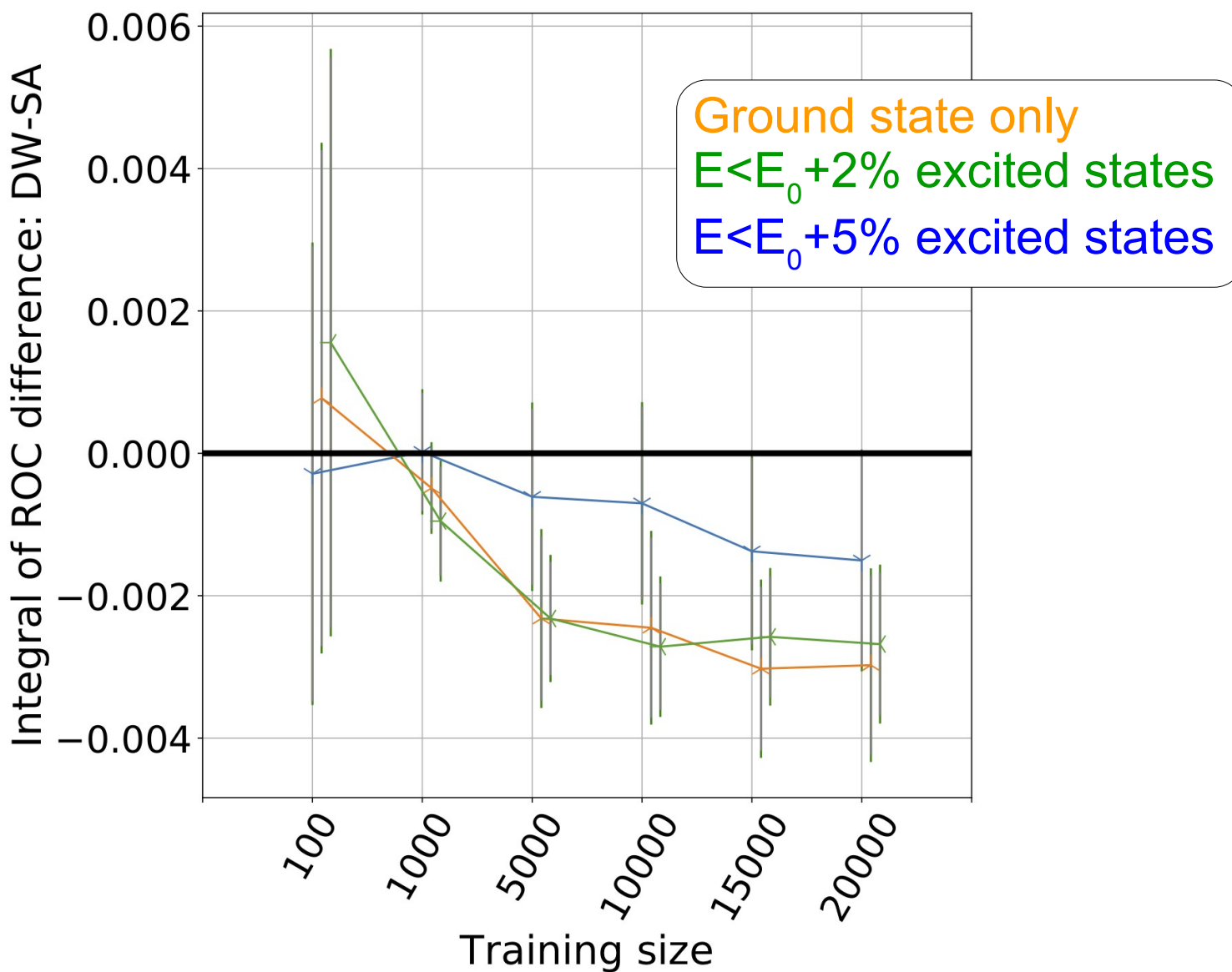
In presence of Excited States



- Excited states bring new mask of weak classifier to build discriminators
- Building a discriminator including excited states
 - Take $x\%$ of the levels above the ground state
 - For each evaluate the average ROC on the training folds
 - At each signal efficiency, pick the energy state that has the most rejecting discriminator

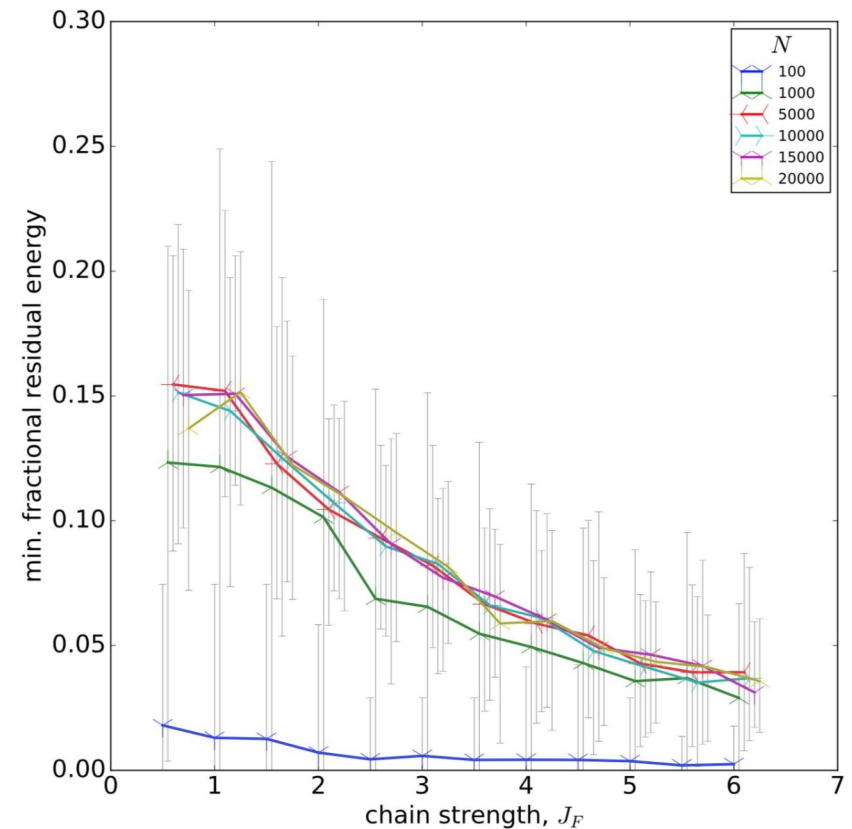
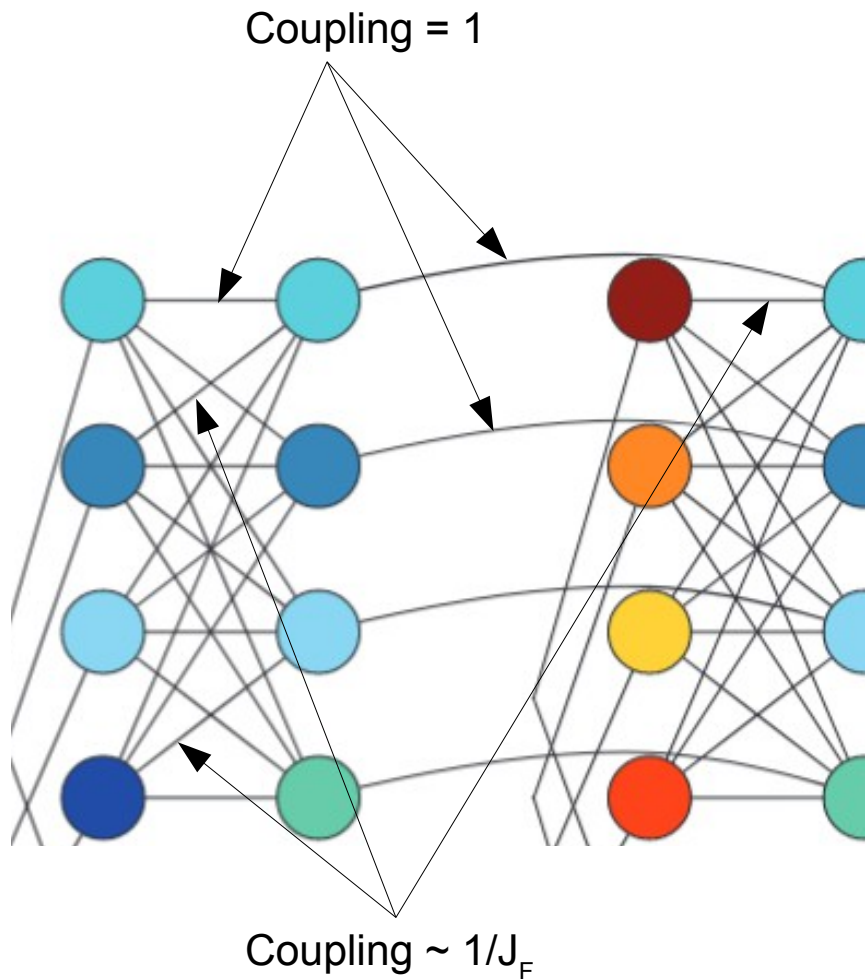
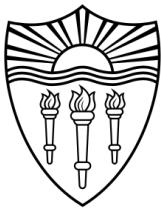


Comparison DW/SA

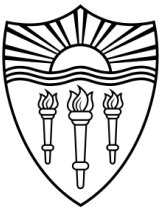




Chimera Spin Chains



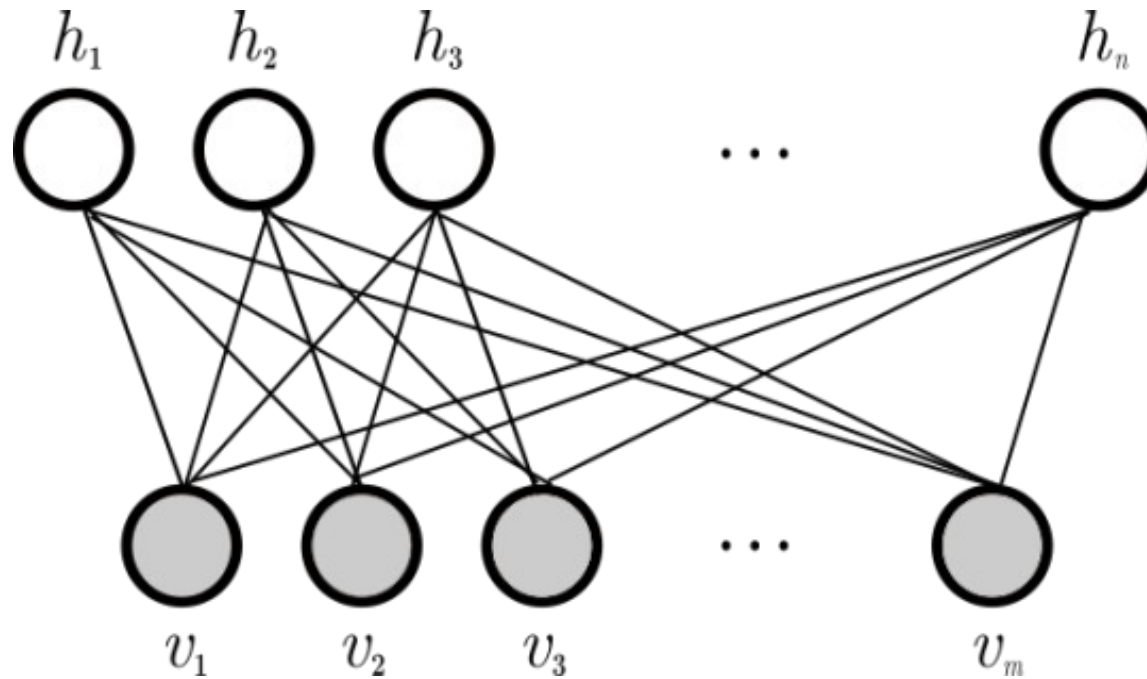
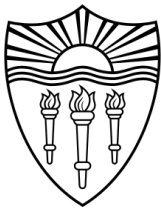
- Quantum Annealing gets better at finding the **ground truth** ground state with strong chain strength.
- SA does not include spin chains.
- Feedback to machine developers



Application Outlook



Boltzmann Machine

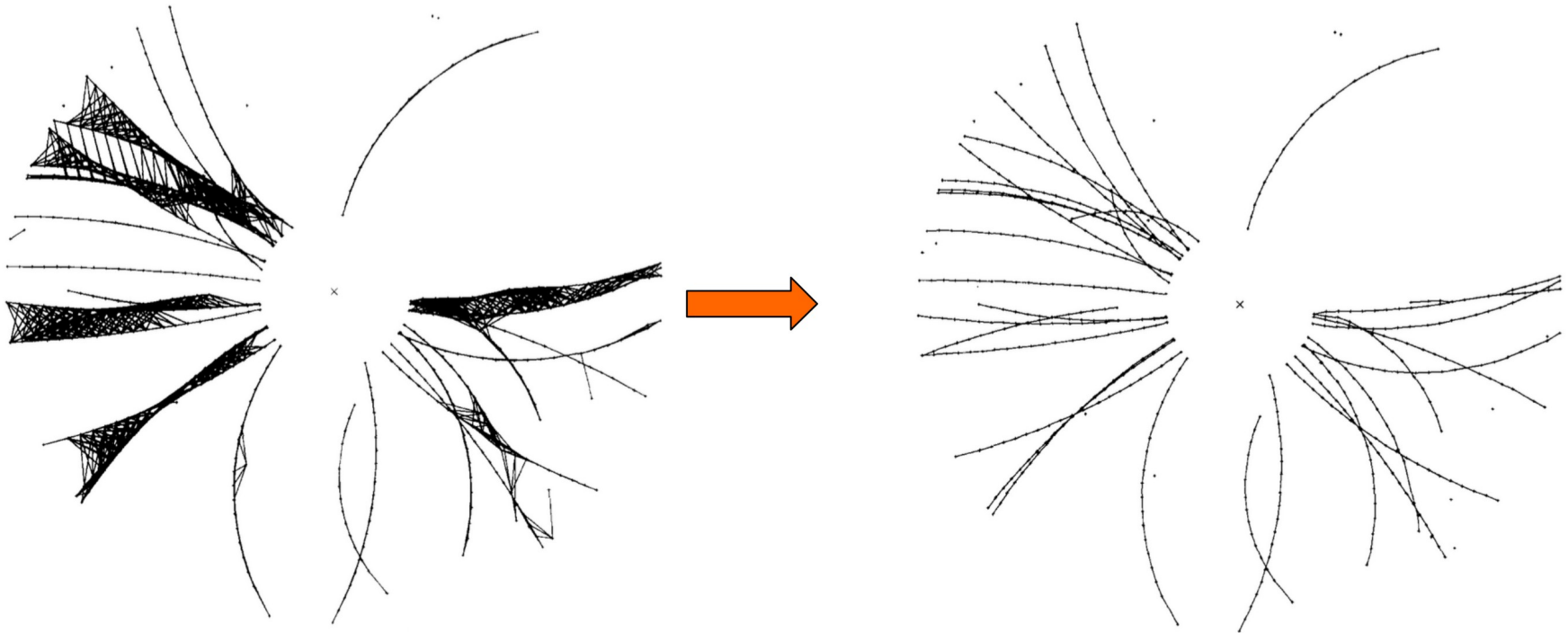
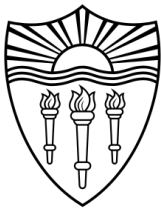


$$E = - \left(\sum_{i < j} w_{ij} s_i s_j + \sum_i \theta_i s_i \right)$$

<https://arxiv.org/abs/1601.02036>



Hopfield Network

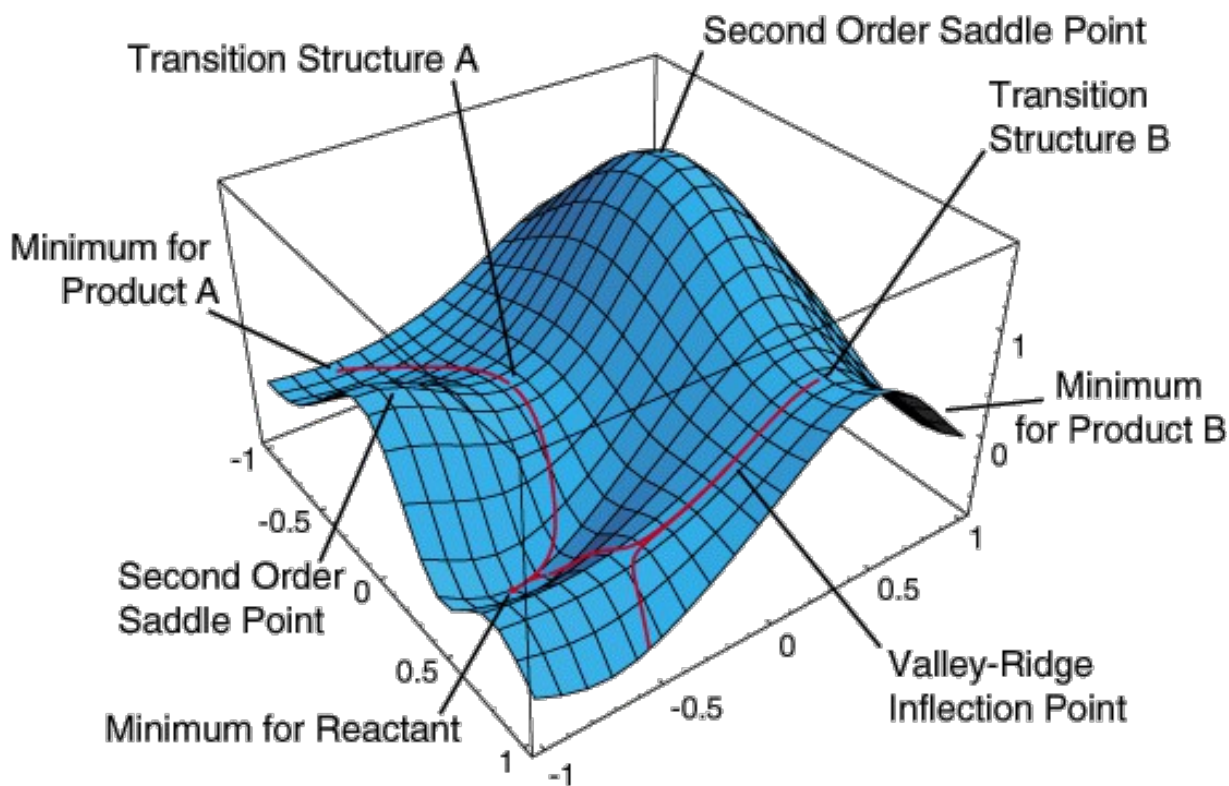


$$E = -\frac{1}{2} \sum_{i,j} w_{ij} s_i s_j + \sum_i \theta_i s_i$$

<https://tinyurl.com/yayglkf4>



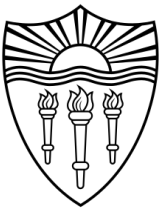
Second Order Optimization



$$y = f(\mathbf{x} + \Delta\mathbf{x}) \approx f(\mathbf{x}) + \nabla f(\mathbf{x})^T \Delta\mathbf{x} + \frac{1}{2!} \Delta\mathbf{x}^T \mathbf{H}(\mathbf{x}) \Delta\mathbf{x}$$



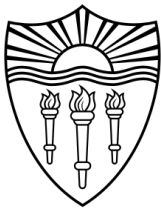
Summary



- First application of D-Wave quantum annealing capability to High Energy Physics use case. Raises interesting questions.
- Scope of the Quantum Annealing on the D-Wave computing device is the solving the Ising model. Limited but powerful.
- Potential applicability of Quantum Annealing to other problems.



Thanks



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