Cosmological Signatures of the SM Higgs Instability: Primordial Black Holes and Gravitational Waves

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Higgs potential beyond the tree level

[ '79 Cabibbo et al.; '89 Sher; '94 Altarelli, Isidori; '96 Casas, Espinosa, Quirós; '07 Espinosa, Giudice, Riotto; '12 Degrassi et al.; '15 Espinosa et al.; '16 East et al.; ... ]

Tree level potential

\[ V(h) = \frac{1}{4} \lambda h^4 - \frac{1}{2} \mu^2 h^2 \]

RG improved potential

\[ V(h) = \frac{1}{4} \lambda(h) h^4 \]

Tunnelling today

Negligible probability, today we are safe. We live in a metastable Universe.
Implications of living in a false vacuum

During inflation

- Background of a scalar field in de Sitter makes quantum jumps $\sim \pm \frac{H}{2\pi}$.
- These fluctuations could lead the Higgs beyond the barrier, and make it roll towards the true vacuum.
- This vacuum has large negative energy $\Rightarrow$ AdS bubble, which can expand at the speed of light $\Rightarrow$ It didn’t happen in our past lightcone.

During reheating

- Higgs interacts with thermal bath of SM particles $\Rightarrow$ stabilisation of the potential

$$V(h) = \frac{1}{4} \lambda(h) h^4 + \frac{1}{2} m_T^2 h^2,$$

$$m_T^2 = 0.12 T^2 \exp \left( -\frac{h}{2\pi T} \right)$$

- If $T_{RH}$ is high enough and $h$ is not too far, thermal corrections can “rescue” the Higgs back to 0.
Implications of living in a false vacuum

### During inflation

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### During reheating

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- If $T_{RH}$ is high enough and $h$ is not too far, thermal corrections can “rescue” the Higgs back to 0.
Are there possible observational signatures of the Higgs instability?

- Tunnelling today: not here, until this afternoon.
  
  If this time is on the order of $10^9 \text{ yr}$, we have occasion for anxiety.
  
  This would be the appropriate case to study if we were currently living in a false vacuum whose apocalyptic decay is yet to occur. [Coleman]

- What if the Higgs probed the unstable region at the end of inflation, and was rescued back in time by thermal corrections at reheating?

- Two possible signatures:
  1. Primordial Black Holes as Dark Matter
  2. Background of Gravitational Waves
We assume that $V(h) < 0$ at some scale.

During inflation, the background field $h_c(t)$ has random fluctuations

$$\Delta_q h_c \sim \pm \frac{H}{2\pi}.$$
If classical evolution prevails over quantum fluctuations, $h_c$ begins to slow roll down the negative potential.
Evolution of the background of the Higgs

At the end of inflation, if the reheating temperature is high enough and Higgs has not gone too far down the potential, thermal corrections can rescue $h$ and bring it back around 0.

- Decay to radiation: in a very short time the Higgs field decays to radiation.
Evolution of Higgs background and fluctuations

\[
\begin{align*}
\text{end of inflation} & \quad \rightarrow t_* \quad \rightarrow t_k \\
|\bar{h}_c| & \quad k^{3/2} |\delta h_k| & \quad T
\end{align*}
\]

units of $H$

$N$
Power spectrum of curvature perturbations

$m_h = 125.09 \pm 0.24 \text{ GeV}$

$m_t = 172.47 \pm 0.5 \text{ GeV}$
Large overfluctuations, when they re-enter the Hubble radius, can collapse into a PBH.

\[ N = \ln \frac{a}{H^{-1}} \text{CMB} \approx 60 \approx 40-17 t_{\text{form}} \]

\[ k_{\text{PBH}}^{-1} \]

\[ t_{\text{end}} \]
**Position** of the peak: depends on \( t_* \), thus on the slope of \( V(h) \).

\[
m_{\text{Higgs}} = 125.09 \text{ GeV} \\
m_{\text{top}} = 172 \text{ GeV}
\]

**Height** of the peak: depends on how much the fluctuations grew. Finely tuned, as in any model for PBH formation.

In SM + inflation, DM is provided by this mechanism \( \implies \) anthropic explanations: without DM there would be no Large Scale Structures.
Tensor perturbations are excited in presence of large scalar perturbations when they cross the Hubble radius, then survive (redshifted) until today. This background of GWs can be detected by Advanced-LIGO, Einstein Telescope and especially LISA. We characterise its power spectrum and bispectrum.
Power spectrum of Gravitational Waves

\[ \Omega_{GW}(f) \simeq 3 \cdot 10^{-8} (f/f_\star)^{n_T} \]

\[ M_{\text{PBH}} [M_\odot] \]
Bispectrum of Gravitational Waves

Equilateral

Folded

\( k_3 = 2 \cdot k_* \)

\((-2\sigma), (0\sigma)\)

pol. \((+++)\)

\((-2\sigma), (0\sigma)\)

pol. \((+\times\times)\)

\((-2\sigma), (0\sigma)\)

pol. \((\times\times+)\)

\((-2\sigma), (0\sigma)\)

pol. \((\times\times+)\)
The implications of the metastability of the Higgs potential could shed light on BSM Physics thanks to Cosmology.

How could we observe indirectly its presence?

If the Higgs probed the unstable region at the end of inflation and was rescued back by reheating, large fluctuations at small scales would be generated.

PBHs and GWs could be an outcome.

Today we have the possibility to probe extensively PBHs as a possible explanation for Dark Matter, and an exciting era for GWs is ahead of us.
Merci pour votre attention!
1. Generation of Primordial Black Holes
2. Generation of Gravitational Waves
3. Homogeneity and fine-tuning
Implications of living in a false vacuum

Tunnelling today

Negligible probability, today we are safe. We live in a metastable Universe.

[1307.3536 Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia]
We assume that $V(h) < 0$ at some scale.

During inflation, the background field $h_c(t)$ has random fluctuations

$$\Delta_q h_c \sim \pm \frac{H}{2\pi}.$$ 

If classical evolution prevails over quantum fluctuations, $h_c$ begins to slow roll down the negative potential.

From this starting point $t_*$ we follow the classical evolution of $h_c$. 
Fluctuations of the Higgs field

- Define the fluctuations of the Higgs, $h(t, x) = h_c(t) + \delta h(t, x)$.

\[
\ddot{\delta h}_k + 3H\dot{\delta h}_k + \frac{k^2}{a^2} \delta h_k + V''(h_c) \delta h_k = 0
\]

Oscillation term dominates.

Sub-Hubble

Super-Hubble

After the Hubble crossing $k \sim aH$, which happens at $t_k$, dominates. $V''(h_c) < 0 \Rightarrow$ source term which drives $\delta h_k$. 

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Cosmological signatures of Higgs Instability: PBHs and GW
\( \delta h_k \) grows as \( \dot{h}_c \)

- We can find a relation between \( \delta h_k \) and \( h_c \):

  - e.o.m. for \( \delta h_k \) (super-Hubble)
    \[
    \ddot{\delta h}_k + 3H \dot{\delta h}_k + V''(h_c)\delta h_k = 0
    \]
  - e.o.m. for \( h_c \)
    \[
    \ddot{h}_c + 3H \dot{h}_c + V'(h_c) = 0
    \]

  - \( \frac{d}{d t} \)
    \[
    (\dot{h}_c)'' + 3H (\dot{h}_c) + V''(h_c) \dot{h}_c = 0
    \]

- \( \delta h_k \) at super-Hubble scales follows the same evolution of \( \dot{h}_c \), they are proportional:

  \[
  \delta h_k(t) = C(k) \dot{h}_c(t)
  \]

- The constant \( C(k) \) is found by matching with the standard sub-Hubble wave solution at \( t_k \):

  \[
  C(k) = \frac{H}{h_c(t_k)\sqrt{2k^3}}.
  \]

- Fluctuations of the Higgs that leave the Hubble radius towards the end of inflation grow a lot.
Perturbations are quantified by the gauge-invariant *comoving curvature perturbation* $\zeta$, which is conserved on super-horizon scales.

In flat gauge,

\[
\zeta = H \frac{\delta \rho_{\text{tot}}}{\dot{\rho}} = \frac{\rho_{\text{st}}}{\rho_{\text{tot}}} \zeta_{\text{st}} + H \frac{\delta \rho_h}{\dot{\rho}_{\text{tot}}}
\]

- Standard inflation contr.
- Contr. from $h$ at small $k$

\[
\delta \rho_h = \rho(h_c + \delta h) - \rho(h_c)
\]

On super-horizon scales, using the e.o.m. we find

\[
\zeta_{h,k} (k \gtrsim k_*) = \left( H C(k) \right).
\]

The largest contribution to $\zeta$, in the last $e$-folds of inflation, comes from the Higgs.
Generation of PBH

\[ k \leftrightarrow M_{\text{PBH}} \rightarrow P_\zeta(k) \rightarrow \Delta(t, x) \rightarrow \sigma_\Delta(M) \rightarrow \beta(M) \rightarrow \Omega_{\text{PBH}} \]
Generation of PBH

\[ k \leftrightarrow M_{\text{PBH}} \rightarrow \mathcal{P}_\zeta(k) \rightarrow \Delta(t, x) \rightarrow \sigma_\Delta(M) \rightarrow \beta(M) \rightarrow \Omega_{\text{PBH}} \]

- \( M_{\text{PBH}} \): mass contained in a sphere of volume \( \sim H^{-3} \) at the time when \( k \) re-enters the Hubble radius.

\[
M_{\text{PBH}} = \gamma \left( \frac{4\pi}{3} \rho H^{-3} \right) \approx 10^{-15} M_\odot \left[ \frac{k}{\left(10^{-14} \text{ Mpc}\right)^{-1}} \right]^{-2}
\]

\[ H \approx 10^{12} \text{ GeV} \approx M_\odot e^{2(N-36)}. \]
Generation of PBH

\[ k \leftrightarrow M_{\text{PBH}} \rightarrow \mathcal{P}_\zeta(k) \rightarrow \Delta(t, x) \rightarrow \sigma_\Delta(M) \rightarrow \beta(M) \rightarrow \Omega_{\text{PBH}} \]

- Power spectrum \( \mathcal{P}_\zeta = \frac{k^3}{2\pi^2} |\zeta_k|^2 \) on small scales \( k \gtrsim k_* \): much larger than the usual \( \mathcal{O}(10^{-9}) \) of CMB scales.

![Graph showing the power spectrum and scales](image-url)
Generation of PBH

- The density contrast is the relevant quantity [’14 Young, Byrnes, Sasaki]

\[ \Delta(t, x) = \frac{4}{9} \left( \frac{1}{aH} \right)^2 \nabla^2 \zeta(x) , \]

physically corresponding to spatial curvature of the metric.

- Threshold for collapse depends on full spatial profile of \( \Delta(t, x) \). [’18 Yoo, Harada, Garriga, Kohri; ’18 Germani, Musco]

- Approximation: collapse happens when \( \Delta \) crosses a threshold \( \Delta_c \approx 0.45 \).
Generation of PBH

\[ k \leftrightarrow M_{\text{PBH}} \rightarrow P_\zeta(k) \rightarrow \Delta(t, x) \rightarrow \sigma_\Delta(M) \rightarrow \beta(M) \rightarrow \Omega_{\text{PBH}} \]

- How likely it is for \( \Delta \) to cross the threshold \( \Delta_c \)?

- Approximation: Gaussian distribution, with variance \( \sigma_\Delta \).

\[
\sigma_\Delta(M(k)) = \sqrt{\frac{16}{81} \int \left( \frac{q^2}{2\pi^2} |\zeta_q|^2 \right) \left( \frac{q}{k} \right)^4 e^{-\frac{q^2}{k^2}} d\ln q}
\]

![Graph showing the distribution of PBH mass](image)
Formation rate: probability of exceeding threshold $\Delta_c$.\(^a\)

$$\beta(M) = \int_{\Delta_c}^{\infty} \frac{d\Delta}{\sqrt{2\pi} \sigma_{\Delta}(M)} \exp\left(-\frac{\Delta^2}{2\sigma_{\Delta}^2(M)}\right)$$

---

\(^a\)Non-Gaussianities play an important role. ['18 Franciolini, Kehagias, Matarrese, Riotto]
After formation, $\rho_{\text{PBH}} \sim a^{-3}$ and behave as collisionless CDM. After equality, they scale as the rest of matter.

$$f(M) \equiv \frac{\Omega_{\text{PBH}}(M)}{\Omega_{\text{CDM}}} = \frac{\beta(M)}{1.6 \cdot 10^{-16}} \left( \frac{\gamma}{0.2} \right)^{3/2} \left( \frac{g_*(T_f)}{106.75} \right)^{-1/4} \left( \frac{M}{10^{-15} M_\odot} \right)^{-1/2}.$$  

For PBHs of mass $\sim 10^{-15} M_\odot$, the right abundance is achieved for $\sigma_\Delta \sim 0.05$.  

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Cosmological signatures of Higgs Instability: PBHs and GW
Non-Gaussianity: account for skewness $S_3 \equiv \langle \Delta^3 \rangle / \sigma^4_{\Delta}$

$$\beta(M) = \frac{1}{\sqrt{2\pi} \nu} \exp \left[ -\frac{1}{2} \nu^2 \left( 1 - S_3 \frac{\sigma_{\Delta}}{3} \left( \nu - 2 - \frac{1}{\nu^2} \right) \right) \right], \quad \nu \equiv \frac{\Delta_c}{\sigma_{\Delta}}.$$
Formation rate as a function of $\sigma_\Delta$

$\beta(\sigma_\Delta) = 1.6 \cdot 10^{-16}$

$\Delta_c = 0.45$
Fine tuning of $\Omega_{\text{PBH}}$
Equation of motion for gravity waves:

\[ h_{ij}'' + 2\mathcal{H} h_{ij}' - \nabla^2 h_{ij} = -4\mathcal{T}_{ij}^{lm} S_{lm}, \]  

(1)

where the transverse projector and the source are

\[ \tilde{T}_{ij}^{lm}(k) = e_{ij}^{(+)}(k) e^{(+)}_{lm}(k) + e_{ij}^{(\times)}(k) e^{(\times)}_{lm}(k), \]  

(2)

\[ S_{ij} = 4\Psi \partial_i \partial_j \Psi + 2\partial_i \Psi \partial_j \Psi - \frac{4}{3(1 + w)} \partial_i \left( \frac{\Psi'}{\mathcal{H}} + \Psi \right) \partial_j \left( \frac{\Psi'}{\mathcal{H}} + \Psi \right). \]  

(3)

Solution with the Green function method in RD:

\[ h_s^k(\eta) = \frac{1}{a(\eta)} \int^{\eta} d\tilde{\eta} g_k(\eta, \tilde{\eta}) a(\tilde{\eta}) \hat{S}_s(\tilde{\eta}, k), \]  

(4)

\[ g_k(\eta, \tilde{\eta}) = \frac{\sin[k(\eta - \tilde{\eta})]}{k} \theta(\eta - \tilde{\eta}). \]  

(5)

Transfer function \( T(\eta, k) \) in RD:

\[ \hat{\Psi}(\eta, k) = \frac{2}{3} T(\eta, k) \zeta(k), \]  

(6)

\[ T(\eta, k) = T(k\eta), \quad T(z) = \frac{9}{z^2} \left[ \frac{\sin(z/\sqrt{3})}{z/\sqrt{3}} - \cos(z/\sqrt{3}) \right]. \]  

(7)
Solution for gravitational waves

The solution reads

\[ h^s_k(\eta) = \frac{4}{9} \int \frac{d^3p}{(2\pi)^3} \frac{1}{k^3 \eta} e^s(k,p) \zeta(p) \zeta(k-p) \left[ I_c(x,y) \cos(k\eta) + I_s(x,y) \sin(k\eta) \right] \]

where the polarisation tensor is

\[ e^s(k,p) \equiv e^{s,ij}(k)p_ip_j = \begin{cases} \frac{1}{\sqrt{2}} p^2 \sin^2 \theta \cos 2\phi & \text{for } s = (+), \\ \frac{1}{\sqrt{2}} p^2 \sin^2 \theta \sin 2\phi & \text{for } s = (\times), \end{cases} \]

and we have defined the dimensionless integrals

\[ I_c(x,y) = \int_1^\infty d\tau \tau (-\sin \tau) \cdot 4 \left\{ 2T(x\tau)T(y\tau) + \left[ T(x\tau) + x\tau T'(x\tau) \right] \left[ T(y\tau) + y\tau T'(y\tau) \right] \right\}, \]

\[ I_s(x,y) = \int_1^\infty d\tau \tau (\cos \tau) \cdot 4 \left\{ 2T(x\tau)T(y\tau) + \left[ T(x\tau) + x\tau T'(x\tau) \right] \left[ T(y\tau) + y\tau T'(y\tau) \right] \right\}. \]
Solution for Gravitational Waves

\[ h^s_k(\eta) = \frac{4}{9} \frac{1}{k^3 \eta} \int \frac{d^3p}{(2\pi)^3} \zeta(p)\zeta(k-p) e^s(k, p). \]

\[ h \sim \mathcal{P}_\zeta \]

Dimensionless integral over time that we compute analytically, as a function of \( p/k, |k - p|/k \):

Integration over momenta

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Cosmological signatures of Higgs Instability: PBHs and GW
From the expression for $h_k$ we can compute

$$\langle h_{k_1} h_{k_2} \rangle \sim \langle \zeta \zeta \zeta \zeta \rangle$$

$$\langle h_{k_1} h_{k_2} h_{k_3} \rangle \sim \langle \zeta \zeta \zeta \zeta \zeta \zeta \rangle$$

which involve just an integration over momenta (2D and 3D numerical integrals).

- 2-pt function $\leftrightarrow \Omega_{GW}$, energy density of GW.
- Final result: $\Omega_{GW} \sim a^{-4}$ as radiation, so we write it in terms of $\Omega_{r,0}$ as

$$\Omega_{GW}(\eta_0, k) = \frac{\Omega_{r,0}}{972} \int \int \mathcal{F} \, dx \, dy \, \frac{x^2}{y^2} \left[ 1 - \frac{(1 + x^2 - y^2)^2}{4x^2} \right]^2 \cdot \mathcal{P}_\zeta(kx) \mathcal{P}_\zeta(ky) \left[ \mathcal{I}_c(x, y)^2 + \mathcal{I}_s(x, y)^2 \right]$$
The dimensionless power spectrum is defined as:

\[
\langle h^r(\eta, k_1)h^s(\eta, k_2) \rangle \equiv (2\pi)^3 \delta^{(3)}(k_1 + k_2) \delta^{rs} \frac{2\pi^2}{k_1^3} \mathcal{P}_h(k_1)
\]  

(11)

which gives a time-averaged power spectrum \( \mathcal{P}_h(\eta, k) \)

\[
\mathcal{P}_h(\eta, k) = \frac{2}{81} \frac{1}{k^2 \eta^2} \int \int dxdy \frac{x^2}{y^2} \left[ 1 - \frac{(1 + x^2 - y^2)^2}{4x^2} \right]^2 
\cdot \mathcal{P}_\zeta(kx)\mathcal{P}_\zeta(ky) \sqrt{\mathcal{I}_c(x, y)^2 + \mathcal{I}_s(x, y)^2}.
\]  

(12)

The corresponding energy density reads

\[
\Omega_{GW}(\eta_0, k) = \Omega_{r,0} \Omega_{GW}(\eta_f, k) = \frac{\Omega_{r,0}}{24} \frac{k^2}{\mathcal{H}(\eta_f)^2} \mathcal{P}_h(\eta_f, k).
\]  

(13)
The peak $f_*$ is sensitive to $m_{\text{Higgs}}$, $m_{\text{top}}$ and thus to the instability scale $\Lambda$. 

$$\Lambda = 3.3 \times 10^{11} \left( \frac{f_*}{\text{Hz}} \right)^{-0.65} \text{ GeV}$$
From the 3-pt function of GW we introduce the bispectrum $B_h$

$$\langle h^r(\eta, k_1) h^s(\eta, k_2) h^t(\eta, k_3) \rangle \equiv (2\pi)^3 \delta^3(k_1 + k_2 + k_3) B_{h}^{rst}(k_1, k_2, k_3)$$

The dimensionless normalised shape $S_{h}^{rst}(k_1, k_2, k_3)$

$$S_{h}^{rst}(k_1, k_2, k_3) = k_1^2 k_2^2 k_3^2 \frac{B_{h}^{rst}(k_1, k_2, k_3)}{\sqrt{\mathcal{P}_h(k_1) \mathcal{P}_h(k_2) \mathcal{P}_h(k_3)}}$$

identifies the strength of the bispectrum signal.

$S_h(k_1, k_2, k_3)$ is a “fingerprint” of the GW signal.
The bispectrum is defined through
\[
\langle h^r(\eta, k_1)h^s(\eta, k_2)h^t(\eta, k_3) \rangle \equiv (2\pi)^3 \delta^3(k_1 + k_2 + k_3) B_{h}^{rst}(k_1, k_2, k_3).
\] (14)
and its envelope over time reads
\[
B_{h}^{rst}(k_1, k_2, k_3) = 8 \left( \frac{4}{9} \right)^3 \pi^3 \int d^3 p_1 \frac{1}{k_1^3 k_2^3 k_3^3 \eta^3} e^r(k_1, p_1) e^s(k_2, p_2) e^t(k_3, p_3) .
\]
\[
\cdot \frac{\mathcal{P}_\zeta(p_1)}{p_1^3} \frac{\mathcal{P}_\zeta(p_2)}{p_2^3} \frac{\mathcal{P}_\zeta(p_3)}{p_3^3} \sqrt{\mathcal{I}_c \left( \frac{p_1}{k_1}, \frac{p_2}{k_1} \right)^2 + \mathcal{I}_s \left( \frac{p_1}{k_1}, \frac{p_2}{k_1} \right)^2} .
\]
\[
\cdot \sqrt{\mathcal{I}_c \left( \frac{p_2}{k_2}, \frac{p_3}{k_2} \right)^2 + \mathcal{I}_s \left( \frac{p_2}{k_2}, \frac{p_3}{k_2} \right)^2} \sqrt{\mathcal{I}_c \left( \frac{p_3}{k_3}, \frac{p_1}{k_3} \right)^2 + \mathcal{I}_s \left( \frac{p_3}{k_3}, \frac{p_1}{k_3} \right)^2}.\] (15)

\[
\int d^3 p_1 \longrightarrow \int_{-\infty}^{+\infty} d\ell \int_0^{+\infty} r \, dr \int_0^{2\pi} d\alpha,
\]
\[
p_1 = (r \cos \alpha, r \sin \alpha, \ell),
\]
\[
p_2 = (-k_1 x + r \cos \alpha, -k_1 y + r \sin \alpha, \ell),
\]
\[
p_3 = (k_3 + r \cos \alpha, r \sin \alpha, \ell),
\]
\[
p_i^2 \sin^2 \theta_i = p_i^2 - \frac{|p_i \cdot k_i|^2}{k_i^2}, \quad \sin \phi_i = \frac{\ell k_i}{|p_i \times k_i|}.
\]
Bispectrum of Gravitational Waves

\[ k_3 = 2. k_\ast \]
\[ \Sigma \text{pol.} \]

\[ (-2\sigma), (0\sigma) \]
The shape of the bispectrum peaks for momenta of order of $k_\ast = a(t_\ast)H$, and for folded and equilateral configurations:

\[
\begin{align*}
S_{h}^{+++} &= O(-600) \quad \text{for folded configurations,} \\
S_{h}^{+\times\times} &= O(-600) \quad \text{for equilaterial configurations,} \\
\sum_{\text{pol}} S_{h} &= O(-1000) \quad \text{for equilateral configurations.}
\end{align*}
\]

In case of detection of the 3-pt function, its shape could help to identify the origin of the signal.
Approximate homogeneity on exponentially large scales

- How large can be the region we are describing?

- A scalar field has similar values on scales as large as the particle horizon:
  \[ \text{correlation length } \sim H^{-1} \exp H t_i \]

- We can assume that \( h_c \) has nearly the same value on a patch larger than our observable Universe.

[Note: Diagram showing time evolution of \( h_c \) with \( t \), correlation length \( \ell \), and \( t_* \) and \( t_{\text{end}} \) marked.]
Inexact homogeneity: small fluctuations on scales $H^{-1}$

- Small fluctuations of order $H/(2\pi) \sim 0.16H$ occur on a scale $H^{-1}$.
- These are larger than the precision required on the initial value $h_\star$.
- A small overshooting on $h_\star$ prevents the reheating from rescuing $h$, leaving an expanding AdS region.
Inexact homogeneity: small fluctuations on scales $H^{-1}$

- Small fluctuations of order $H/(2\pi) \sim 0.16H$ occur on a scale $H^{-1}$.
- These are larger than the precision required on the initial value $h_\ast$.
- A small overshooting on $h_\ast$ prevents the reheating from rescuing $h$, leaving an expanding AdS region.

[1803.10242 Gross, Polosa, Strumia, Urbano, Xue]
A scenario without AdS regions at all

- We do not need the same $P_\zeta$ everywhere: inhomogeneities in the PBH distribution are OK, given that they are generated strongly clustered.
- For AdS regions it’s different: they can’t be in our past lightcone.
- In a multiverse, “small” probability does not mean much, but it is true that the lifetime of anthropically selected regions could be short.
- If $\lambda(h)$ returns positive at some high scale, the mechanism can still work and AdS regions are avoided.
A scenario without AdS regions at all

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- In a multiverse, “small” probability does not mean much, but it is true that the lifetime of anthropically selected regions could be short.
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A scenario without AdS regions at all

Example of BSM contributions which can lift $\lambda(h)$ to positive values: extra scalar $S$ interacting with the Higgs, yielding a positive contribution to $\lambda$ above $m_S$ [1203.0237, Elias-Miró, Espinosa, Giudice, Lee, Strumia].

- Higgs potential has an absolute minimum at $\sim m_S$.
- Assume $m_S \lesssim T_{RH}$, so that the Higgs is always rescued at reheating. No fine-tuning here: we can allow variations $\sim (10 - 20)\%$ for $m_S$, not to spoil the PBH generation.
Evolution in the presence of an absolute minimum

- If $h_c$ undershoots (starts too late, or too low), nothing changes.
- If $h_c$ overshoots (starts too early, or too high), it reaches the minimum, where it oscillates. During this phase $h$ and $\delta h_k$ slowly decrease as matter.

![Graph showing evolution of Higgs field](image)
Imagine to fix $h_*$ and vary $N_*$. What is the final $\mathcal{P}_\zeta$?
Evolution in the presence of an absolute minimum
SM + an extra scalar $S$

- As an example of BSM contributions which can lift $\lambda(h)$ to positive values, consider a scalar $S$ interacting with the SM Higgs through [1203.0237, Elias-Miró, Espinosa, Giudice, Lee, Strumia]

$$V = \lambda_S \left( |S|^2 - \frac{\omega^2}{2} \right)^2 + 2\lambda_{HS} \left( |S|^2 - \frac{\omega^2}{2} \right) \left( |\varphi_H|^2 - \frac{v^2}{2} \right)$$

- Most general Lagrangian preserving a global $U(1)$ symmetry, can arise in various BSM extensions.

- $S$ takes a large vev $\omega/\sqrt{2}$, and $m_S = \sqrt{\lambda_S} \omega$. When integrating out $S$ at scales below $m_S$, one finds

$$V_{\text{eff}}(h) = \left( \lambda_H - \frac{\lambda_{HS}^2}{\lambda_S} \right) \left( |\varphi_H|^2 - \frac{v^2}{2} \right)^2 .$$

- For $h > m_S$, $\lambda$ jumps up of a factor $\delta \lambda \sim \frac{\lambda_{HS}^2}{\lambda_S}$.

- Only requirement for $\lambda_{SH}$ and $\lambda_S$ is that they yield $\lambda(h \gtrsim m_S) > 0$, corresponding to $|\delta \lambda| \gtrsim 0.008$. 