Cosmological Signatures of the SM Higgs Instability: Primordial Black Holes and Gravitational Waves

with José Ramón Espinosa and Antonio Riotto

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Higgs Hunting – Paris

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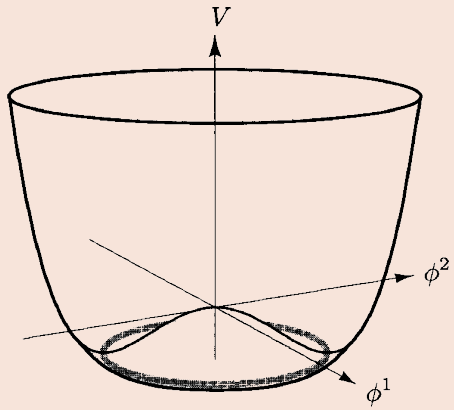
Higgs potential beyond the tree level

['79 Cabibbo et al.; '89 Sher; '94 Altarelli, Isidori; '96 Casas, Espinosa, Quirós; '07 Espinosa, Giudice, Riotto; '12 Degrassi et al.; '15 Espinosa et al.; '16 East et al.; . . .]

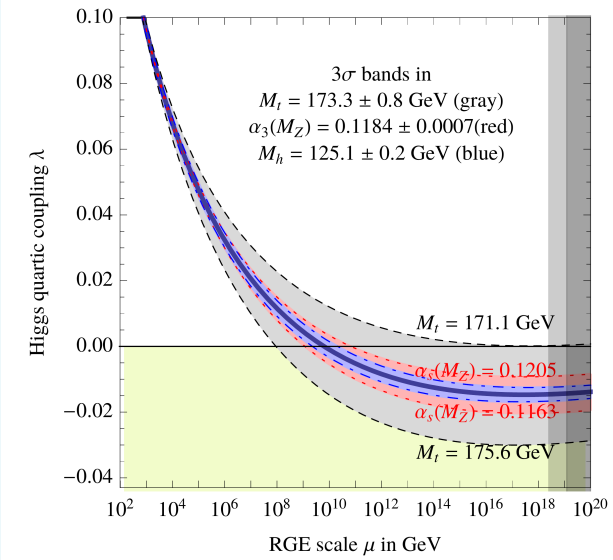
Tree level potential

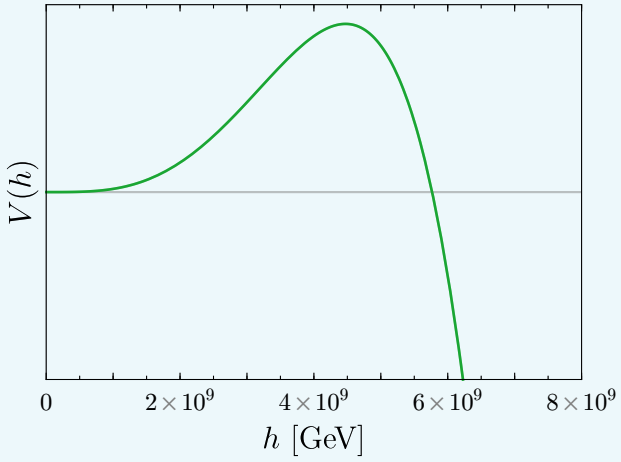
$$V(h) = \frac{1}{4}\lambda\, h^4 - \frac{1}{2}\mu^2 h^2$$



RG improved potential

$$V(h) = \frac{1}{4}\lambda(h)\, h^4$$





Tunnelling today

Negligible probability, today we are safe. We live in a metastable Universe.

Implications of living in a false vacuum

During inflation

- Background of a scalar field in de Sitter makes quantum jumps $\sim \pm \frac{H}{2\pi}$.
- These fluctuations could lead the Higgs beyond the barrier, and make it roll towards the true vacuum.
- This vacuum has large negative energy $\Longrightarrow$ AdS bubble, which can expand at the speed of light $\Longrightarrow$ $\boxed{\text{It didn't happen in our past lightcone.}}$

During reheating

- Higgs interacts with thermal bath of SM particles $\Longrightarrow$ stabilisation of the potential

$$V(h) = \tfrac{1}{4}\lambda(h)h^4 \boxed{+\tfrac{1}{2}m_T^2 h^2},$$

$$m_T^2 = 0.12\, T^2 \exp\left(-\tfrac{h}{2\pi T}\right)$$

- If T_{RH} is high enough and h is not too far, thermal corrections can "rescue" the Higgs back to 0.

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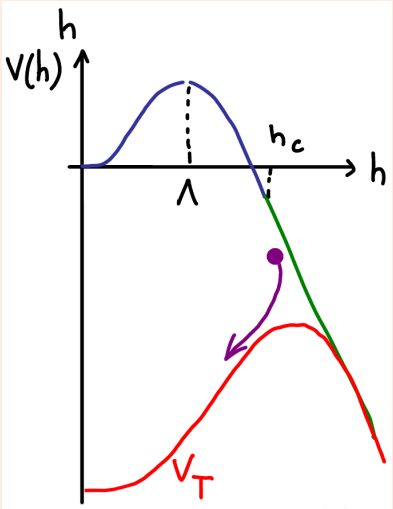


Possible observational evidences?

Are there possible observational signatures of the Higgs instability?

- Tunnelling today: not here, until this afternoon.

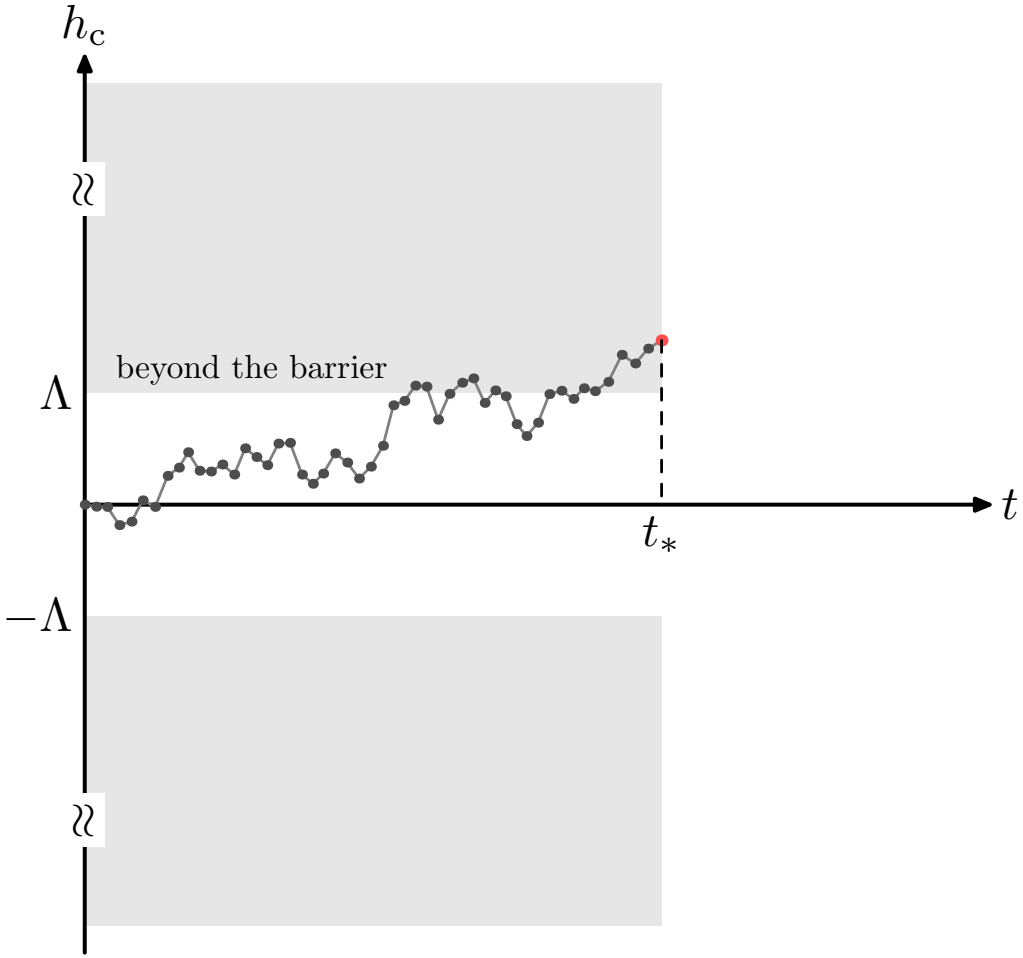
If this time is on the order of 10^9 yr, we have occasion for anxiety.

This would be the appropriate case to study if we were currently living in a false vacuum whose apocalyptic decay is yet to occur.

[Coleman]

- What if the Higgs probed the unstable region at the end of inflation, and was rescued back in time by thermal corrections at reheating?
- Two possible signatures:
  1. Primordial Black Holes as Dark Matter
  2. Background of Gravitational Waves

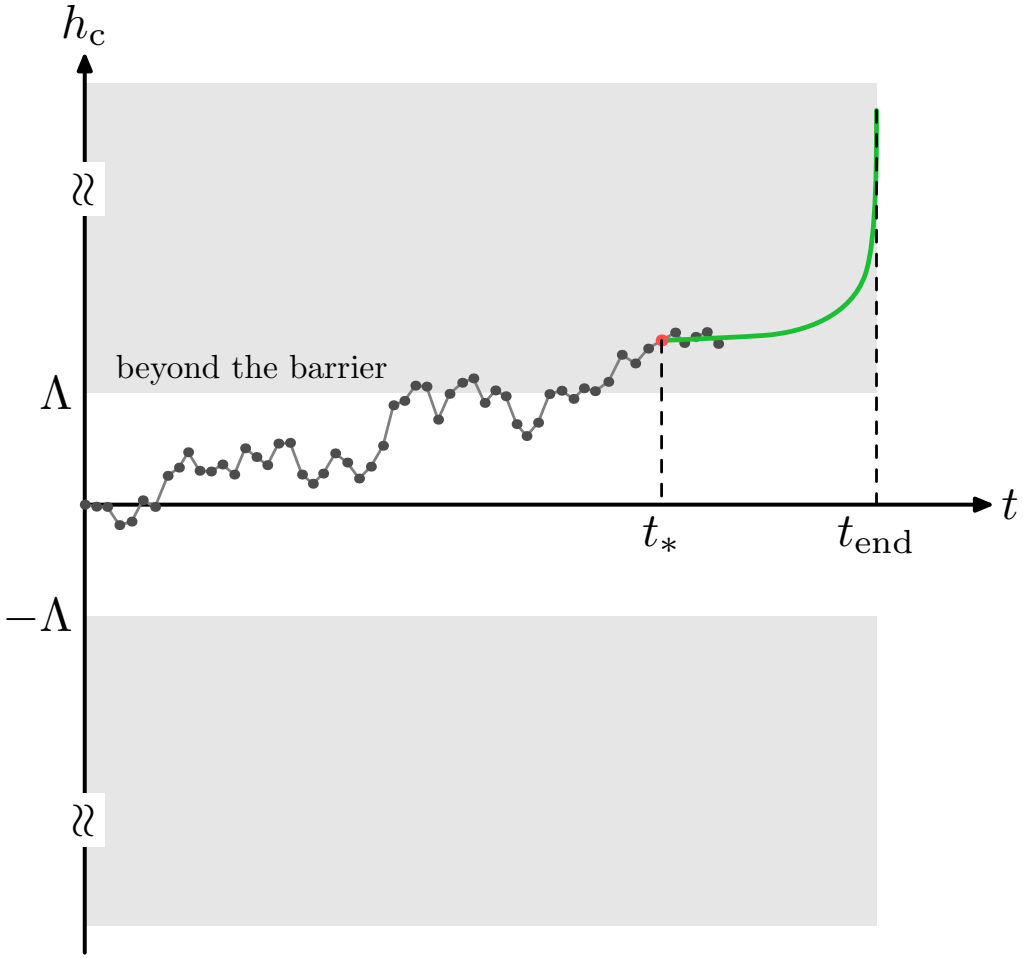
Evolution of the background of the Higgs



- We assume that $V(h) < 0$ at some scale.
- During inflation, the background field $h_c(t)$ has random fluctuations

$$\Delta_q h_c \sim \pm \frac{H}{2\pi} \, .$$

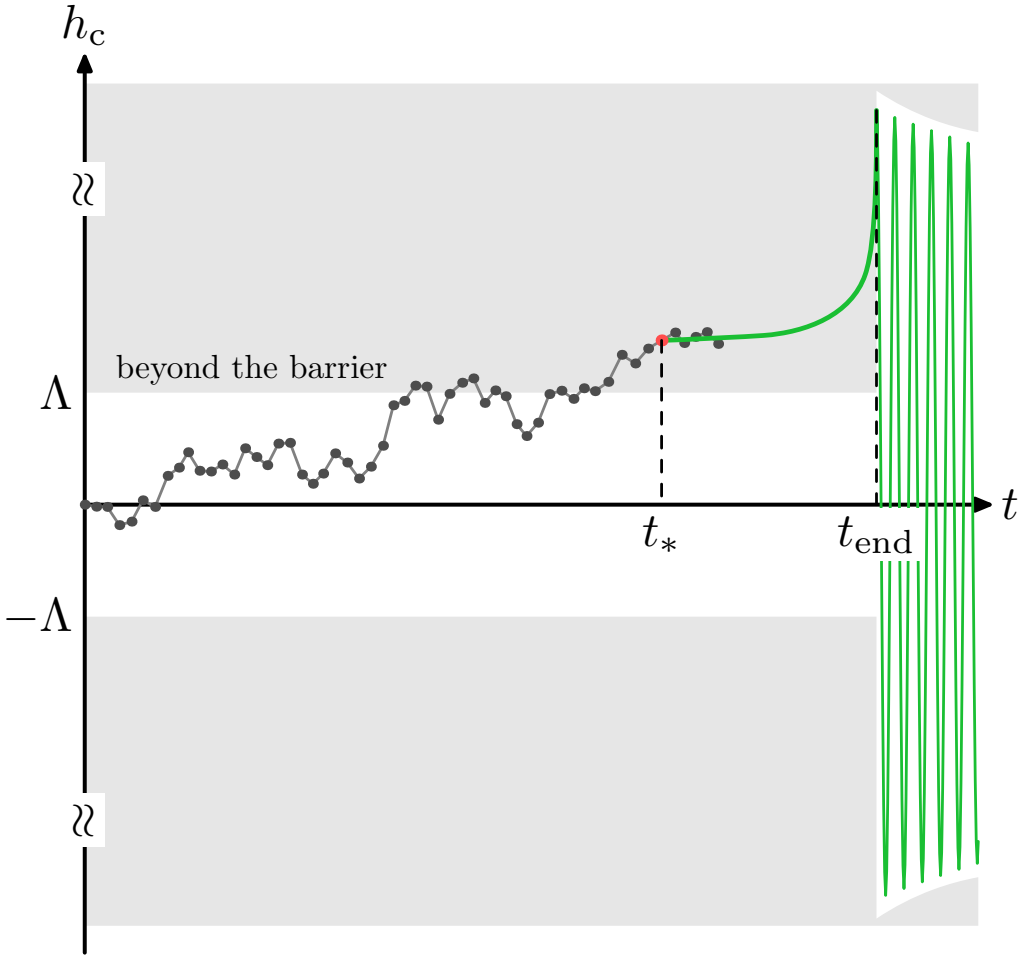
Evolution of the background of the Higgs



- If classical evolution prevails over quantum fluctuations, h_c begins to slow roll down the negative potential.

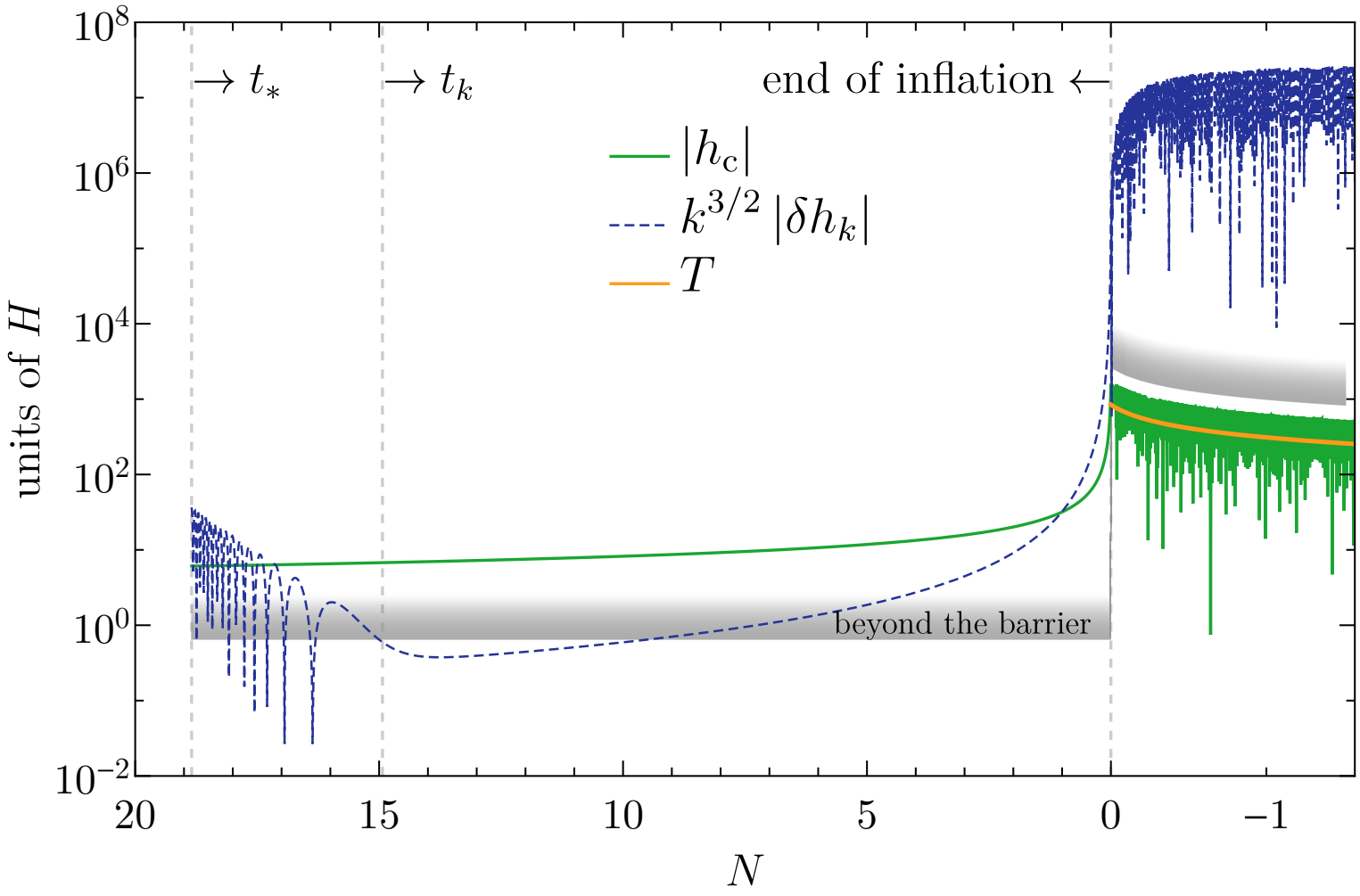
$$\overbrace{\dot{h}_c \Delta t}^{\text{classical}} \gtrsim \overbrace{\frac{H}{2\pi}}^{\text{quantum}}$$

Evolution of the background of the Higgs

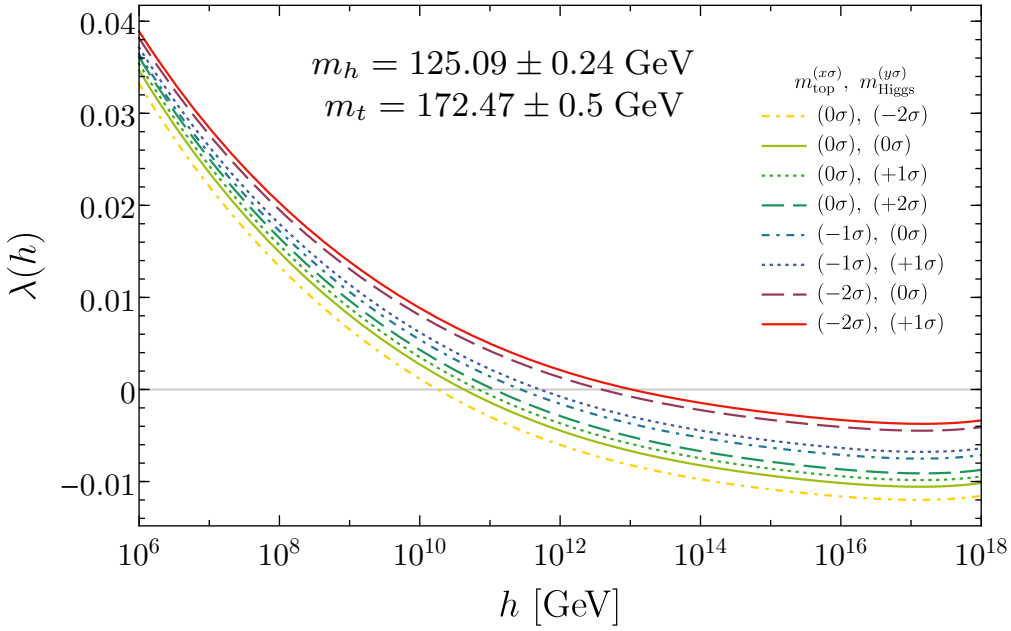


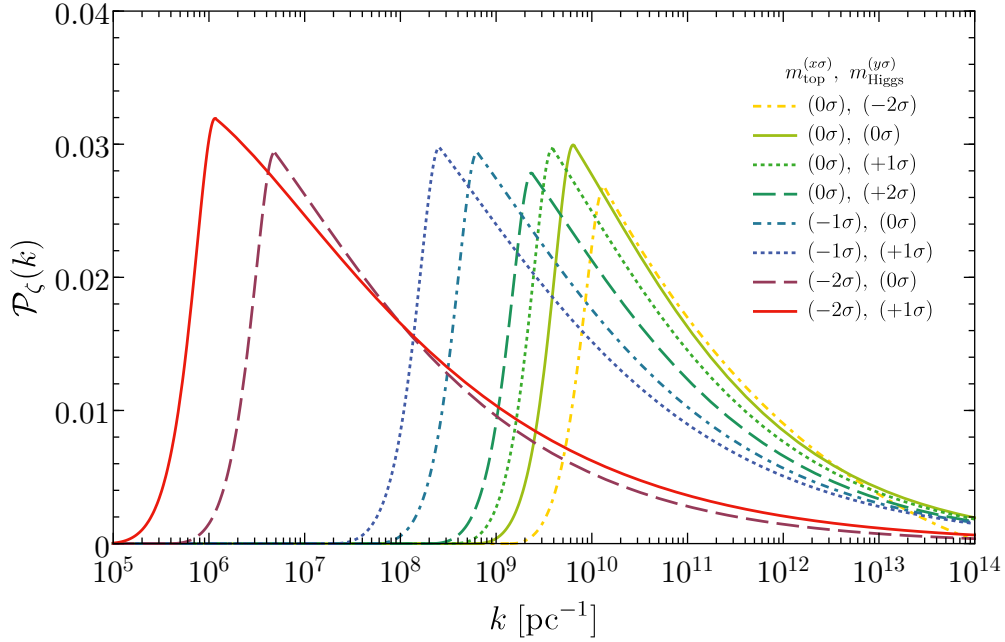
- At the end of inflation, if the reheating temperature is high enough and Higgs has not gone too far down the potential, thermal corrections can rescue h and bring it back around 0.
- Decay to radiation: in a very short time the Higgs field decays to radiation.

Evolution of Higgs background and fluctuations



Power spectrum of curvature perturbations

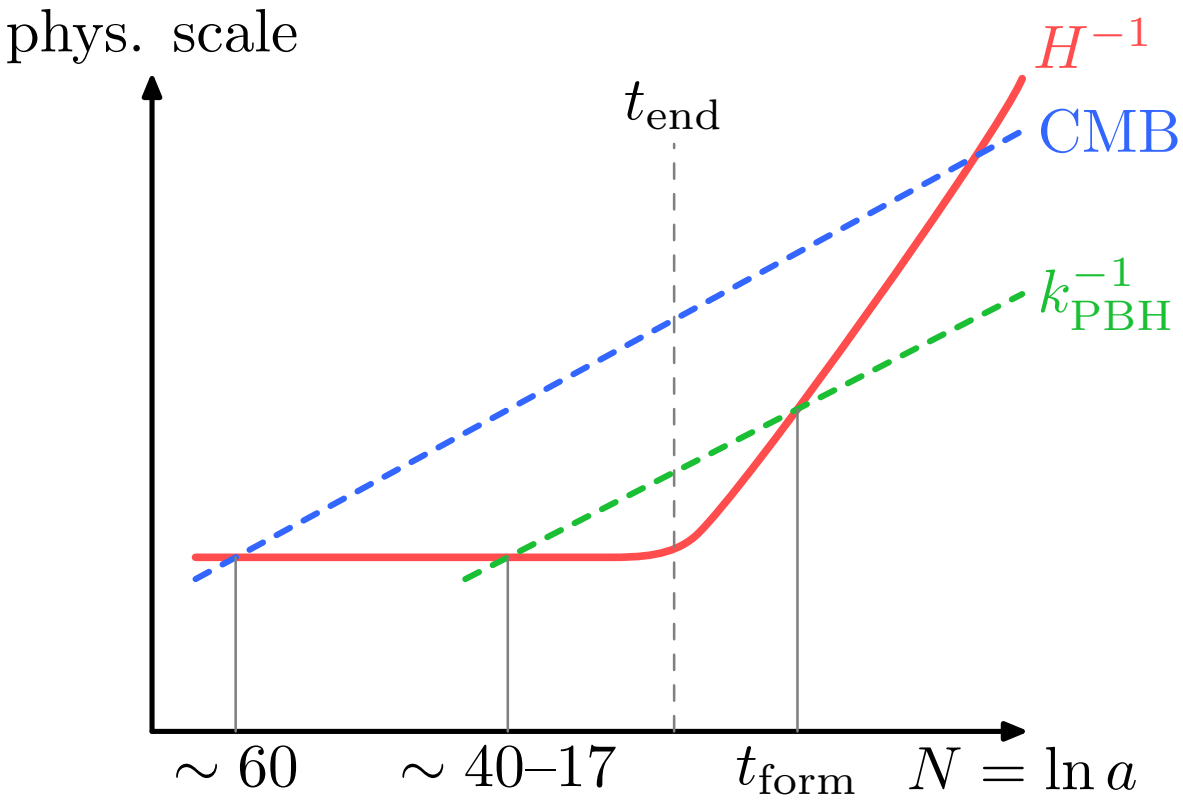




1st signature: Primordial Black Holes

['67 Zeldovich, Novikov; '71 Hawking; '75 Chapline; '96 Garcia-Bellido, Linde, Wands; '97 Green, Liddle; '16 Carr, Kühnel, Sandstad (and refs therein); '17 Ballesteros, Taoso; '18 Carr, Silk; '18 Sasaki et al. (and refs therein); . . .]

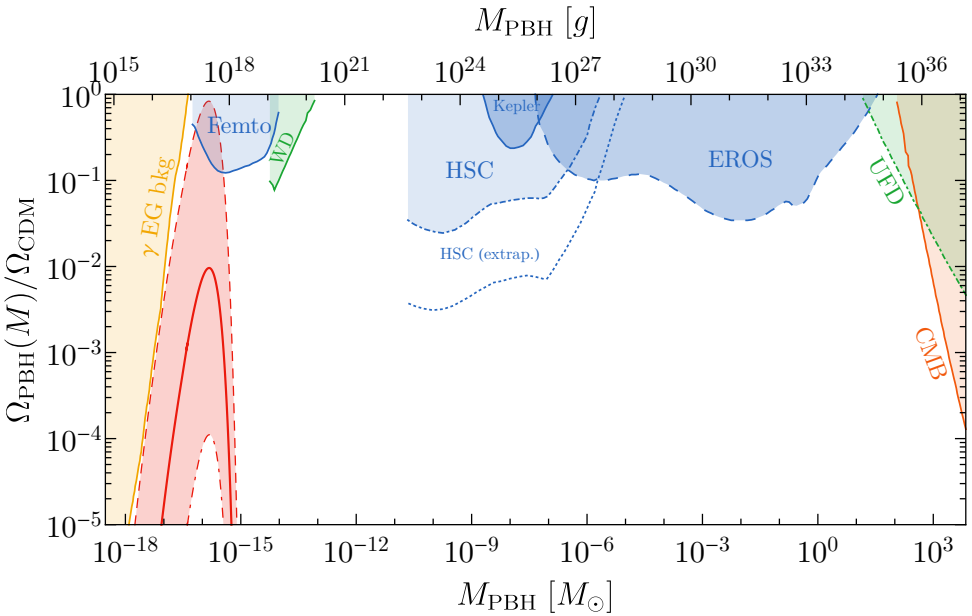
- Large overfluctuations, when they re-enter the Hubble radius, can collapse into a PBH.



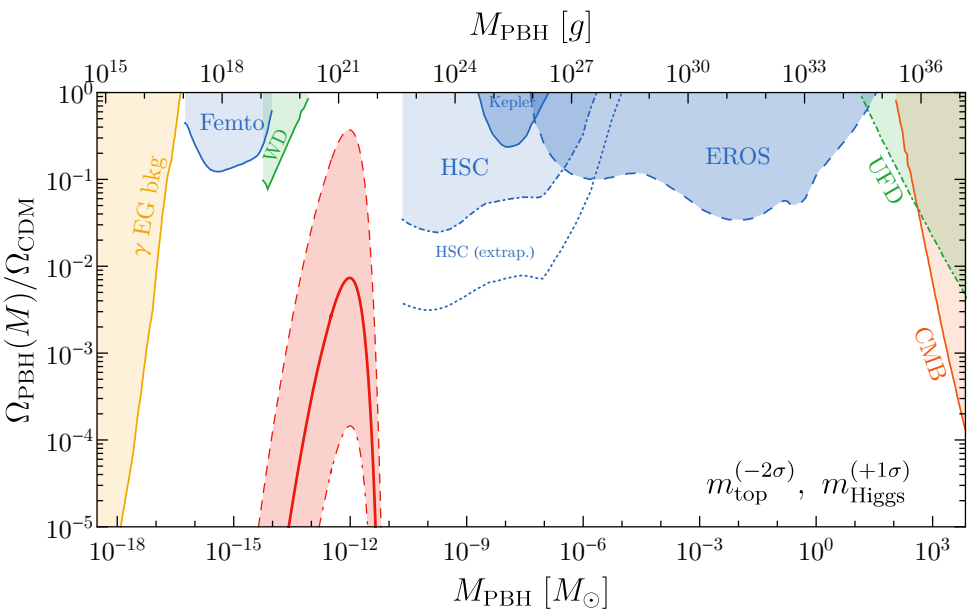
Mass function of Primordial Black Holes

- Position of the peak: depends on t_*, thus on the slope of $V(h)$.

$m_{\text{Higgs}} = 125.09$ GeV
$m_{\text{top}} = 172$ GeV



$m_{\text{Higgs}} = 125.33$ GeV
$m_{\text{top}} = 171.47$ GeV

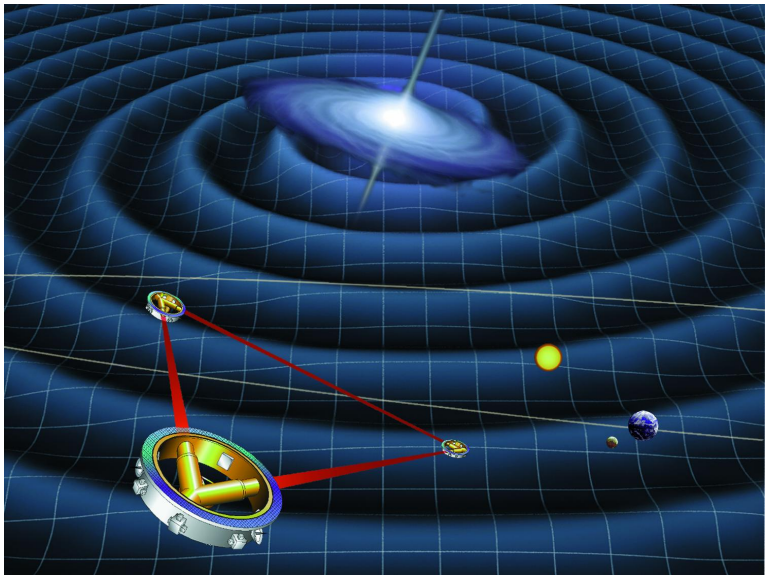


- Height of the peak: depends on how much the fluctuations grew.
  Finely tuned, as in any model for PBH formation.
  In SM + inflation, DM is provided by this mechanism $\Longrightarrow$ anthropic explanations: without DM there would be no Large Scale Structures.

2nd signature: Gravitational Waves

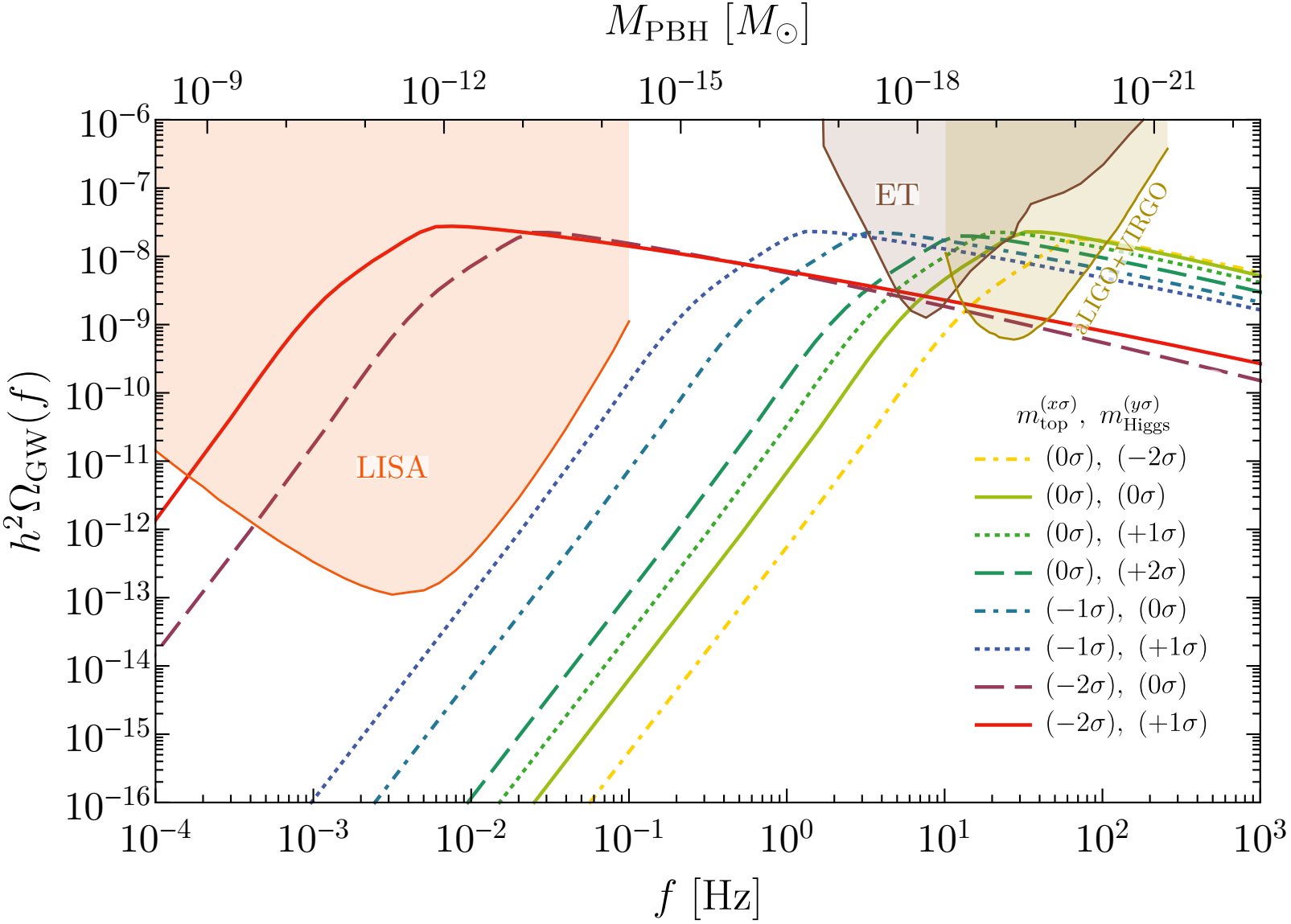
['02 Acquaviva et al.; '03 Mollerach, Harari, Matarrese; '06 Ananda, Clarkson, Wands; '07 Baumann et al.; '17 Ando et al.; '18 Espinosa, DR, Riotto]

- Tensor perturbations are excited in presence of large scalar perturbations when they cross the Hubble radius, then survive (redshifted) until today.
- This background of GWs can be detected by Advanced-LIGO, Einstein Telescope and especially LISA.
- We characterise its power spectrum and bispectrum.

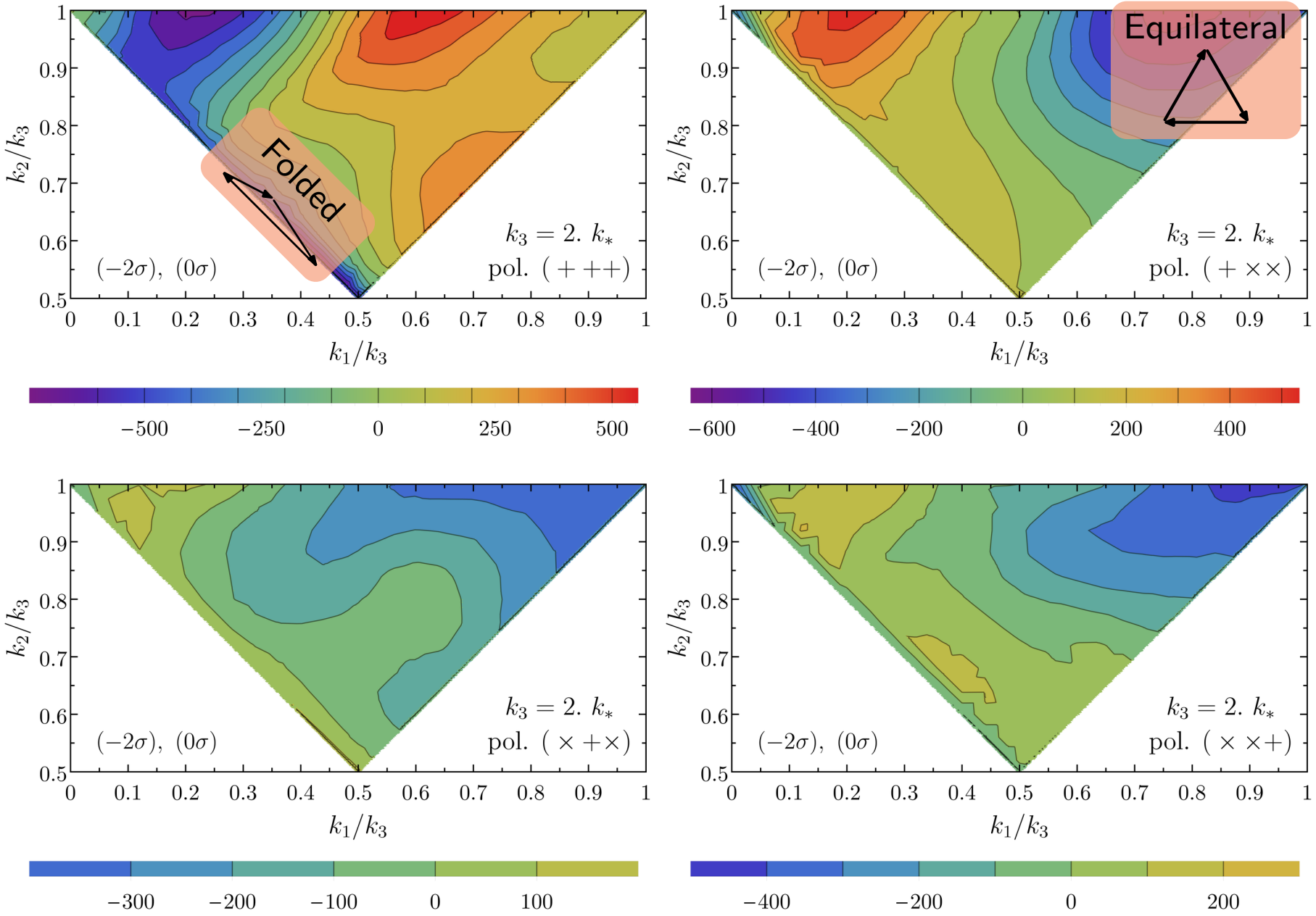


Power spectrum of Gravitational Waves

$$\Omega_{\mathsf{GW}}(f) \simeq 3 \cdot 10^{-8} \, (f/f_*)^{n_T}$$



Bispectrum of Gravitational Waves



Conclusions

- The implications of the metastability of the Higgs potential could shed light on BSM Physics thanks to Cosmology.
- How could we observe indirectly its presence?
- If the Higgs probed the unstable region at the end of inflation and was rescued back by reheating, large fluctuations at small scales would be generated.
  $\Longrightarrow$ PBHs and GWs could be an outcome.
- Today we have the possibility to probe extensively PBHs as a possible explanation for Dark Matter, and an exciting era for GWs is ahead of us.

Merci pour votre attention!

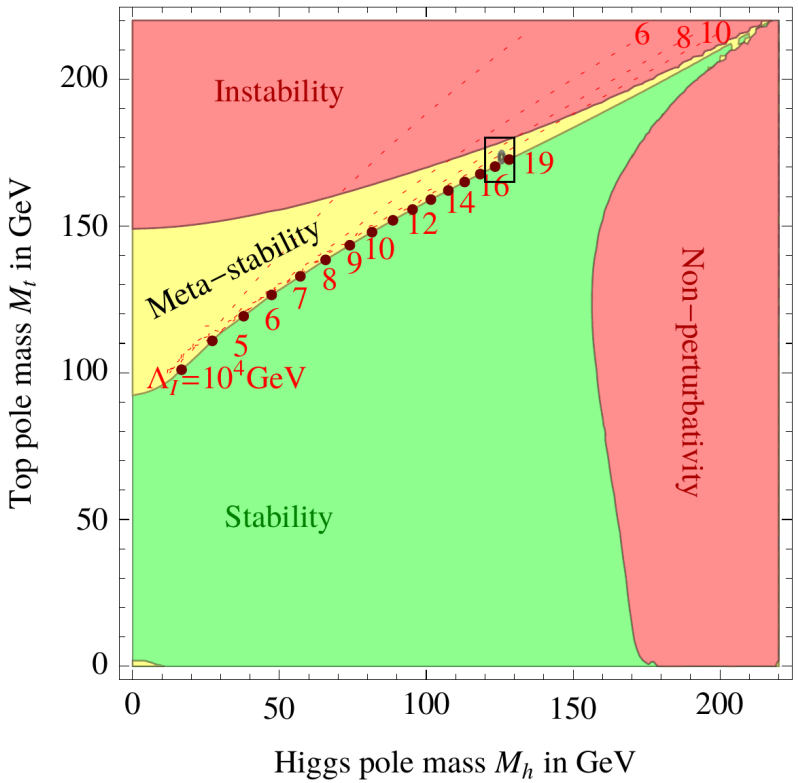
BACKUP SLIDES

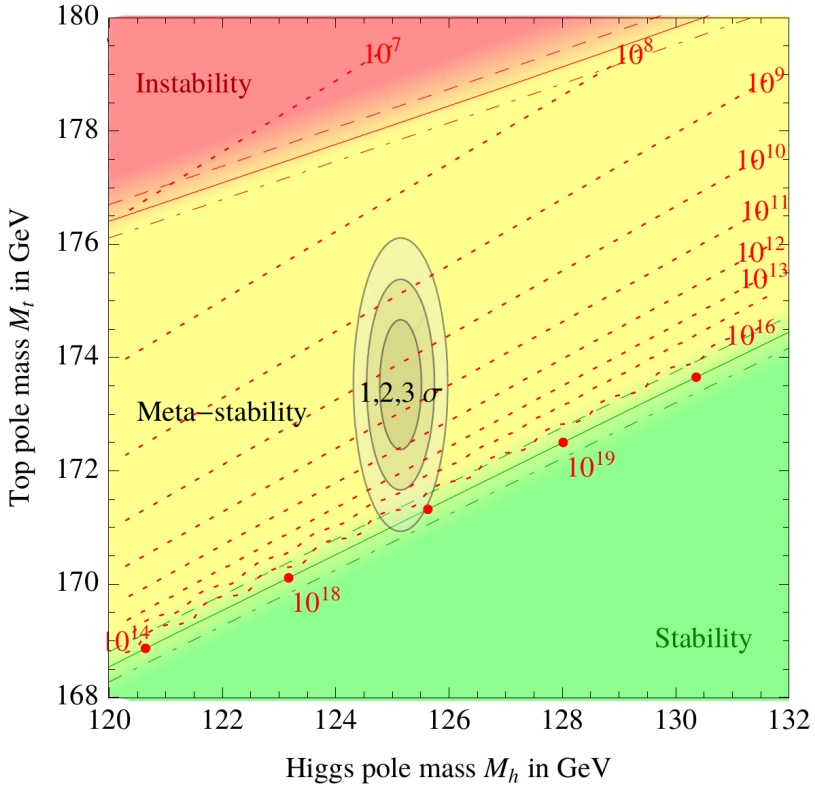
1. Generation of Primordial Black Holes
2. Generation of Gravitational Waves
3. Homogeneity and fine-tuning

Implications of living in a false vacuum

Tunnelling today

Negligible probability, today we are safe. We live in a metastable Universe.





[1307.3536 Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia]

Higgs instability and quantum fluctuations

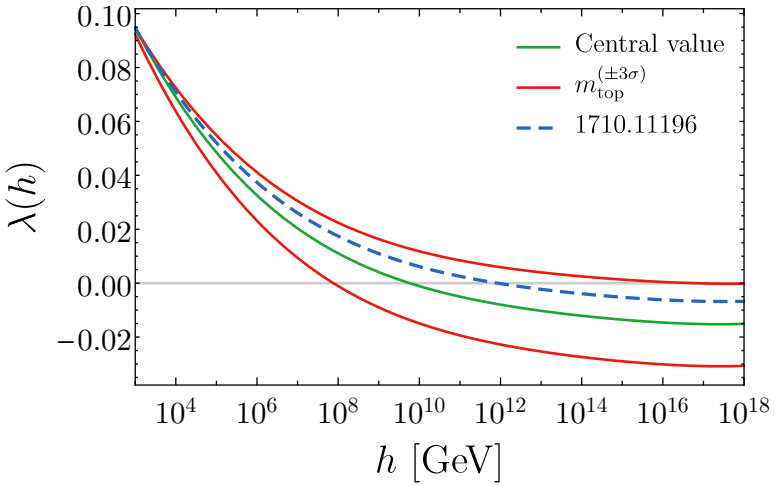
- We assume that $V(h) < 0$ at some scale.
- During inflation, the background field $h_c(t)$ has random fluctuations

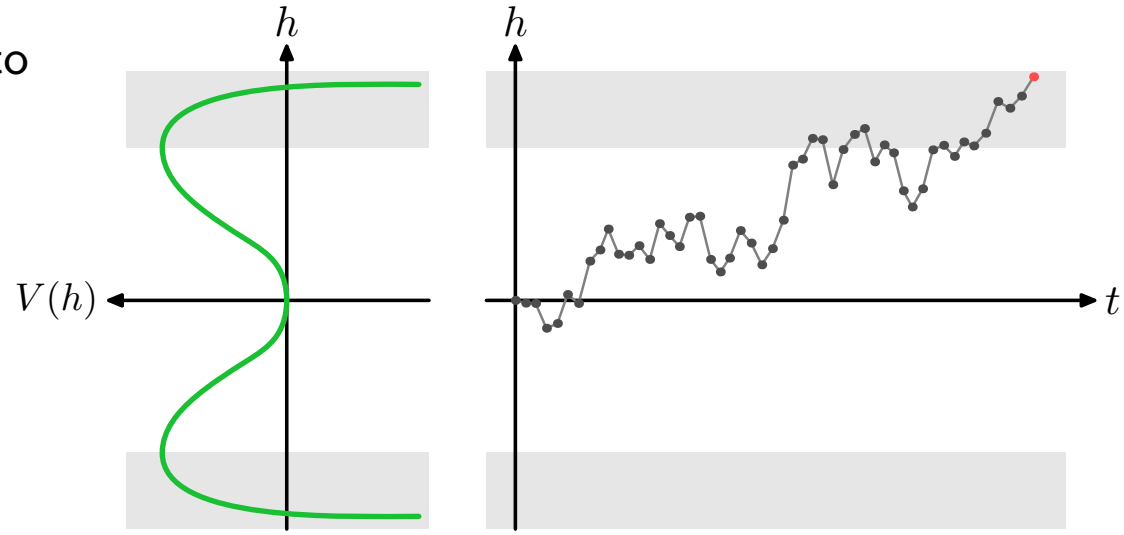
$$\Delta_q h_c \sim \pm \frac{H}{2\pi} \, .$$

- If classical evolution prevails over quantum fluctuations, h_c begins to slow roll down the negative potential.

$$\overbrace{\dot{h}_c \Delta t \sim \frac{V'(h_c)}{3H^2}}^{\text{classical}} \gtrsim \overbrace{\frac{H}{2\pi}}^{\text{quantum}}$$

- From this starting point t_* we follow the classical evolution of h_c.





Fluctuations of the Higgs field

- Define the fluctuations of the Higgs, $h(t, \mathbf{x}) = h_c(t) + \delta h(t, \mathbf{x})$.

$$\delta \ddot{h}_k + 3H\delta \dot{h}_k + \frac{k^2}{a^2}\,\delta h_k + V''(h_c)\,\delta h_k = 0$$

Sub-Hubble

Oscillation term dominates.

Super-Hubble

After the Hubble crossing $k \sim aH$, which happens at t_k, dominates.
$V''(h_c) < 0 \Rightarrow$ source term which drives δh_k.

δh_k grows as $\dot{h}_c$

- We can find a relation between δh_k and h_c:

e.o.m. for δh_k (super-Hubble)	$\delta\ddot{h}_k + 3H\delta\dot{h}_k + V''(h_c)\delta h_k = 0$
e.o.m. for h_c	$\ddot{h}_c + 3H\dot{h}_c + V'(h_c) = 0$
$\to \mathrm{d}/\mathrm{d}t$	$\left(\dot{h}_c\right)^{\cdot\cdot} + 3H\left(\dot{h}_c\right)^{\cdot} + V''(h_c)\,\dot{h}_c = 0$

- δh_k at super-Hubble scales follows the same evolution of $\dot{h}_c$, they are proportional:

$$\delta h_k(t) = C(k)\,\dot{h}_c(t)$$

- The constant $C(k)$ is found by matching with the standard sub-Hubble wave solution at t_k:

$$C(k) = \frac{H}{\dot{h}_c(t_k)\sqrt{2k^3}}\,.$$

- Fluctuations of the Higgs that leave the Hubble radius towards the end of inflation grow a lot.

Comoving curvature perturbation ζ

- Perturbations are quantified by the gauge-invariant *comoving curvature perturbation* ζ, which is conserved on super-horizon scales.
- In flat gauge,

$$\zeta = H\frac{\delta\rho_{\text{tot}}}{\dot{\rho}} = \underbrace{\frac{\rho_{\text{st}}}{\rho_{\text{tot}}}\zeta_{\text{st}}}_{\text{standard inflation contr.}} + \underbrace{H\frac{\delta\rho_h}{\dot{\rho}_{\text{tot}}}}_{\text{contr. from } h \text{ at small } k}$$

$$\delta\rho_h = \rho(h_{\text{c}} + \delta h) - \rho(h_{\text{c}})$$

- On super-horizon scales, using the e.o.m. we find

$$\zeta_{h_k}(k \gtrsim k_*) = \Big(H\,C(k)\Big)\,.$$

- The largest contribution to ζ, in the last e-folds of inflation, comes from the Higgs.

Generation of PBH

$$k \leftrightarrow M_{\rm PBH} \rightarrow \mathcal{P}_\zeta(k) \rightarrow \Delta(t, \mathbf{x}) \rightarrow \sigma_\Delta(M) \rightarrow \beta(M) \rightarrow \Omega_{\rm PBH}$$

Generation of PBH

$k \leftrightarrow M_{\rm PBH}$ $\rightarrow$ $\mathcal{P}_\zeta(k)$ $\rightarrow$ $\Delta(t,\mathbf{x})$ $\rightarrow$ $\sigma_\Delta(M)$ $\rightarrow$ $\beta(M)$ $\rightarrow$ $\Omega_{\rm PBH}$

- $M_{\rm PBH}$: mass contained in a sphere of volume $\sim H^{-3}$ at the time when k re-enters the Hubble radius.

$$M_{\rm PBH} = \underbrace{\gamma}_{\sim 0.2} \frac{4\pi}{3}\,\rho\, H^{-3} \approx 10^{-15} M_\odot \left[\frac{k}{(10^{-14}\ \text{Mpc})^{-1}}\right]^{-2}$$

$$\overset{H\sim 10^{12}\ \text{GeV}}{\approx} M_\odot\, e^{2(N-36)}\,.$$

Generation of PBH

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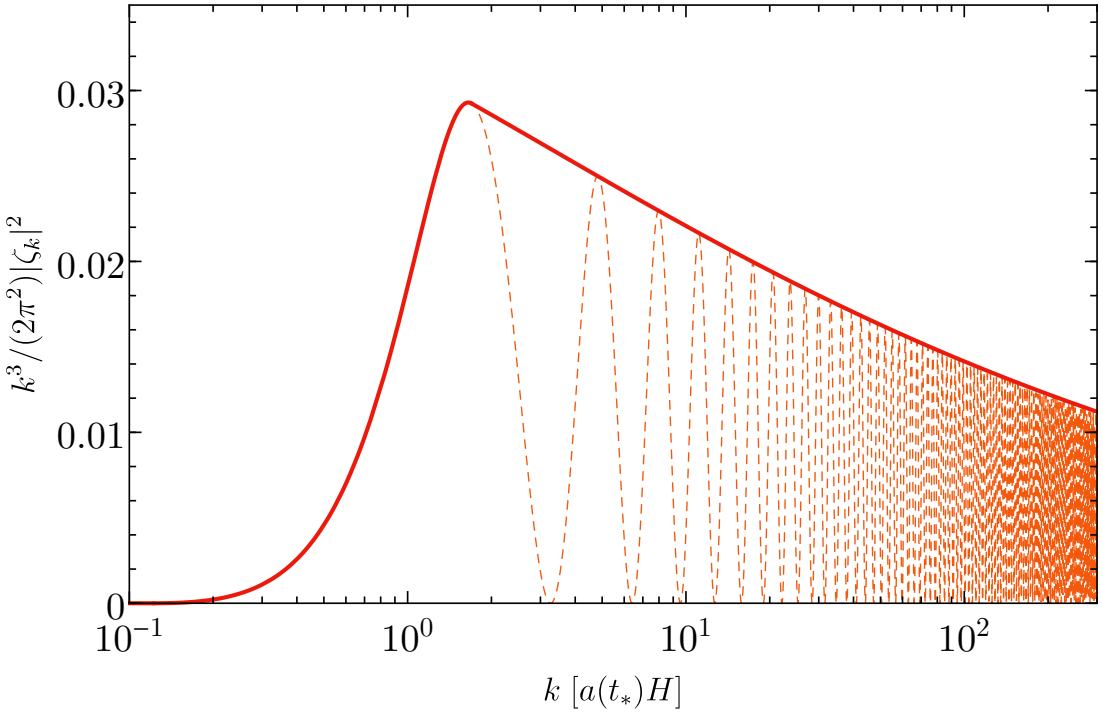
- Power spectrum $\mathcal{P}_\zeta = \dfrac{k^3}{2\pi^2} |\zeta_k|^2$ on small scales $k \gtrsim k_*$: much larger than the usual $\mathcal{O}(10^{-9})$ of CMB scales.



Generation of PBH

$k \leftrightarrow M_{\rm PBH}$ $\rightarrow$ $\mathcal{P}_\zeta(k)$ $\rightarrow$ $\boldsymbol{\Delta(t,\mathbf{x})}$ $\rightarrow$ $\sigma_\Delta(M)$ $\rightarrow$ $\beta(M)$ $\rightarrow$ $\Omega_{\rm PBH}$

- The density contrast is the relevant quantity ['14 Young, Byrnes, Sasaki]

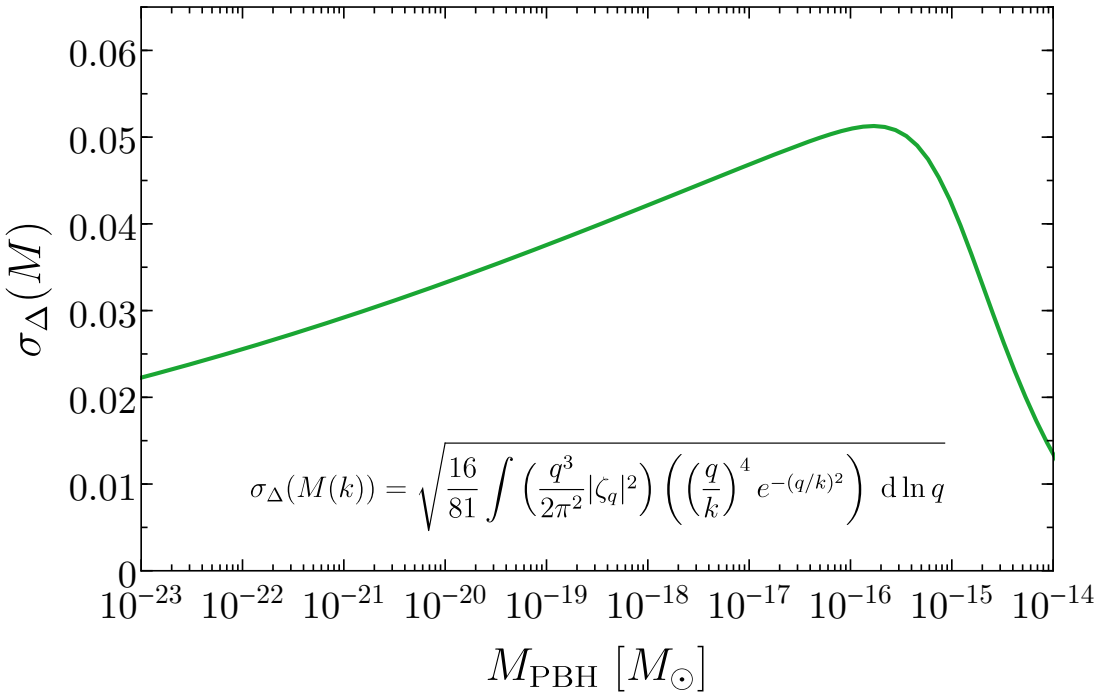
$$\Delta(t,\mathbf{x}) = \frac{4}{9}\left(\frac{1}{aH}\right)^2 \nabla^2\zeta(\mathbf{x})\,,$$

  physically corresponding to spatial curvature of the metric.
- Threshold for collapse depends on full spatial profile of $\Delta(t,\mathbf{x})$. ['18 Yoo, Harada, Garriga, Kohri; '18 Germani, Musco]
- Approximation: collapse happens when Δ crosses a threshold $\Delta_{\rm c} \approx 0.45$.

Generation of PBH

$k \leftrightarrow M_{\rm PBH} \rightarrow \mathcal{P}_\zeta(k) \rightarrow \Delta(t, \mathbf{x}) \rightarrow \sigma_\Delta(M) \rightarrow \beta(M) \rightarrow \Omega_{\rm PBH}$

- How likely it is for Δ to cross the threshold $\Delta_{\rm c}$?
- Approximation: Gaussian distribution, with variance σ_Δ.

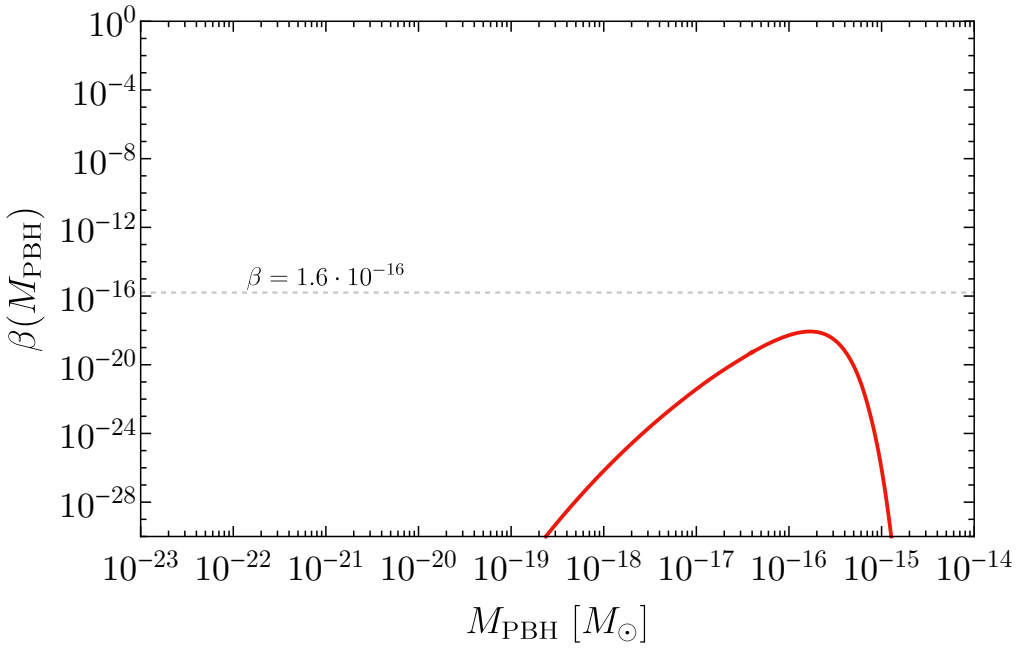


Generation of PBH

$k \leftrightarrow M_{\rm PBH} \rightarrow \mathcal{P}_\zeta(k) \rightarrow \Delta(t,\mathbf{x}) \rightarrow \sigma_\Delta(M) \rightarrow \beta(M) \rightarrow \Omega_{\rm PBH}$

- Formation rate: probability of exceeding threshold $\Delta_{\rm c}$.[a]

$$\beta(M) = \int_{\Delta_{\rm c}}^{\infty} \frac{{\rm d}\Delta}{\sqrt{2\pi}\,\sigma_\Delta(M)} \exp\left(-\frac{\Delta^2}{2{\sigma_\Delta}^2(M)}\right)$$



[a] Non-Gaussianities play an important role. ['18 Franciolini, Kehagias, Matarrese, Riotto]

Generation of PBH

$k \leftrightarrow M_{\rm PBH}$ $\rightarrow$ $\mathcal{P}_\zeta(k)$ $\rightarrow$ $\Delta(t, \mathbf{x})$ $\rightarrow$ $\sigma_\Delta(M)$ $\rightarrow$ $\beta(M)$ $\rightarrow$ $\Omega_{\rm PBH}$

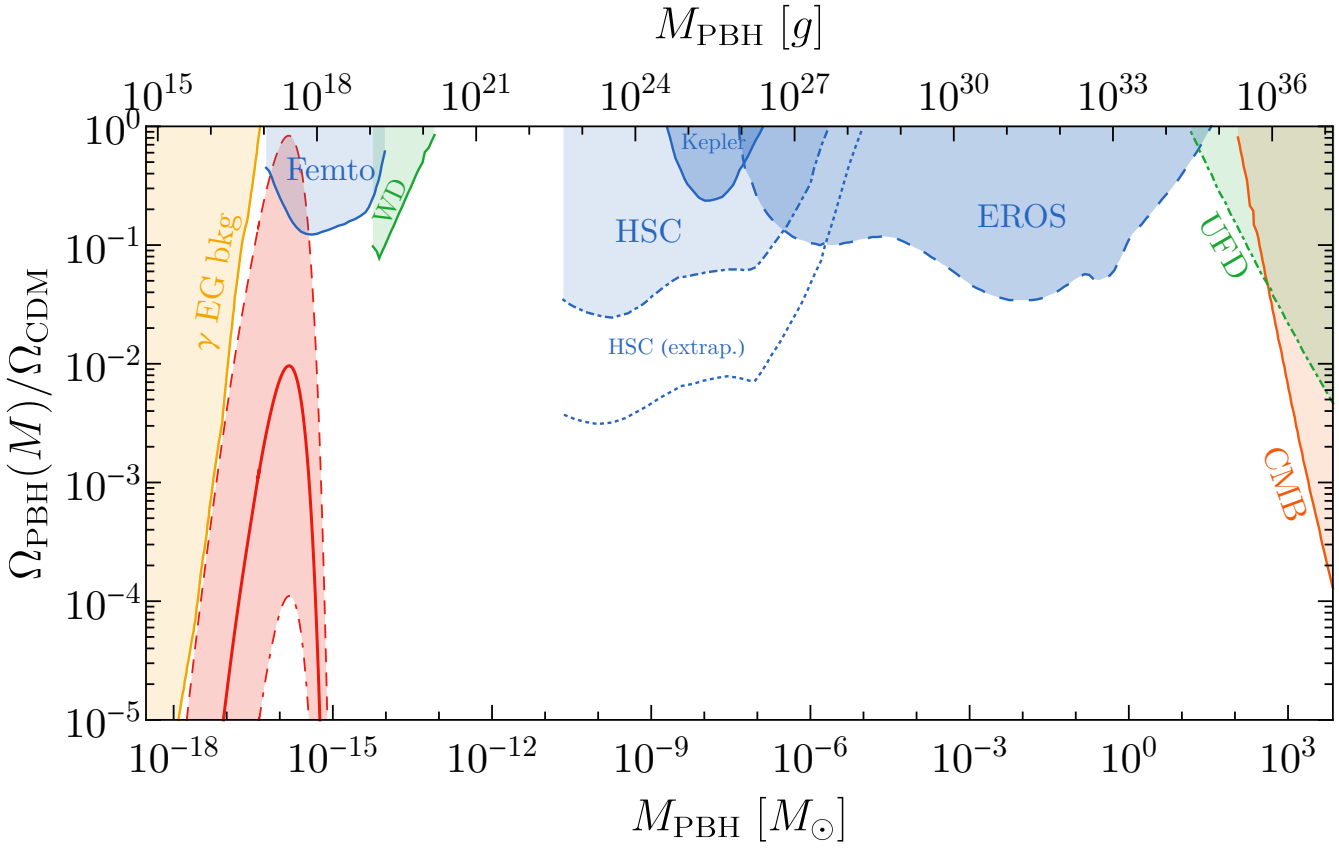
- After formation, $\rho_{\rm PBH} \sim a^{-3}$ and behave as collisionless CDM. After equality, they scale as the rest of matter.

$$f(M) \equiv \frac{\Omega_{\rm PBH}(M)}{\Omega_{\rm CDM}} = \frac{\beta(M)}{1.6 \cdot 10^{-16}} \left(\frac{\gamma}{0.2}\right)^{\frac{3}{2}} \left(\frac{g_*(T_f)}{106.75}\right)^{-\frac{1}{4}} \left(\frac{M}{10^{-15} M_\odot}\right)^{-\frac{1}{2}} .$$

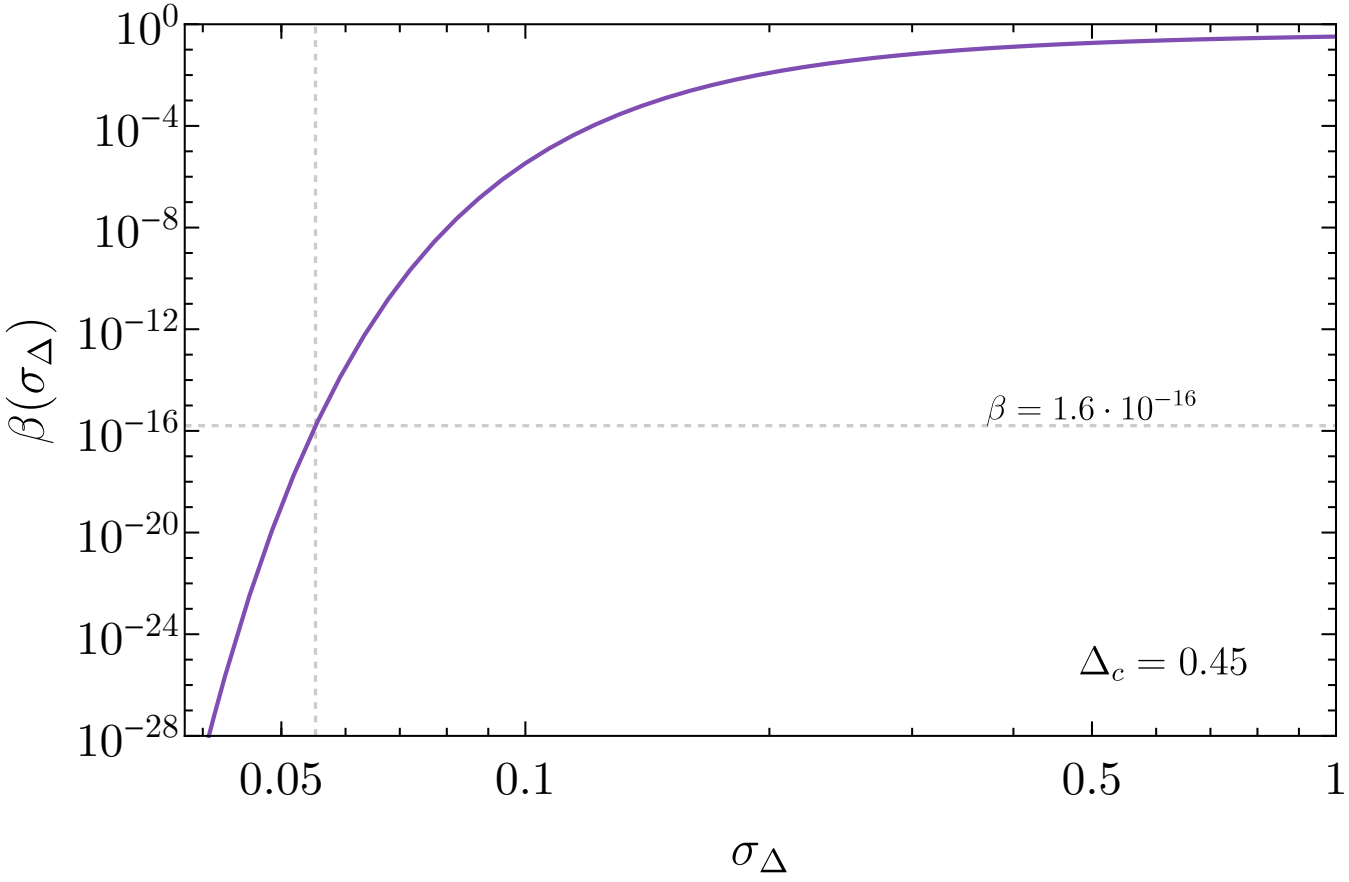
- For PBHs of mass $\sim 10^{-15} M_\odot$, the right abundance is achieved for $\sigma_\Delta \sim 0.05$.

Non-Gaussianity: account for skewness $S_3 \equiv \langle \Delta^3 \rangle / \sigma_\Delta^4$

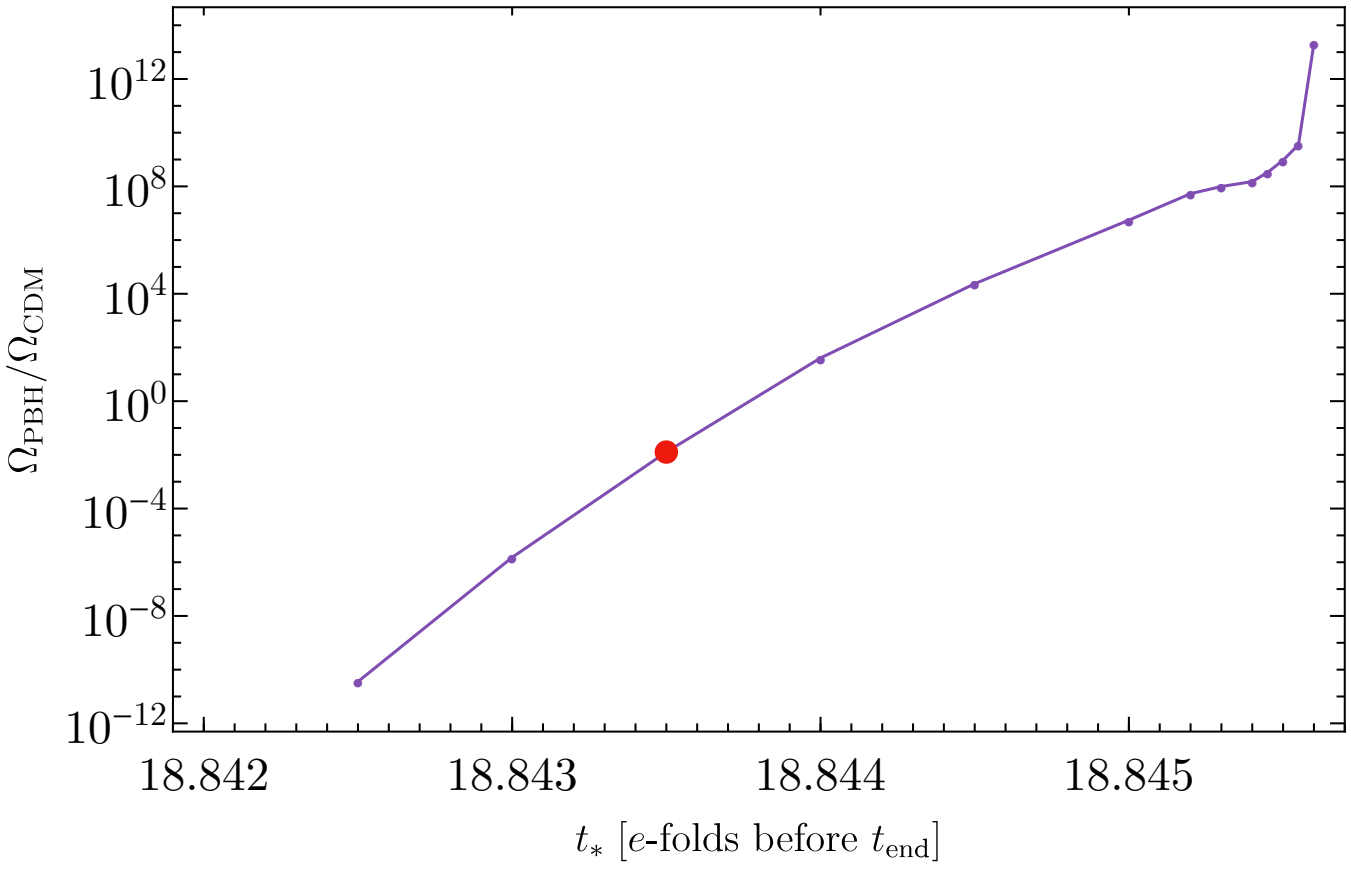
$$\beta(M) = \frac{1}{\sqrt{2\pi}\,\nu} \exp\left[-\frac{1}{2}\nu^2\left(1 - S_3\frac{\sigma_\Delta}{3}\left(\nu - 2 - \tfrac{1}{\nu^2}\right)\right)\right], \qquad \nu \equiv \frac{\Delta_c}{\sigma_\Delta}.$$



Formation rate as a function of σ_Δ



Fine tuning of $\Omega_{\rm PBH}$



Equation of motion for gravitational waves

Equation of motion for gravity waves:

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = -4\mathcal{T}_{ij}{}^{lm}\mathcal{S}_{lm}, \tag{1}$$

where the transverse projector and the source are

$$\widehat{\mathcal{T}}_{ij}{}^{lm}(\mathbf{k}) = e_{ij}^{(+)}(\mathbf{k})\, e^{(+)lm}(\mathbf{k}) + e_{ij}^{(\times)}(\mathbf{k})\, e^{(\times)lm}(\mathbf{k})\,, \tag{2}$$

$$\mathcal{S}_{ij} = 4\Psi\partial_i\partial_j\Psi + 2\partial_i\Psi\partial_j\Psi - \frac{4}{3(1+w)}\partial_i\left(\frac{\Psi'}{\mathcal{H}} + \Psi\right)\partial_j\left(\frac{\Psi'}{\mathcal{H}} + \Psi\right). \tag{3}$$

Solution with the Green function method in RD:

$$h^s_{\mathbf{k}}(\eta) = \frac{1}{a(\eta)}\int^{\eta} \mathrm{d}\widetilde{\eta}\, g_{\mathbf{k}}(\eta,\widetilde{\eta})\, a(\widetilde{\eta})\, \widehat{\mathcal{S}}^s(\widetilde{\eta},\mathbf{k}), \tag{4}$$

$$g_{\mathbf{k}}(\eta,\widetilde{\eta}) = \frac{\sin\left[k(\eta-\widetilde{\eta})\right]}{k}\,\theta(\eta-\widetilde{\eta})\,. \tag{5}$$

Transfer function $T(\eta,k)$ in RD:

$$\widehat{\Psi}(\eta,\mathbf{k}) = \frac{2}{3}T(\eta,k)\zeta(\mathbf{k}), \tag{6}$$

$$T(\eta,k) = \mathcal{T}(k\eta), \quad \mathcal{T}(z) = \frac{9}{z^2}\left[\frac{\sin(z/\sqrt{3})}{z/\sqrt{3}} - \cos(z/\sqrt{3})\right]. \tag{7}$$

Solution for gravitational waves

The solution reads

$$h^s_{\mathbf{k}}(\eta) = \frac{4}{9}\int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{k^3\eta}e^s(\mathbf{k},\mathbf{p})\zeta(\mathbf{p})\zeta(\mathbf{k}-\mathbf{p})\Big[\mathcal{I}_c(x,y)\cos(k\eta)+\mathcal{I}_s(x,y)\sin(k\eta)\Big] \tag{8}$$

where the polarisation tensor is

$$e^s(\mathbf{k},\mathbf{p}) \equiv e^{s,ij}(\mathbf{k})p_ip_j = \begin{cases} \frac{1}{\sqrt{2}}p^2\sin^2\theta\cos 2\phi & \text{for } s=(+), \\ \frac{1}{\sqrt{2}}p^2\sin^2\theta\sin 2\phi & \text{for } s=(\times), \end{cases} \tag{9}$$

and we have defined the dimensionless integrals

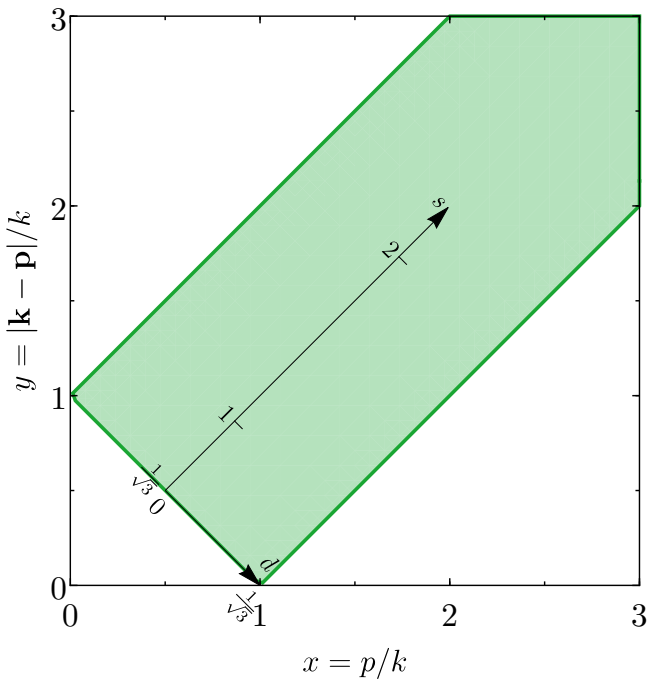
$$\begin{aligned} \mathcal{I}_c(x,y) &= \int_1^\infty \mathrm{d}\tau\,\tau(-\sin\tau)\cdot 4\Big\{2\mathcal{T}(x\tau)\mathcal{T}(y\tau)+\Big[\mathcal{T}(x\tau)+x\tau\,\mathcal{T}'(x\tau)\Big]\Big[\mathcal{T}(y\tau)+y\tau\,\mathcal{T}'(y\tau)\Big]\Big\}, \\ \mathcal{I}_s(x,y) &= \int_1^\infty \mathrm{d}\tau\,\tau(\cos\tau)\cdot 4\Big\{2\mathcal{T}(x\tau)\mathcal{T}(y\tau)+\Big[\mathcal{T}(x\tau)+x\tau\,\mathcal{T}'(x\tau)\Big]\Big[\mathcal{T}(y\tau)+y\tau\,\mathcal{T}'(y\tau)\Big]\Big\}. \end{aligned} \tag{10}$$

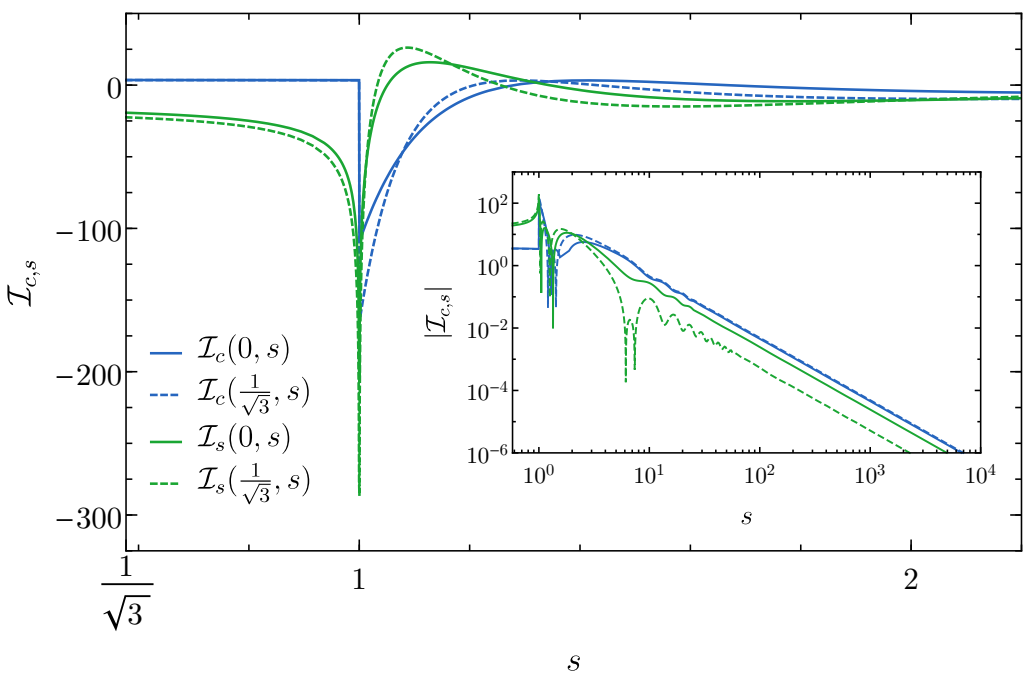
Solution for Gravitational Waves

$$h^s_{\mathbf{k}}(\eta) = \frac{4}{9}\frac{1}{k^3\eta}\int \frac{\mathrm{d}^3p}{(2\pi)^3}\,\zeta(\mathbf{p})\zeta(\mathbf{k}-\mathbf{p})\; e^s(\mathbf{k},\mathbf{p})\cdot$$

$$\cdot\left[\int_{k^{-1}}^{\eta} \mathrm{d}\widetilde{\eta}\Big(\text{Green \& Transfer functions}\Big)(p,|\mathbf{k}-\mathbf{p}|)\right]$$

1. $h \sim \mathcal{P}_\zeta$
2. Dimensionless integral over time that we compute analytically, as a function of $p/k, |\mathbf{k}-\mathbf{p}|/k$:





3. Integration over momenta

Power spectrum of Gravitational Waves

- From the expression for $h_{\mathbf{k}}$ we can compute

$$\langle h_{\mathbf{k}_1} h_{\mathbf{k}_2} \rangle \sim \langle \zeta\zeta\zeta\zeta \rangle$$

$$\langle h_{\mathbf{k}_1} h_{\mathbf{k}_2} h_{\mathbf{k}_3} \rangle \sim \langle \zeta\zeta\zeta\zeta\zeta\zeta \rangle$$

  which involve just an integration over momenta (2D and 3D numerical integrals).
- 2-pt function $\leftrightarrow$ Ω_{GW}, energy density of GW.
- Final result: $\Omega_{\text{GW}} \sim a^{-4}$ as radiation, so we write it in terms of $\Omega_{r,0}$ as

$$\Omega_{\text{GW}}(\eta_0, k) = \frac{\Omega_{r,0}}{972} \iint_{\mathscr{S}} \mathrm{d}x\, \mathrm{d}y \frac{x^2}{y^2} \left[1 - \frac{(1 + x^2 - y^2)^2}{4x^2} \right]^2 \cdot \\ \cdot \mathcal{P}_\zeta(kx)\, \mathcal{P}_\zeta(ky) \left[\mathcal{I}_c(x,y)^2 + \mathcal{I}_s(x,y)^2 \right]$$

Power spectrum of gravitional waves

The dimensionless power spectrum is defined as

$$\langle h^r(\eta, \mathbf{k}_1) h^s(\eta, \mathbf{k}_2)\rangle \equiv (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2)\, \delta^{rs}\, \frac{2\pi^2}{k_1^3} \mathcal{P}_h(k_1) \tag{11}$$

which gives a time-averaged power spectrum $\overline{\mathcal{P}_h(\eta, k)}$

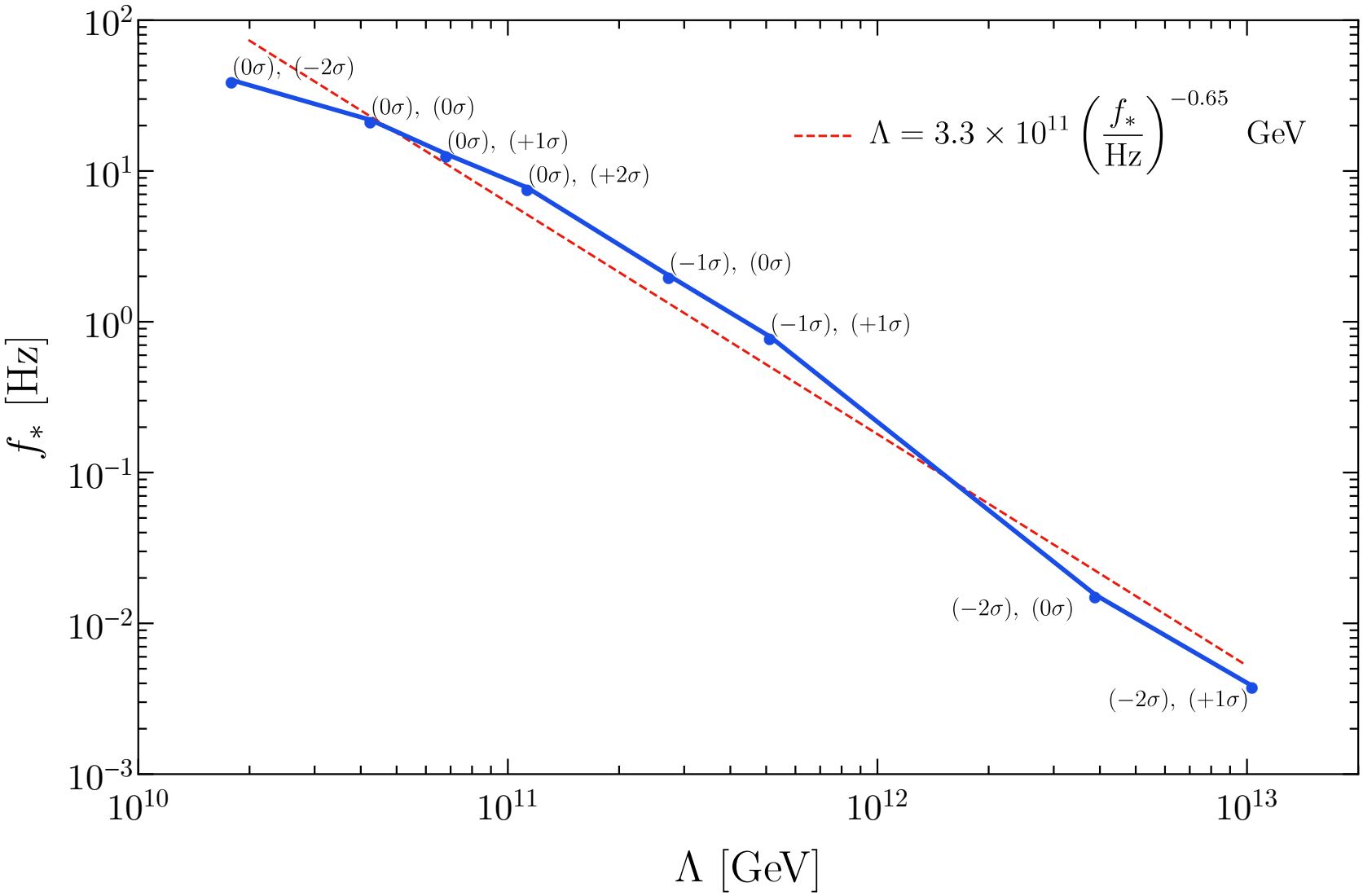
$$\overline{\mathcal{P}_h(\eta, k)} = \frac{2}{81} \frac{1}{k^2\eta^2} \iint_{\mathscr{S}} \mathrm{d}x\, \mathrm{d}y \frac{x^2}{y^2} \left[1 - \frac{(1 + x^2 - y^2)^2}{4x^2}\right]^2 \cdot$$
$$\cdot\, \mathcal{P}_\zeta(kx) \mathcal{P}_\zeta(ky) \sqrt{\mathcal{I}_c(x, y)^2 + \mathcal{I}_s(x, y)^2}\,. \tag{12}$$

The corresponding energy density reads

$$\Omega_{\mathsf{GW}}(\eta_0, k) = \Omega_{r,0}\, \Omega_{\mathsf{GW}}(\eta_f, k) = \frac{\Omega_{r,0}}{24} \frac{k^2}{\mathcal{H}(\eta_f)^2} \overline{\mathcal{P}_h(\eta_f, k)}\,. \tag{13}$$

"Hearing" the instability scale

The peak f_* is sensitive to m_{Higgs}, m_{top} and thus to the instability scale Λ.



Bispectrum of Gravitational Waves

- From the 3-pt function of GW we introduce the bispectrum B_h

$$\left\langle h^r(\eta,\mathbf{k}_1)h^s(\eta,\mathbf{k}_2)h^t(\eta,\mathbf{k}_3)\right\rangle \equiv (2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\, B_h^{rst}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$$

- The dimensionless normalised shape $S_h^{rst}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$

$$S_h^{rst}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = k_1^2k_2^2k_3^2\frac{B_h^{rst}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)}{\sqrt{\mathcal{P}_h(k_1)\mathcal{P}_h(k_2)\mathcal{P}_h(k_3)}}$$

  identifies the strength of the bispectrum signal.
- $S_h(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$ is a "fingerprint" of the GW signal.

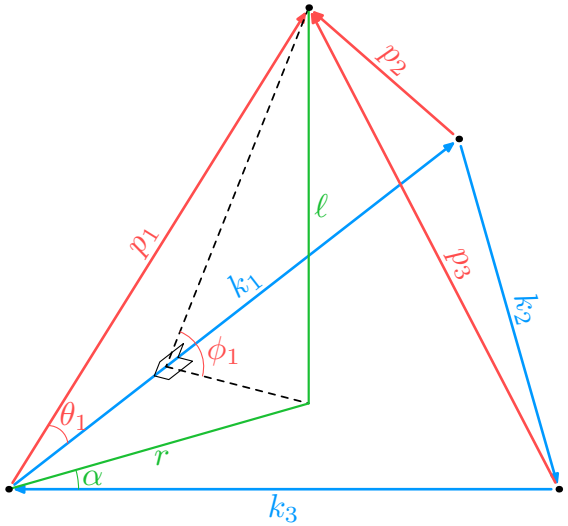
Bispectrum of gravitional waves

The bispectrum is defined through

$$\left\langle h^r(\eta,\mathbf{k}_1)h^s(\eta,\mathbf{k}_2)h^t(\eta,\mathbf{k}_3)\right\rangle \equiv (2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\, B_h^{rst}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3). \tag{14}$$

and its envelope over time reads

$$\begin{aligned} B_h^{rst}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = 8\left(\frac{4}{9}\right)^3\pi^3\int \mathrm{d}^3p_1\,\frac{1}{k_1^3k_2^3k_3^3\eta^3}\,e^r(\mathbf{k}_1,\mathbf{p}_1)e^s(\mathbf{k}_2,\mathbf{p}_2)e^t(\mathbf{k}_3,\mathbf{p}_3)\cdot \\ \cdot\,\frac{\mathcal{P}_\zeta(p_1)}{p_1^3}\,\frac{\mathcal{P}_\zeta(p_2)}{p_2^3}\,\frac{\mathcal{P}_\zeta(p_3)}{p_3^3}\sqrt{\mathcal{I}_c\left(\tfrac{p_1}{k_1},\tfrac{p_2}{k_1}\right)^2+\mathcal{I}_s\left(\tfrac{p_1}{k_1},\tfrac{p_2}{k_1}\right)^2}\cdot \\ \cdot\sqrt{\mathcal{I}_c\left(\tfrac{p_2}{k_2},\tfrac{p_3}{k_2}\right)^2+\mathcal{I}_s\left(\tfrac{p_2}{k_2},\tfrac{p_3}{k_2}\right)^2}\sqrt{\mathcal{I}_c\left(\tfrac{p_3}{k_3},\tfrac{p_1}{k_3}\right)^2+\mathcal{I}_s\left(\tfrac{p_3}{k_3},\tfrac{p_1}{k_3}\right)^2} \end{aligned} \tag{15}$$



$$\int \mathrm{d}^3p_1 \longrightarrow \int_{-\infty}^{+\infty}\mathrm{d}\ell\int_0^{+\infty} r\,\mathrm{d}r\int_0^{2\pi}\mathrm{d}\alpha\,,$$

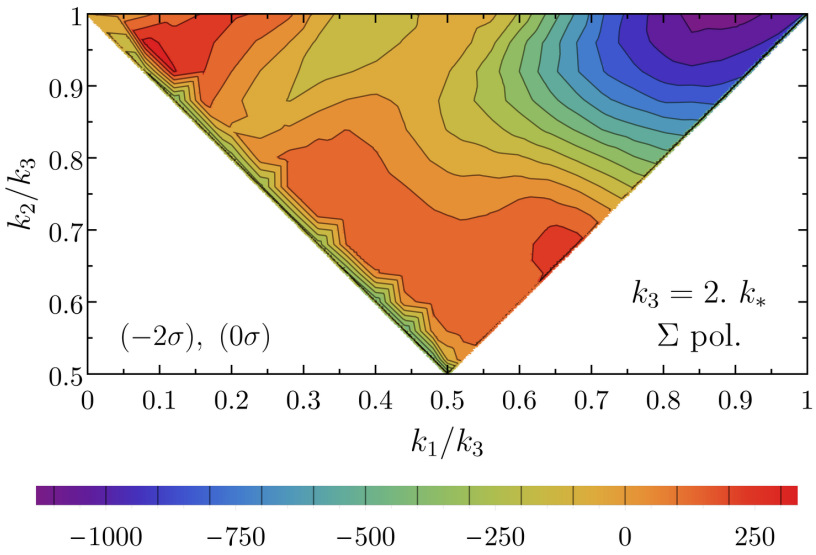
$$\mathbf{p}_1 = (r\cos\alpha, r\sin\alpha, \ell)\,,$$

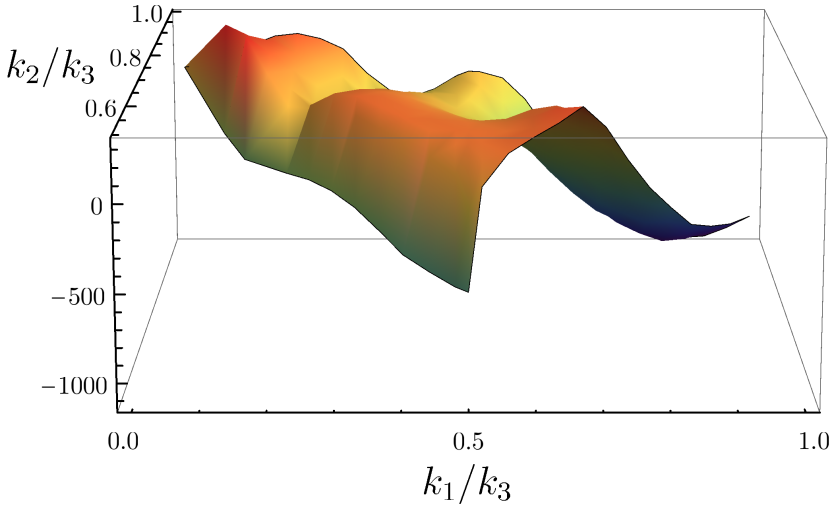
$$\mathbf{p}_2 = (-k_{1x}+r\cos\alpha, -k_{1y}+r\sin\alpha, \ell)\,,$$

$$\mathbf{p}_3 = (k_3+r\cos\alpha, r\sin\alpha, \ell)\,,$$

$$p_i^2\sin^2\theta_i = p_i^2 - \frac{|\mathbf{p}_i\cdot\mathbf{k}_i|^2}{k_i^2}, \qquad \sin\phi_i = \frac{\ell\, k_i}{|\mathbf{p}_i\times\mathbf{k}_i|}.$$

Bispectrum of Gravitational Waves





Bispectrum of Gravitational Waves

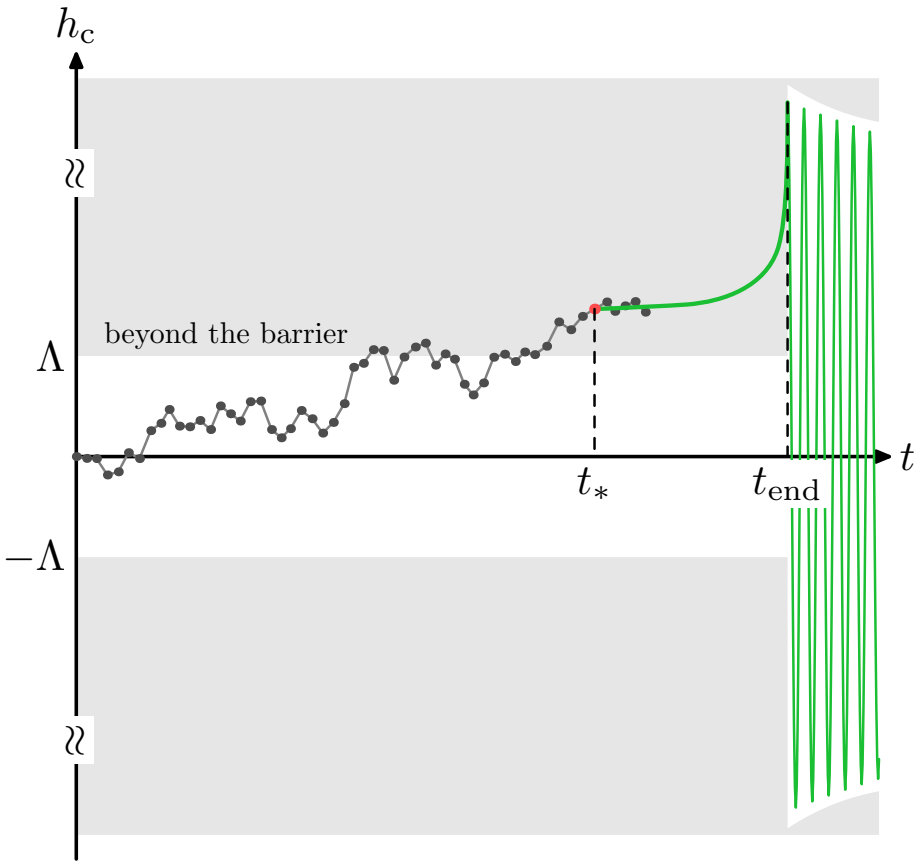
- The shape of the bispectrum peaks for momenta of order of $k_* = a(t_*)H$, and for folded and equilateral configurations:

$$\boxed{\begin{array}{ll} S_h^{+++} = \mathcal{O}(-600) & \text{for folded configurations,} \\ S_h^{+\times\times} = \mathcal{O}(-600) & \text{for equilateral configurations,} \\ \sum_{\text{pol}} S_h = \mathcal{O}(-1000) & \text{for equilateral configurations.} \end{array}}$$

- In case of detection of the 3-pt function, its shape could help to identify the origin of the signal.

Approximate homogeneity on exponentially large scales

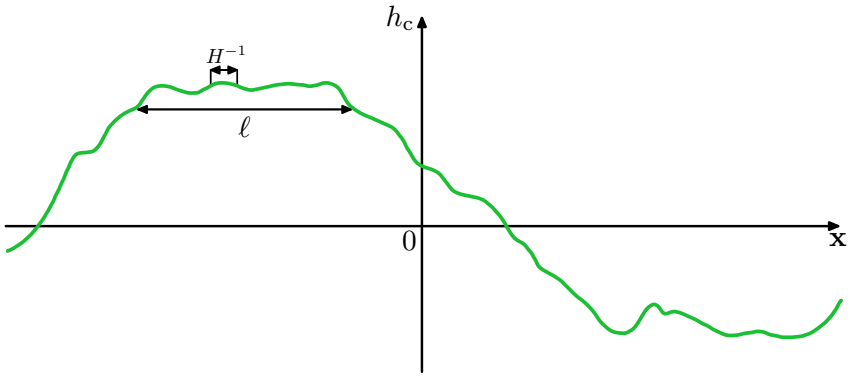
- How large can be the region we are describing?



- A scalar field has similar values on scales as large as the *particle horizon*:

$$\text{correlation length} \sim H^{-1} \exp H t_i$$

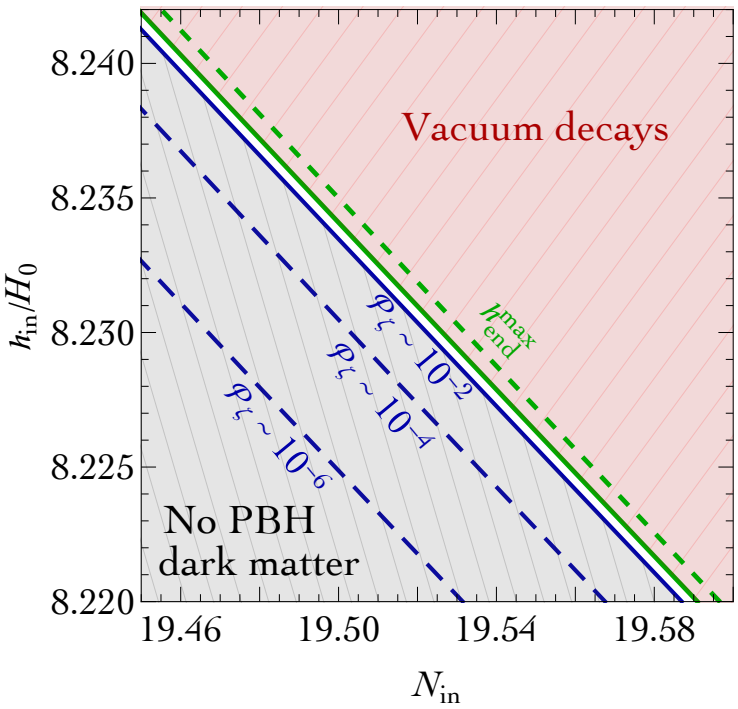
- We can assume that h_c has nearly the same value on a patch larger than our observable Universe.



[Linde, *Particle Physics and Inflationary Cosmology*, 1990]

Inexact homogeneity: small fluctuations on scales H^{-1}

- Small fluctuations of order $H/(2\pi) \sim 0.16H$ occur on a scale H^{-1}.
- These are larger than the precision required on the initial value h_*.
- A small overshooting on h_* prevents the reheating from rescuing h, leaving an expanding AdS region.



[1803.10242 Gross, Polosa,Strumia, Urbano, Xue]

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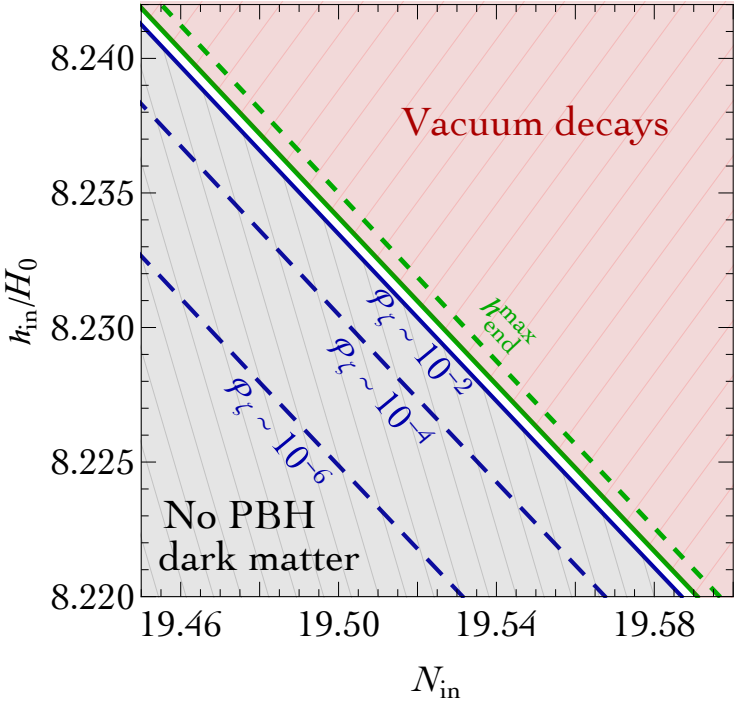

[1803.10242 Gross, Polosa,Strumia, Urbano, Xue]

A scenario without AdS regions at all

- We do not need the same $\mathcal{P}_\zeta$ everywhere: inhomogeneities in the PBH distribution are OK, given that they are generated strongly clustered.
- For AdS regions it's different: they can't be in our past lightcone.
- In a multiverse, "small" probability does not mean much, but it is true that the lifetime of anthropically selected regions could be short.
- If $\lambda(h)$ returns positive at some high scale, the mechanism can still work and AdS regions are avoided.

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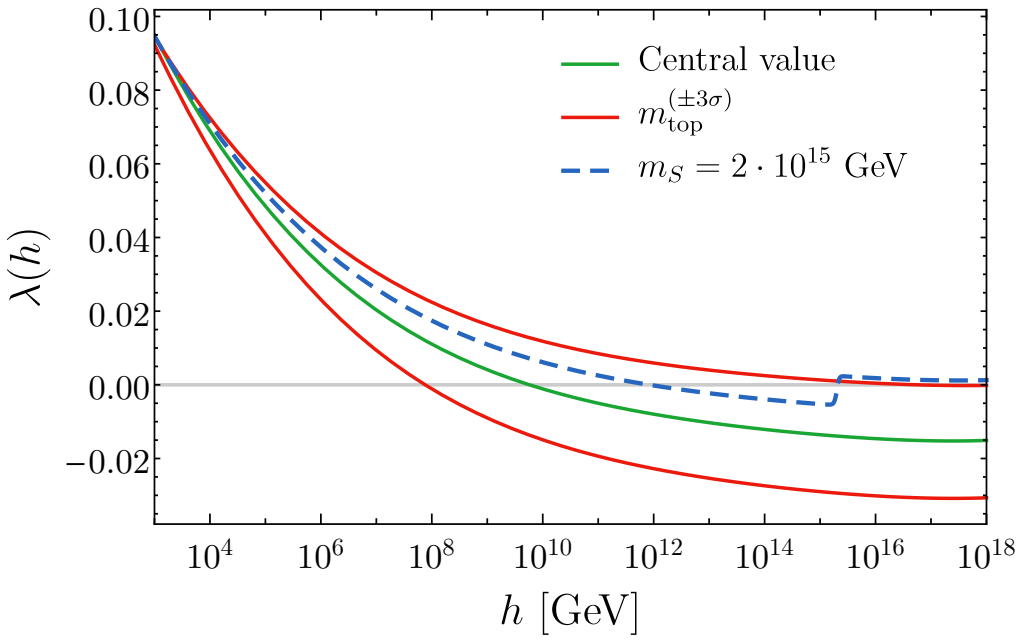

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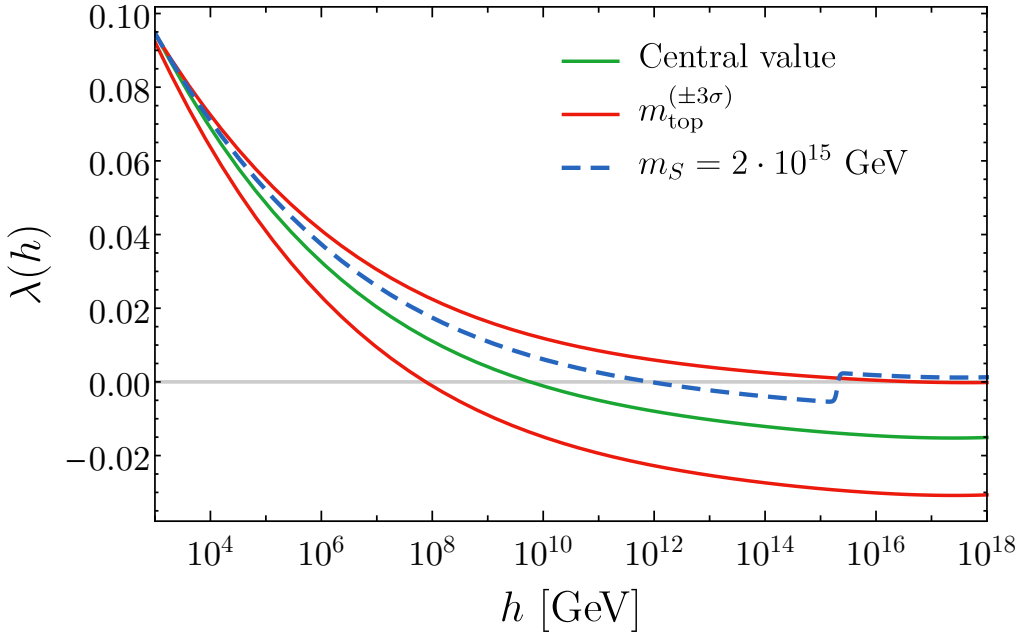
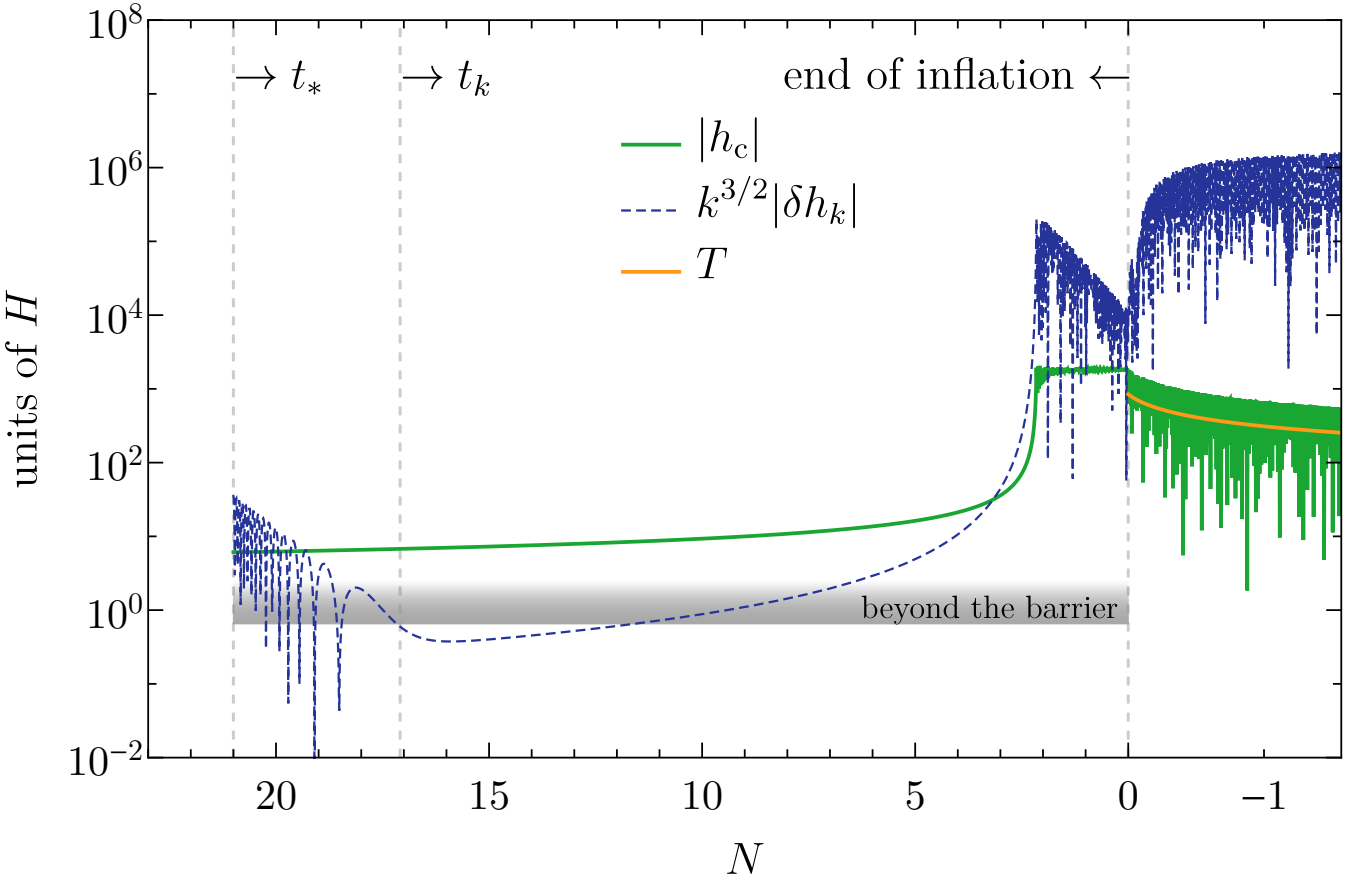


- Example of BSM contributions which can lift $\lambda(h)$ to positive values: extra scalar S interacting with the Higgs, yielding a positive contribution to λ above m_S [1203.0237, Elias-Miró, Espinosa, Giudice, Lee, Strumia].
- Higgs potential has an absolute minimum at $\sim m_S$.
- Assume $m_S \lesssim T_{\rm RH}$, so that the Higgs is always rescued at reheating. No fine-tuning here: we can allow variations $\sim (10-20)\%$ for m_S, not to spoil the PBH generation.

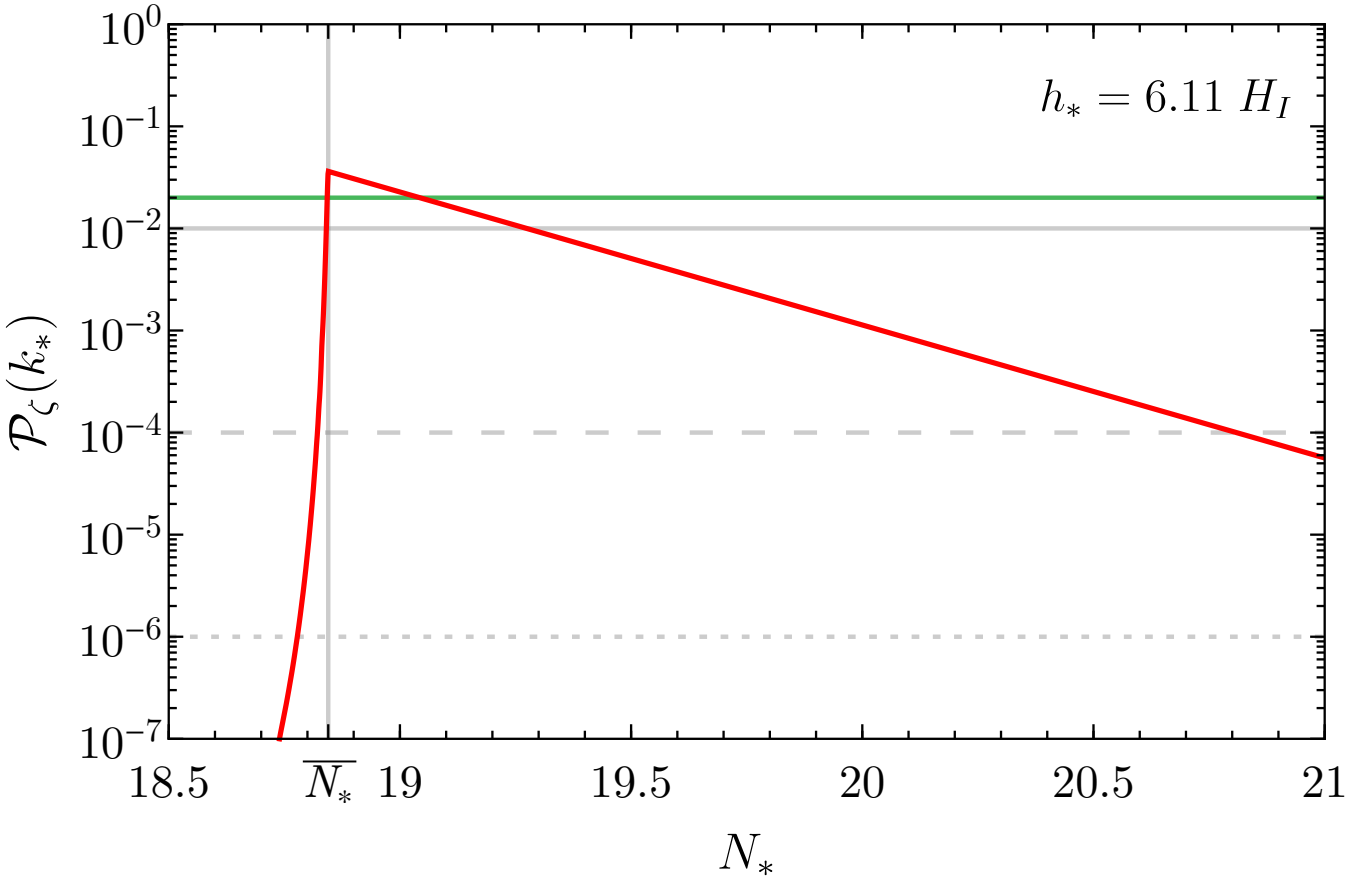
Evolution in the presence of an absolute minimum

- If h_c undershoots (starts too late, or too low), nothing changes.
- If h_c overshoots (starts too early, or too high), it reaches the minimum, where it oscillates. During this phase h and δh_k slowly decrease as matter.

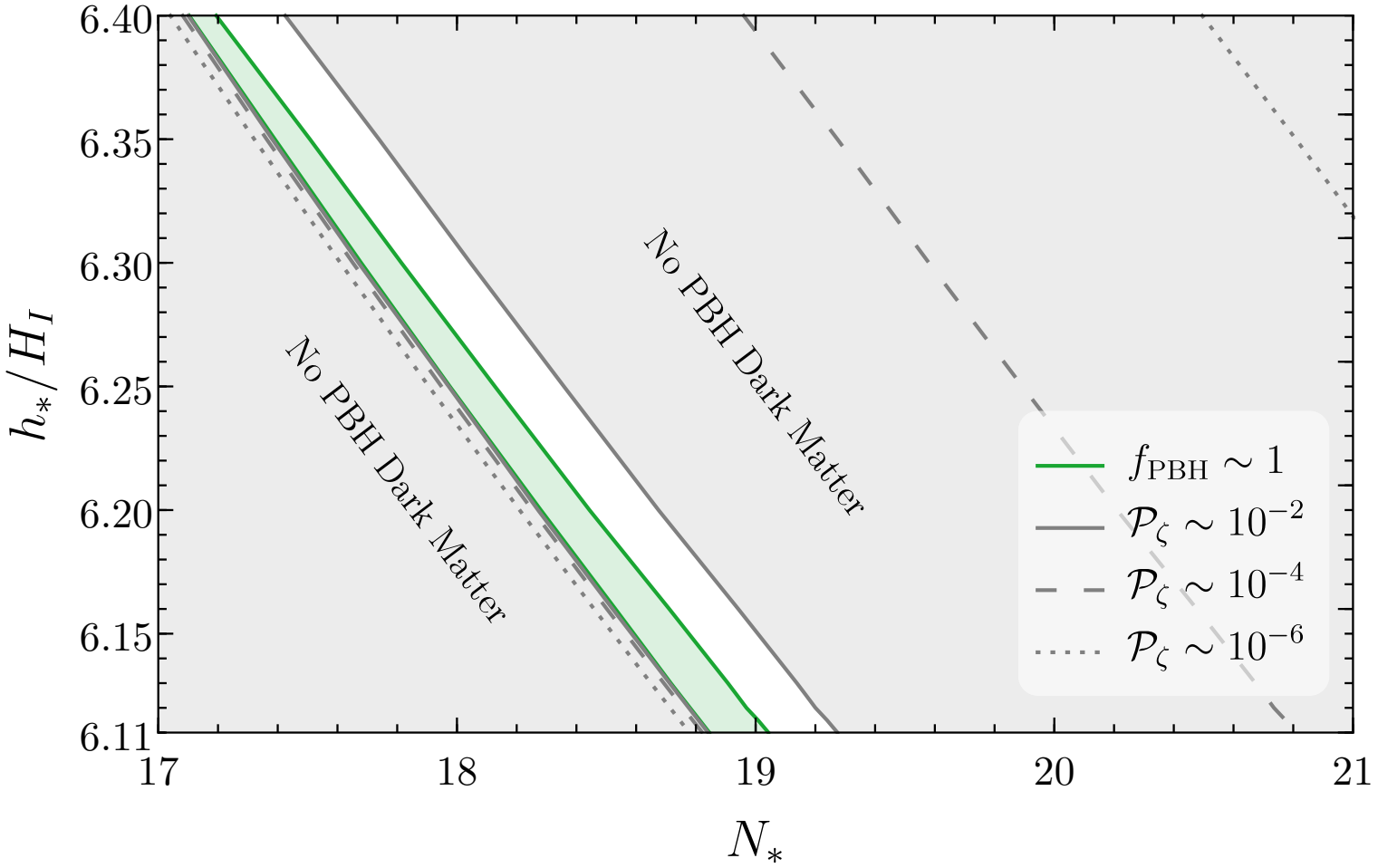


Evolution in the presence of an absolute minimum

- Imagine to fix h_* and vary N_*. What is the final $\mathcal{P}_\zeta$?



Evolution in the presence of an absolute minimum



SM + an extra scalar S

- As an example of BSM contributions which can lift $\lambda(h)$ to positive values, consider a scalar S interacting with the SM Higgs through [1203.0237, Elias-Miró, Espinosa, Giudice, Lee, Strumia]

$$V = \lambda_S \left(|S|^2 - \frac{\omega^2}{2} \right)^2 + 2\lambda_{HS} \left(|S|^2 - \frac{\omega^2}{2} \right) \left(|\varphi_H|^2 - \frac{v^2}{2} \right)$$

- Most general Lagrangian preserving a global $U(1)$ symmetry, can arise in various BSM extensions.
- S takes a large vev $\omega/\sqrt{2}$, and $m_S = \sqrt{\lambda_S}\omega$. When integrating out S at scales below m_S, one finds

$$V_{\text{eff}}(h) = \left(\lambda_H - \frac{\lambda_{HS}^2}{\lambda_S} \right) \left(|\varphi_H|^2 - \frac{v^2}{2} \right)^2 .$$

- For $h > m_S$, λ jumps up of a factor $\delta\lambda \sim \dfrac{\lambda_{HS}^2}{\lambda_S}$.
- Only requirement for λ_{SH} and λ_S is that they yield $\lambda(h \gtrsim m_S) > 0$, corresponding to $|\delta\lambda| \gtrsim 0.008$.