Constraining the Higgs potential at $e^+e^-$ colliders

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The Higgs boson

- Discovered at LHC, 4th July, 2012.
- Properties measured at LHC.
- Next step: measuring Higgs self-couplings to probe the Higgs potential.

\[ e^+ e^- \text{ colliders} \]
The SM Higgs potential is fixed by gauge symmetry and renormalisability:

$$V_{SM}^{SM}(\Phi) = -\mu^2(\Phi^\dagger \Phi) + \lambda(\Phi^\dagger \Phi)^2$$  \hspace{1cm} (1)$$

After electroweak symmetry breaking:

$$V_{SM}^{SM}(\Phi) = \frac{1}{2}m_H^2H^2 + \lambda_3^{SM}H^3 + \frac{\lambda_4^{SM}}{4}H^4$$  \hspace{1cm} (2)$$

where $\lambda_3^{SM} = \lambda_4^{SM} = \lambda = \frac{m_H^2}{2v^2}$, fully determined in SM!

Various new physics models lead to non-SM values of $\lambda_3$ and/or $\lambda_4$.

Measuring $\lambda_3$ and $\lambda_4$: the main targets of the current and future colliders!
Direct measurement

- Directly measuring $\lambda_3$: double Higgs production. Loose bound at LHC, good precision at future $e^+e^-$ and hadron colliders.

- Directly measuring $\lambda_4$: triple Higgs production. Only loose bounds at 100TeV pp collider, e.g. $\lambda_4/\lambda_4^{\text{SM}} \in [-20, 30]$ [Chen et.al. ’15]

$O(1)ab$ at $e^+e^-$, too tiny for measurements, but sensitive on $\lambda_4$!
Indirectly probe

- $\lambda_3$: single Higgs at one (more) loop
  Attract lots of attention recently!
  
  McCullough '14  Gorbahn and Haisch '16  Degrassi et.al. '16  Bizon et.al. '16  Di Vita et.al. '17  Maltoni et.al. '17

Two-loop effects in electroweak precision measurement:

  Degrassi et.al. '17  Kribs et.al. '17

How about $\lambda_4$? Double Higgs production at one-loop!
Previously only $\lambda_3$ at tree-level of double Higgs and one-loop level of single Higgs are studied.

<table>
<thead>
<tr>
<th></th>
<th>$\lambda_3$</th>
<th>$\lambda_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single Higgs</td>
<td>one-loop</td>
<td>(two-loop)</td>
</tr>
<tr>
<td>Double Higgs</td>
<td>tree-level</td>
<td>one-loop</td>
</tr>
<tr>
<td>Triple Higgs</td>
<td>tree-level</td>
<td>tree-level</td>
</tr>
</tbody>
</table>

We extend to one-loop level of double Higgs, which is sensitive on $\lambda_4$. We also examine triple Higgs, where $\lambda_3$ and $\lambda_4$ appear at tree-level.
EFT parametrization and Renormalization

UV Divergent $\Rightarrow$ EFT for renormalization.

$$V(\Phi) = -\mu^2(\Phi^\dagger\Phi) + \lambda(\Phi^\dagger\Phi)^2 + \sum_{n=3}^\infty \frac{c_{2n}}{\Lambda^{2n-4}} (\Phi^\dagger\Phi - \frac{1}{2}v^2)^2$$  \hspace{1cm} (3)

To vary $\lambda_3$ and $\lambda_4$ independently, we need both $c_6$ and $c_8$.

$$\kappa_3 = \frac{\lambda_3}{\lambda_3^{SM}} = 1 + \frac{c_6 v^2}{\Lambda^2} = 1 + \bar{c}_6$$  \hspace{1cm} (4)

$$\kappa_4 = \frac{\lambda_4}{\lambda_4^{SM}} = 1 + \frac{6c_6 v^2}{\Lambda^2} + \frac{c_8 v^4}{\Lambda^4} = 1 + 6\bar{c}_6 + \bar{c}_8$$  \hspace{1cm} (5)
Bounding the Higgs potential: setup

Two production mechanisms
- Z boson associated production: $ZH^n$
- W boson fusion (WBF).

Colliders:
- Circular colliders: CEPC and FCC-ee. Only single Higgs is available.
- Linear colliders: ILC and CLIC. All processes are available.

Scenarios:
- Scenario 1: $\bar{c}_8$ effects are negligible.
- Scenario 2: $\bar{c}_8$ effects are non-negligible.
Scenario 1: single and double Higgs

\(\bar{c}_8\) effects are negligible, we only have \(\bar{c}_6\).

Left: Single Higgs

“X”-shape band, different center point \(\Rightarrow\) complementary.

The band for double Higgs is thinner than single Higgs.

Right: Double Higgs
Scenario 1: combined

\( \bar{c}_8 \) effects are negligible, we only have \( \bar{c}_6 \).

A good constraint over all region can be obtained at ILC.

CLIC cannot resolve degeneracy around \( \bar{c}_6 = 0.5 \).
Scenario 2: double Higgs

Non-negligible $\bar{c}_8$, assuming SM cross sections:

Combining $ZHH$ and WBF $HH$ yields better bounds.
Scenario 2: triple Higgs

Tiny cross section $\Rightarrow$ zero events once SM is assumed.

- Useless at ILC.
- WBF $HHH$ is important at CLIC, especially at 3TeV.
Scenario 2: combined

ILC is better on $\bar{c}_6$
CLIC is better on $\bar{c}_8$
Conclusion

- $e^+e^-$ colliders
- One-loop for single and double Higgs
- Tree-level for triple Higgs
- $(\bar{c}_6, \bar{c}_8)$: EFT description to cancel UV divergence
- WBF HH and ZHH are complementary
- Strong dependence on $\bar{c}_8$ for triple Higgs
- $\bar{c}_8$, comparable or even better than 100 TeV.
Backup slides
Counter term

\[
\delta c_6 = \frac{\Delta}{16\pi^2} \left[ c_6 \left( 54\lambda - 9 \frac{m_Z^2 + 2m_W^2}{v^2} + 6 \frac{N_c m_t^2}{v^2} \right) \\
+ \frac{c_8 v^2}{\Lambda^2} \left( 64\lambda - 6 \frac{m_Z^2 + 2m_W^2}{v^2} + 4 \frac{N_c m_t^2}{v^2} \right) + \frac{45 c_6^2 v^2}{\Lambda^2} \\
+ \frac{20 c_{10} \lambda v^4}{\Lambda^4} + \frac{36 c_6 c_8 v^4}{\Lambda^4} \right]
\]

where \( \Delta = \frac{1}{\epsilon} - \log(4\pi) + \gamma_E \) is the divergence in \( \overline{\text{MS}} \) convention.
Results for single Higgs

\[ \sigma_{NLO} = \sigma_{LO} + \sigma_{1\text{-loop}}, \quad \sigma_{LO} = \sigma_{LO}^{SM} \] (6)

\[ \sigma_{1\text{-loop}} = \sigma_0 + \sigma_1 \bar{c}_6 + \sigma_2 \bar{c}_6^2 \] (7)

\[ \sigma_{NLO}^{\text{pheno}} = \sigma_{LO} + \sigma_1 \bar{c}_6 + \sigma_2 \bar{c}_6^2 \] (8)

\[ = \sigma_{LO}^{\text{LO}} \left( 1 + (\kappa_3 - 1) C_1 + (\kappa_3^2 - 1) C_2 \right) \] (9)

\[ C_2 = \delta Z_{H}^{SM, \lambda} \approx -0.00154 \] (10)
Results for double Higgs

\[ \sigma_{\text{LO}} = \sigma_0 + \sigma_1 \bar{c}_6 + \sigma_2 \bar{c}_6^2 \]  
(11)

\[ \sigma_{\text{NLO}} = \sigma_{\text{LO}} + \sigma_{1-\text{loop}} \]  
(12)

\[ \sigma_{1-\text{loop}} = \sigma_{00} + \sigma_{10} \bar{c}_6 + \sigma_{20} \bar{c}_6^2 + \sigma_{30} \bar{c}_6^3 + \sigma_{40} \bar{c}_6^4 \]  
(13)

\[ \bar{c}_{10} (\sigma_{01} + \sigma_{11} \bar{c}_6 + \sigma_{21} \bar{c}_6^2) + \bar{c}_{10} (\sigma_{001} + \sigma_{101} \bar{c}_6) \]  
(14)

\( \bar{c}_{10} \) contribution can be written as a kinematically independent shift to \( \bar{c}_6 \):

\[ \bar{c}_6 \rightarrow \bar{c}_6 + \frac{5 \lambda \bar{c}_{10}}{4 \pi^2} (1 - \log \frac{m_H^2}{\mu_r^2}) \]  
(15)

\[ \sigma_{\text{NLO}}^{\text{pheno}} = \sigma_{\text{LO}} + \Delta \bar{c}_6 + \Delta \bar{c}_8 \]  
(16)

\[ \Delta \bar{c}_6 = \sigma_{30} \bar{c}_6^3 + \sigma_{40} \bar{c}_6^4 \]  
(17)

\[ \Delta \bar{c}_8 = \bar{c}_8 (\sigma_{01} + \sigma_{11} \bar{c}_6 + \sigma_{21} \bar{c}_6^2) \]  
(18)
Results for double Higgs

LO cross sections:
Results for double Higgs

The one-loop contributions:

Sensitivity on $\bar{c}_8$ depends on value of $\bar{c}_6$
Results for Triple Higgs

\[ \sigma_{\text{LO}} = \sigma_{00} + \sum_{1 \leq i+2j \leq 4} \sigma_{ij} \bar{c}_6^i \bar{c}_8^j \]  \hspace{1cm} (19)

Although \( \sigma_{\text{SM}} = \sigma_{00} \) is tiny, for WBF \( HHH \) the cross section strongly depends on \((\bar{c}_6, \bar{c}_8)\): large \( \bar{c}_8 \Rightarrow \sigma_{\text{LO}} \approx \bar{c}_8^2 \sigma_{02} \approx \bar{c}_8^2 \sigma_{00} \).
Scenario 2: non-negligible $\bar{c}_8$, BSM cases:
Bounding the Higgs potential: combined

Scenario 2: assuming BSM cases:

\[ \bar{c}_8 = -4 \]

\[ \bar{c}_6 = -2 \]

\[ \bar{c}_6 = -1 \]

\[ \bar{c}_6 = 4 \]

\[ \bar{c}_6 = 2 \]

\[ \bar{c}_6 = 1 \]

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Bounding the Higgs potential: scenarios

<table>
<thead>
<tr>
<th></th>
<th>$\sqrt{s}$ [GeV]</th>
<th>$P(e^-, e^+)$</th>
<th>Luminosity [ab$^{-1}$]</th>
<th>Relevant final states</th>
</tr>
</thead>
<tbody>
<tr>
<td>CEPC</td>
<td>250</td>
<td>(0.0,0.0)</td>
<td>5.0</td>
<td>$ZH$, WBF $H$</td>
</tr>
<tr>
<td>FCC-ee</td>
<td>240</td>
<td>(0.0,0.0)</td>
<td>10.0</td>
<td>$ZH$, WBF $H$</td>
</tr>
<tr>
<td></td>
<td>350</td>
<td>(0.0,0.0)</td>
<td>2.6</td>
<td>$ZH$, WBF $H$</td>
</tr>
<tr>
<td>ILC</td>
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<td>2.0</td>
<td>$ZH$, WBF $H$</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>(-0.8,0.3)</td>
<td>4.0</td>
<td>$ZH$, WBF $H$</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>(-0.8,0.2)</td>
<td>2.0</td>
<td>$ZH$, WBF $H$</td>
</tr>
<tr>
<td>CLIC</td>
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<td>0.5</td>
<td>$ZH$, WBF $H$</td>
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<tr>
<td></td>
<td>1400</td>
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<td>1.5</td>
<td>$ZH$, WBF $H$</td>
</tr>
<tr>
<td></td>
<td>3000</td>
<td>(-0.8,0.0)</td>
<td>2.0</td>
<td>$ZH$, WBF $H(H(H))$</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>$ZHHH$, WBF $H(H(H))$</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>WBF $H(H(H))$</td>
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</tbody>
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- **Scenario 1**: $\bar{c}_8$ effects are negligible. Single, double and triple Higgs are considered. We allow both $\bar{c}_6 = 0$(SM-like) and $\bar{c}_6 \neq 0$(BSM case)

- **Scenario 2**: $\bar{c}_8$ effects are non-negligible. Here only double and triple Higgs are considered, since two-loop for single Higgs are not available. We explore how well we can constrain $\bar{c}_8$ and how much $\bar{c}_8$ can affect the measurement of $\bar{c}_6$. 