MATCHING NLO CALCULATIONS WITH PARTON SHOWER: THE POSITIVE-WEIGHT HARDEST EMISSION GENERATOR

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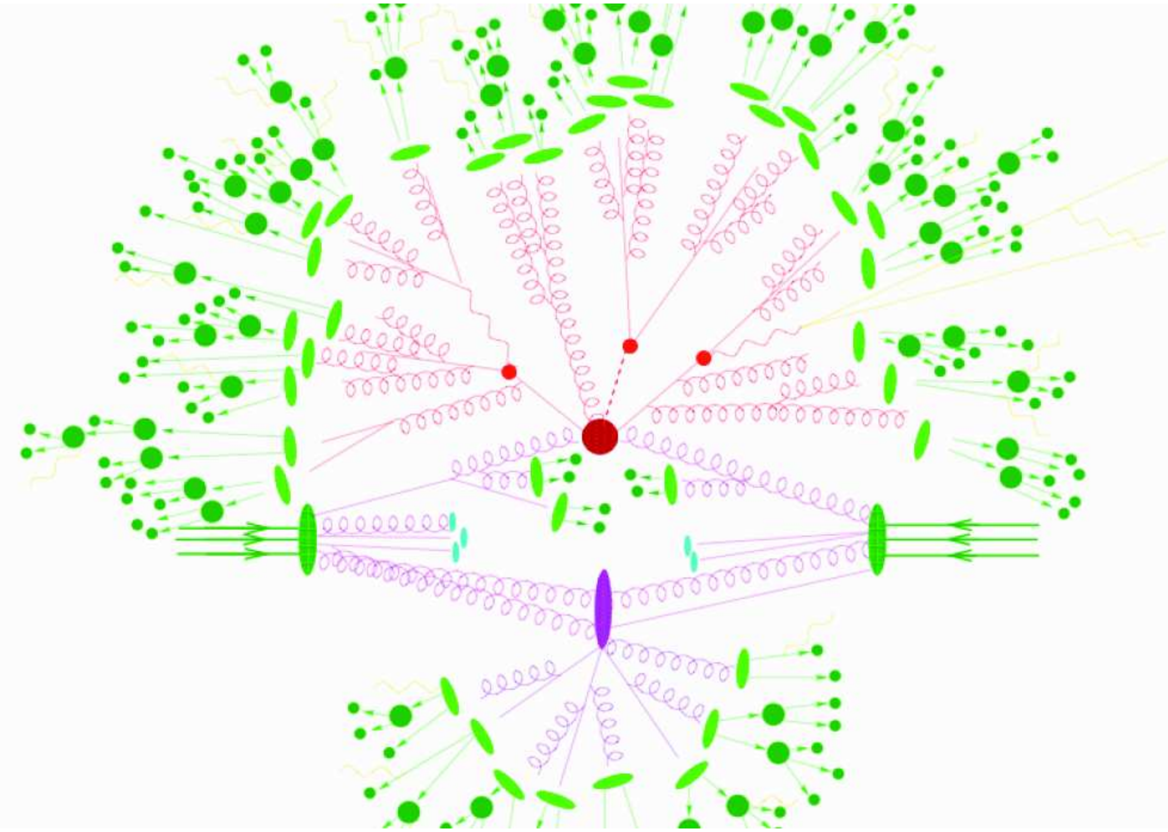
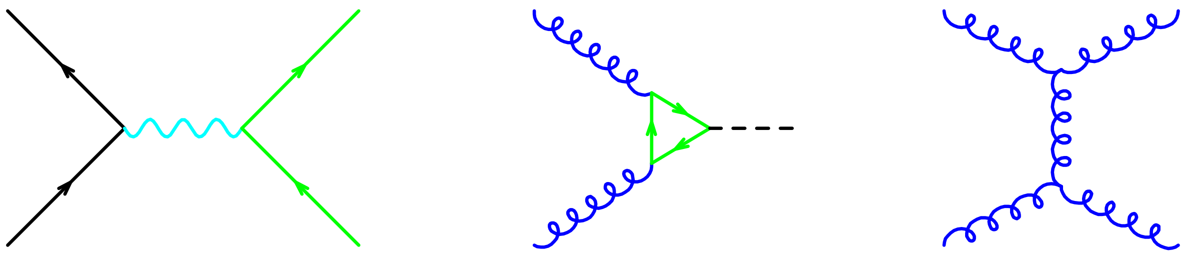
- Basics of Shower Monte Carlo programs
- The POWHEG formalism
- Applications
- Conclusions



High energy collisions

High-energy particle physics deals with the scattering and the production of elementary constituents

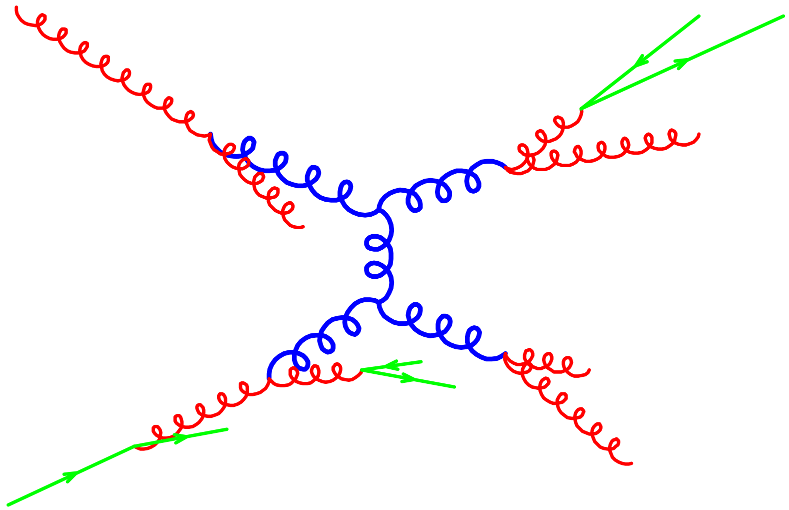


$e^+e^- \longrightarrow q\bar{q}$ $\qquad gg \longrightarrow H$ $\qquad gg \longrightarrow gg$

Ideally, one needs elementary constituents as projectiles and targets, (i.e. a collider for leptons, gluons and quarks) and a final-state detector of leptons, gluons and quarks. Not obvious for quarks and gluons:

- at short distance, due to asymptotic freedom, quarks and gluons behave as free particles
- at long distance, infrared slavery: very strong interactions hide the simplicity of the description of the constituents.

Dominant corrections



Collinear-splitting processes in the initial and final state (always with transverse momenta $> \Lambda_{\rm QCD}$) are strongly enhanced. This is due to the fact that, in perturbation theory, the denominators in the propagators are small.

- The algorithms that evaluate all these enhanced contributions are called shower algorithms.
- Shower algorithms give a description of a hard collision up to distances of order $1/\Lambda_{\rm QCD}$.
- At larger distances, perturbation theory breaks down and we need to resort to non-perturbative methods (i.e. lattice calculations). However, these methods can be applied only to simple systems. The only viable alternative is to use models of hadron formation.

Hadronic final states

IHEP	ID	IDPDG	IST	MO1	MO2	DA1	DA2	P-X	P-Y	P-Z	ENERGY	MASS	V-X	V-Y	V-Z	V-C*T
30	NU_E	12	1	28	23	0	0	64.30	25.12	-1194.4	1196.4	0.00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
31	E+	-11	1	29	23	0	0	-22.36	6.19	-234.2	235.4	0.00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
230	PI0	111	1	155	24	0	0	0.31	0.38	0.9	1.0	0.13	4.209E-11	6.148E-11	-3.341E-11	5.192E-10
231	RHO+	213	197	155	24	317	318	-0.06	0.07	0.1	0.8	0.77	4.183E-11	6.130E-11	-3.365E-11	5.189E-10
232	P	2212	1	156	24	0	0	0.40	0.78	1.0	1.6	0.94	4.156E-11	6.029E-11	-4.205E-11	5.250E-10
233	NBAR	-2112	1	156	24	0	0	-0.13	-0.35	-0.9	1.3	0.94	4.168E-11	6.021E-11	-4.217E-11	5.249E-10
234	PI-	-211	1	157	9	0	0	0.14	0.34	286.9	286.9	0.14	4.660E-13	8.237E-12	1.748E-09	1.749E-09
235	PI+	211	1	157	9	0	0	-0.14	-0.34	624.5	624.5	0.14	4.056E-13	8.532E-12	2.462E-09	2.462E-09
236	P	2212	1	158	9	0	0	-1.23	-0.26	0.9	1.8	0.94	-4.815E-11	1.893E-11	7.520E-12	3.252E-10
237	DLTABR--	-2224	197	158	9	319	320	0.94	0.35	1.6	2.2	1.23	-4.817E-11	1.900E-11	7.482E-12	3.252E-10
238	PI0	111	1	159	9	0	0	0.74	-0.31	-27.9	27.9	0.13	-1.889E-10	9.893E-11	-2.123E-09	2.157E-09
239	RHO0	113	197	159	9	321	322	0.73	-0.88	-19.5	19.5	0.77	-1.888E-10	9.859E-11	-2.129E-09	2.163E-09
240	K+	321	1	160	9	0	0	0.58	0.02	-11.0	11.0	0.49	-1.890E-10	9.873E-11	-2.135E-09	2.169E-09
241	KL_1-	-10323	197	160	9	323	324	1.23	-1.50	-50.2	50.2	1.57	-1.890E-10	9.879E-11	-2.132E-09	2.166E-09
242	K-	-321	1	161	24	0	0	0.01	0.22	1.3	1.4	0.49	4.250E-11	6.333E-11	-2.746E-11	5.211E-10
243	PI0	111	1	161	24	0	0	0.31	0.38	0.2	0.6	0.13	4.301E-11	6.282E-11	-2.751E-11	5.210E-10

High-energy experimental physicists feed this kind of output through their detector-simulation software, and use it to determine efficiencies for signal detection, and perform background estimates.

Analysis strategies are set up using these simulated data.

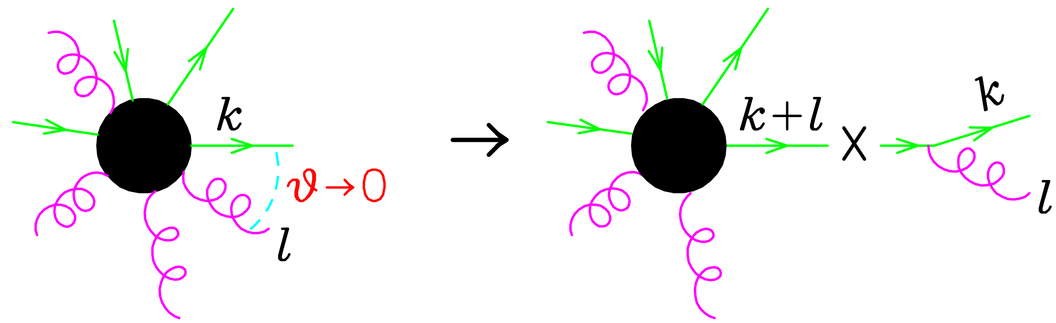
"The Monte Carlo simulation has become the major mean of visualization of not only detector performance but also of physics phenomena. So far so good. But it often happens that the physics simulations provided by the Monte Carlo generators carry the authority of data itself. They look like data and feel like data, and if one is not careful they are accepted as if they were data."

J.D. Bjorken (1992)

Shower basics: collinear factorization

QCD emissions are enhanced near the collinear limit

Cross sections factorize near collinear limit



$$d\mathbf{\Phi}_{n+1} = d\mathbf{\Phi}_n \, d\Phi_r \qquad d\Phi_r \div dt\, dz\, d\varphi$$

$$|M_{n+1}|^2 d\mathbf{\Phi}_{n+1} \Longrightarrow |M_n|^2 \, d\mathbf{\Phi}_n \, \frac{\alpha_s}{2\pi} \, \frac{dt}{t} \, P_{q,qg}(z)\, dz \, \frac{d\varphi}{2\pi} \quad \begin{cases} \dfrac{dt}{t} \approx \dfrac{d\theta}{\theta} & \text{collinear singularity} \\ \dfrac{dz}{1-z} \approx \dfrac{dE_g}{E_g} & \text{soft singularity} \end{cases}$$

t : $(k+l)^2,\ p_T^2,\ E^2\theta^2 \ldots$

$z = k^0/(k^0+l^0)$: energy (or $p_\parallel$ or p^+) fraction of quark

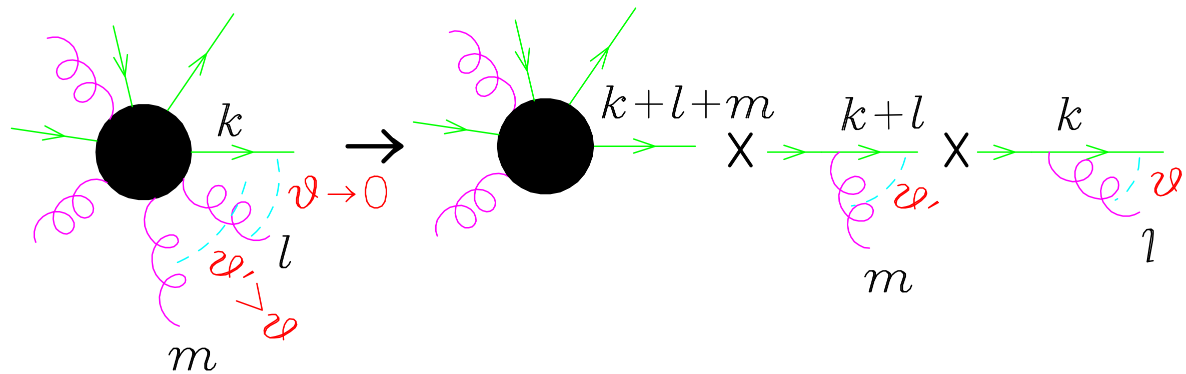
$P_{q,qg}(z) = C_F \dfrac{1+z^2}{1-z}$: Altarelli-Parisi splitting function

(ignore $z \to 1$ IR divergence for now)

Shower basics: collinear factorization

If another gluon becomes collinear, iterate the previous formula

$\theta', \theta \to 0$ with $\theta' > \theta$



$$|M_{n+1}|^2 d\mathbf{\Phi}_{n+1} \implies |M_{n-1}|^2 d\mathbf{\Phi}_{n-1} \times \frac{\alpha_s}{2\pi}\frac{dt'}{t'} P_{q,qg}(z')\, dz' \frac{d\varphi'}{2\pi} \times \frac{\alpha_s}{2\pi}\frac{dt}{t} P_{q,qg}(z)\, dz \frac{d\varphi}{2\pi}\, \theta(t'-t)$$

Collinear partons can be described by a factorized integral ordered in t.

Collinear factorization: multiple emissions

For n collinear emissions, the cross section goes as

$$\sigma \approx \sigma_0 \alpha_s^n \int_{t_0}^{Q^2} \frac{dt_1}{t_1} \frac{dt_2}{t_2} \ldots \frac{dt_n}{t_n} \theta\left(Q^2 > t_1 > t_2 > \ldots > t_n > t_0\right)$$

$$= \sigma_0 \alpha_s^n \int_{t_0}^{Q^2} \frac{dt_1}{t_1} \int_{t_0}^{t_1} \frac{dt_2}{t_2} \ldots \int_{t_0}^{t_{n-1}} \frac{dt_n}{t_n} \approx \sigma_0 \alpha_s^n \frac{1}{n!} \left(\log \frac{Q^2}{t_0}\right)^n$$

- Q^2 is an upper cutoff for the ordering variable t
- $t_0 \approx \Lambda^2 \approx \Lambda^2_{\mathrm{QCD}}$ is an infrared cutoff (quark mass, confinement scale)
- Due to the log dependence, we call it leading-log approximation.
- According to the Kinoshita-Lee-Nauenberg theorem, the virtual corrections, order by order, contribute with a comparable term, with opposite sign.
- The virtual leading-log contribution should be included in order to get sensible results!

Simple probabilistic interpretation of "not-resolved" corrections

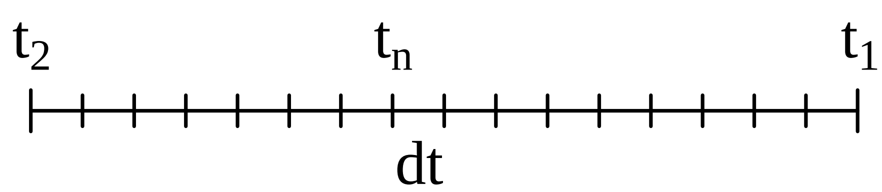
- probability of emission in the interval dt, at order α_s (multiple emissions are of higher orders in α_s)

$$dP_{\rm emis}(t+dt,t) = \frac{dt}{t}\,\frac{\alpha_s(t)}{2\pi}\int dz\, P_{i,jk}(z)$$

- probability of no emission in the interval dt

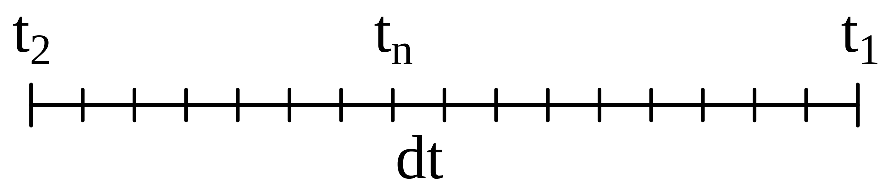
$$dP_{\rm no\ emis}(t+dt,t) = 1 - dP_{\rm emis}(t+dt,t) = 1 - \frac{dt}{t}\,\frac{\alpha_s(t)}{2\pi}\int dz\, P_{i,jk}(z)$$

The "no emission" probability contains, through the 1, all the virtual corrections (in the collinear approximation, that is at the leading-log level).



Simple probabilistic interpretation of “not-resolved” corrections

- divide a finite interval $[t_2, t_1]$ in N small intervals $dt = (t_1 - t_2)/N$.



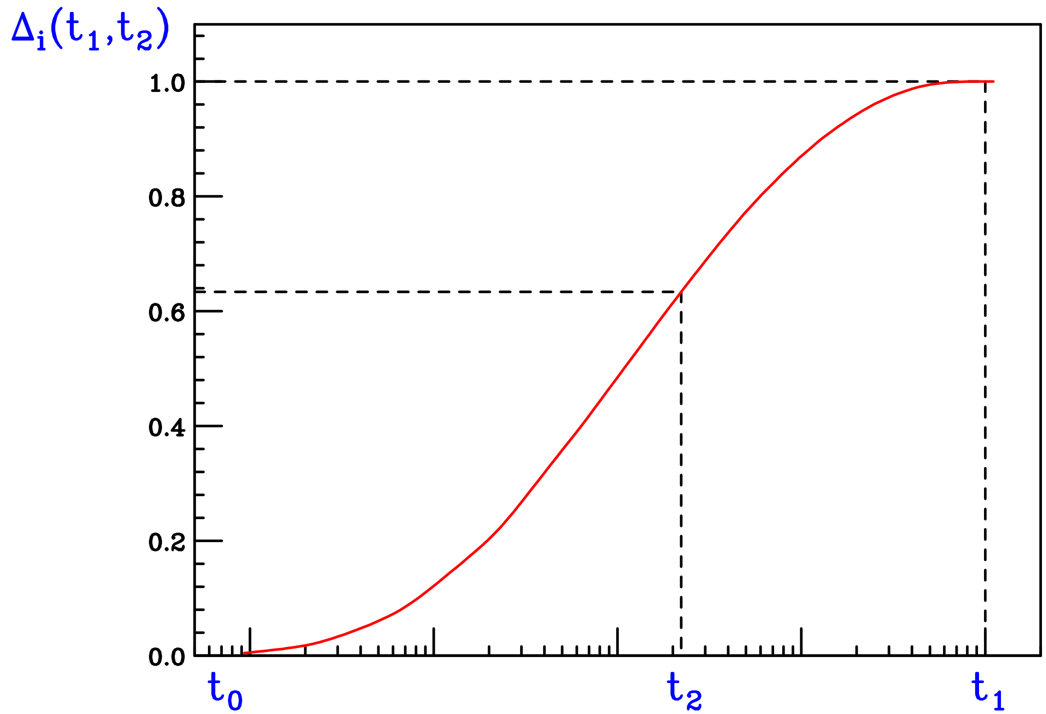
The probability of not emitting radiation between the two ordering scales t_1 and t_2 is given by the product

$$
\begin{aligned}
P_{\text{no emis}}(t_1, t_2) &= \lim_{N \to \infty} \prod_{n=1}^{N} \left[1 - \frac{dt}{t_n} \frac{\alpha_s(t_n)}{2\pi} \int dz\, P_{i,jk}(z) \right] \\
&= \exp\left\{ - \int_{t_2}^{t_1} \frac{dt}{t} \frac{\alpha_s(t)}{2\pi} \int dz\, P_{i,jk}(z) \right\} \\
&\equiv \Delta(t_1, t_2)
\end{aligned}
$$

- The weight $\Delta(t_1, t_2)$ is called Sudakov form factor. It resums all the dominant virtual corrections to the tree graph (in the collinear approximation).

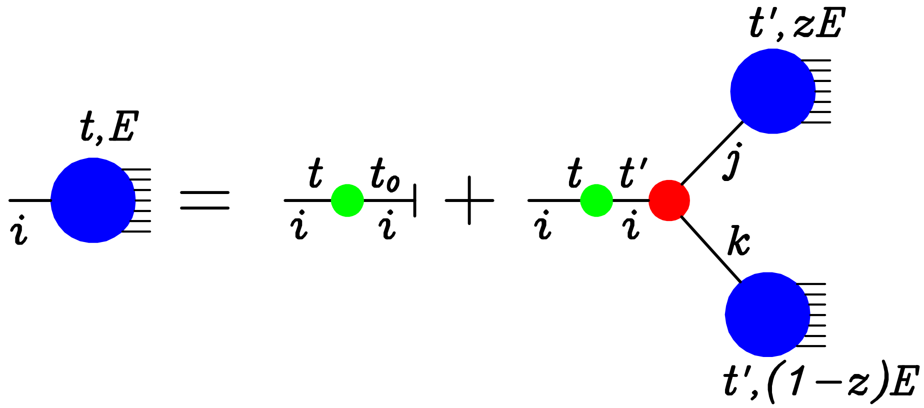
Sudakov form factors

$$\Delta_i(t_1, t_2) = \exp\left\{ -\sum_{jk} \int_{t_2}^{t_1} \frac{dt}{t} \frac{\alpha_s(t)}{2\pi} \int dz\, P_{i,jk}(z) \right\}$$



Notice that, when $t_2 \ll t_1$, $\Delta \to 0$, i.e. the probability that a hard parton turns into a narrow jet, or that it does not radiate at all, is small (it is Sudakov suppressed)

Final recipe



$$\mathcal{S}_i(t,E) = \Delta_i(t,t_0)\,\mathbb{1} + \sum_{(jk)} \int_{t_0}^{t} \frac{\alpha_S(t')}{2\pi}\,\frac{dt'}{t'} \int dz \int \frac{d\varphi}{2\pi}\,\Delta_i(t,t')\,P_{i,jk}(z)\,\mathcal{S}_j(t',zE)\;\mathcal{S}_k(t',(1-z)E)$$

- consider all tree graphs.
- assign values to the radiation variables Φ_r (t, z and φ) to each vertex.
- at each vertex, $i \to jk$, include a factor

$$\frac{dt}{t}\,dz\,\frac{\alpha_S(t)}{2\pi}\,P_{i,jk}(z)\,\frac{d\varphi}{2\pi}$$

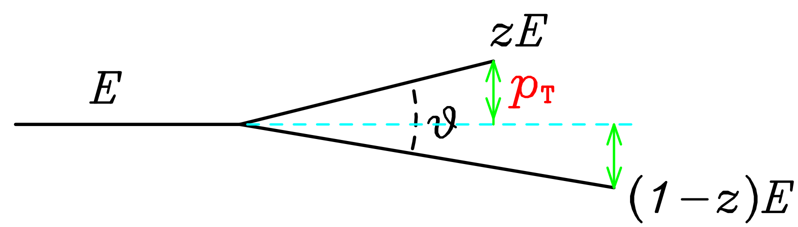
- include a factor $\Delta_i(t_1,t_2)$ to each internal parton i, from hardness t_1 to hardness t_2.
- include a factor $\Delta_i(t,t_0)$ on final lines (t_0 = IR cutoff)

Accuracy: soft divergences and double-log regions

$z \to 1$ ($z \to 0$) region problematic. In fact, for $z \to 1$, $P_{qq}, P_{gg} \div 1/(1-z)$

The choice of the ordering variable t makes a difference

$$\begin{aligned} \text{virtuality:} \quad & t \equiv E^2 z(1-z) \overbrace{\theta^2}^{2(1-\cos\theta)} \\ p_T^2: \quad & t \equiv E^2 z^2 (1-z)^2 \theta^2 \\ \text{angle:} \quad & t \equiv E^2 \theta^2 \end{aligned}$$



$$\text{virtuality}: \; z(1-z) > t/E^2 \quad \Longrightarrow \quad \int \frac{dt}{t} \int_{\sqrt{t}/E}^{1-\sqrt{t}/E} \frac{dz}{1-z} \approx \frac{1}{4} \log^2 \frac{t}{E^2}$$

$$p_T^2: \; z^2(1-z)^2 > t/E^2 \quad \Longrightarrow \quad \int \frac{dt}{t} \int_{t/E^2}^{1-t/E^2} \frac{dz}{1-z} \approx \frac{1}{2} \log^2 \frac{t}{E^2}$$

$$\text{angle}: \quad \Longrightarrow \quad \int \frac{dt}{t} \int_0^1 \frac{dz}{1-z} \approx \log t \, \log \Lambda$$

Sizable difference in double-log structure!

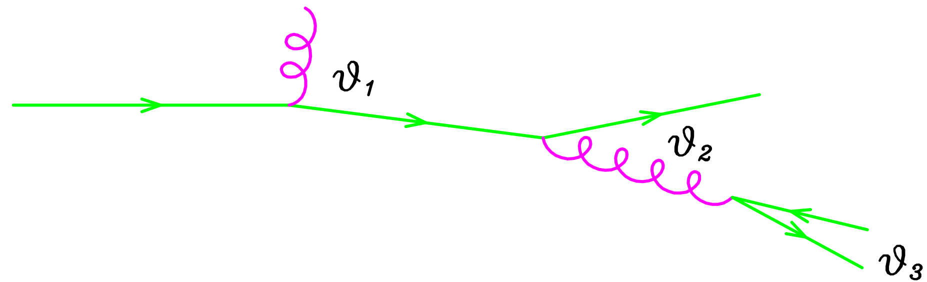
Angular ordering and color coherence

Mueller (1981) showed that angular ordering is the correct choice

$$\frac{d\theta}{\theta}\,\frac{\alpha_s\left(p_T^2\right)}{2\pi}\,P(z)\,dz$$

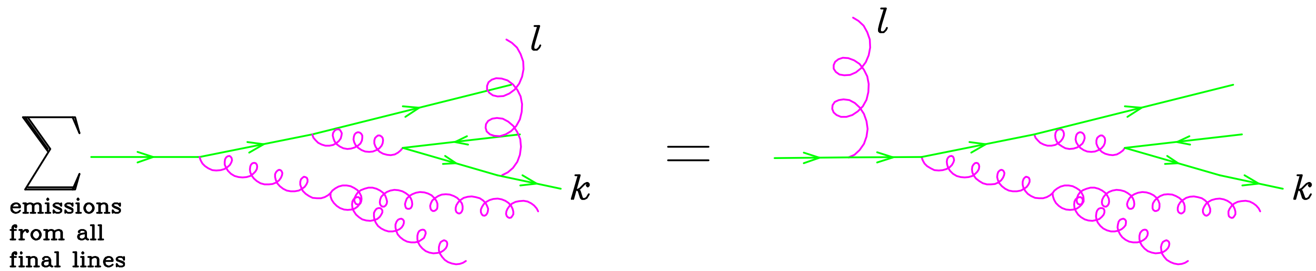
$$\theta_1 > \theta_2 > \theta_3 \ldots$$

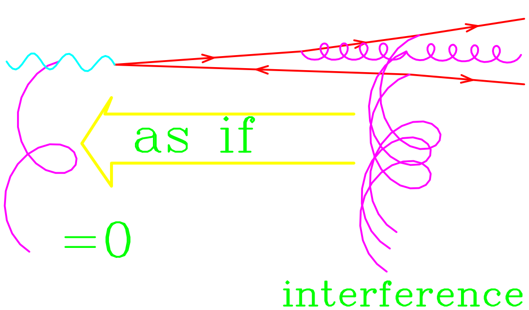
$$p_T^2 = E^2 z^2 (1-z)^2 \theta^2$$



$\alpha_s(p_T^2)$ for a correct treatment of charge renormalization in soft region

Soft gluons emitted at large angles from final-state partons add coherently





- angular ordering accounts for soft gluon interference.
- intensity for photon jets $= 0$
- intensity for gluon jets $= C_A$ instead of $2\,C_F + C_A$

Some available codes and accuracy

	collinear	soft-collinear	soft large-N_c	soft
PYTHIA	leading	partial	no	no
HERWIG	leading	leading	no	no
ARIADNE	partial	partial	leading	no
PYTHIA6.4	partial	partial	leading	no
SHERPA	leading	partial	no	no

One can realistically aim at

leading collinear, leading double log, leading soft in large-N_c limit

Soft effects for finite N_c require matrix exponentiation in the Sudakov form factor.

NLO + Parton Shower

LO-ME good for shapes. Uncertain absolute normalization

$$\alpha_s^n(2\mu) \approx \alpha_s^n(\mu)\left(1 - b_0\alpha_s(\mu)\log(4)\right)^n \approx \alpha_s^n(\mu)\left(1 - n\alpha_s(\mu)\right)$$

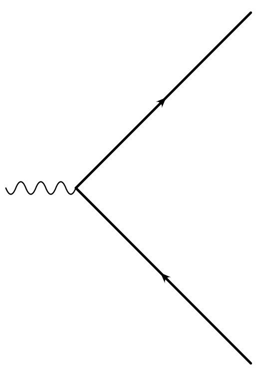
For $\mu = 100$ GeV, $\alpha_s = 0.12$, normalization uncertainty:

$W + 1J$	$W + 2J$	$W + 3J$
$\pm 12\%$	$\pm 24\%$	$\pm 36\%$

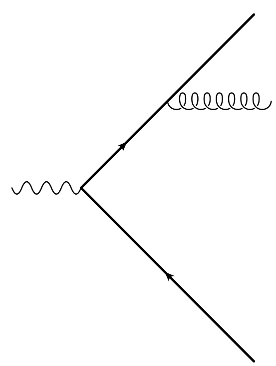
To improve on this, we need to go to NLO

The main problem in merging a NLO result and a Parton Shower is not to double-count radiation: the shower might produce some radiation already present at the NLO level.

LO:



NLO:



POWHEG: how it works

1. POWHEG, POsitive Weight Hardest Emission Generator, [Nason, hep-ph/0409146], generates first a partonic event with just one single emission, at NLO level, and with the correct probability in order not to have double-counting coming from (subsequent) radiation. The p_T of the produced radiation works as an upper cutoff for the p_T's of the entire subsequent shower.

2. The event is written on a file using the standard Les Houches Interface and is processed by the Parton Shower program (HERWIG, PYTHIA...), that showers the event, but with a p_T less than the p_T generated by POWHEG (p_T veto).

   - if the shower is ordered in p_T (for example PYTHIA), nothing else needs to be done
   - if the shower is ordered in angle (for example HERWIG), we need to generate correctly soft radiation at large angle.
     - pair up the partons that are nearest in p_T
     - generate an angular-ordered shower associated with the paired parton, stopping at the angle of the paired partons (truncated shower)
     - generate all subsequent vetoed showers

POsitive-Weight Hardest Emission Generator

✓ it is independent from parton-shower programs. POWHEG can be interfaced with both PYTHIA and HERWIG, or with your favorite showering program, if the vetoed shower is implemented, according to the Les Houches Interface.

✓ it can use existing NLO results

✓ it generates events with positive weights

✓ As far as the hardest emission is concerned, POWHEG guarantees:

- NLO accuracy on integrated quantities
- collinear, double-log (soft-collinear), large-N_c-soft single-log of the Sudakov (in fact, corrections that exponentiates are obviously OK)

✓ As far as subsequent (less hard) emissions, the output has the accuracy of the SMC one is using.

✗ no truncated shower implemented up to now. But this is a problem that affects all the angular-ordered SMC when the shower is initiated by a relatively complex matrix element.

Existing implementations

The POWHEG method has already been successfully used in

- $pp \to ZZ$ [Nason and Ridolfi, hep-ph/0606275]
- $e^+e^- \to$ hadrons [Latunde-Dada, Gieseke and Webber, hep-ph/0612281]
  $e^+e^- \to t\bar{t}$ with top decay [Latunde-Dada, arXiv:0806.4560]
- $pp \to Q\overline{Q}$ ($c\bar{c}$, $b\bar{b}$, $t\bar{t}$) with spin correlations [Frixione, Nason and Ridolfi, arXiv:0707.3088].
- $pp \to W/Z$ with spin correlations [Alioli, Nason, Oleari and Re, arXiv:0805.4802; Hamilton, Richardson and Tully, arXiv:0806.0290].
- $pp \to H$ [Alioli, Nason, Oleari and Re, arXiv:0812.0578]

All POWHEG implementations for hadronic colliders have been interfaced to both PYTHIA and HERWIG.

To appear soon

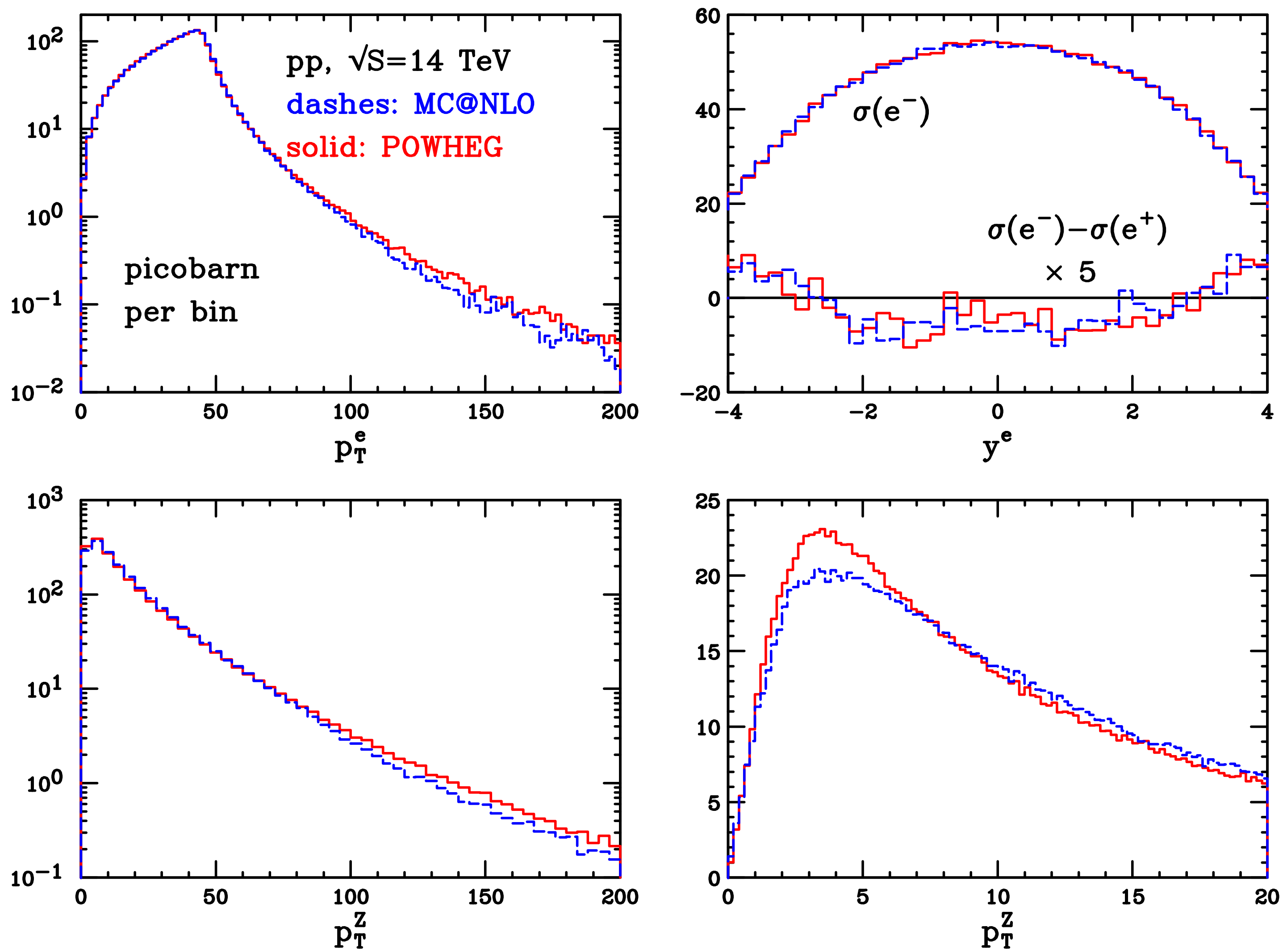
- $pp \to H$ [Hamilton, Richardson and Tully, HERWIG++ group]
- single top production [Alioli, Nason, Oleari and Re]
- $pp \to W/Z + 1$ jet [Alioli, Nason, Oleari and Re]

We are working now on a general framework for the implementation of any NLO process into the POWHEG formalism.

Given the Born, real and virtual amplitudes, combine them automatically to produce POWHEG events.

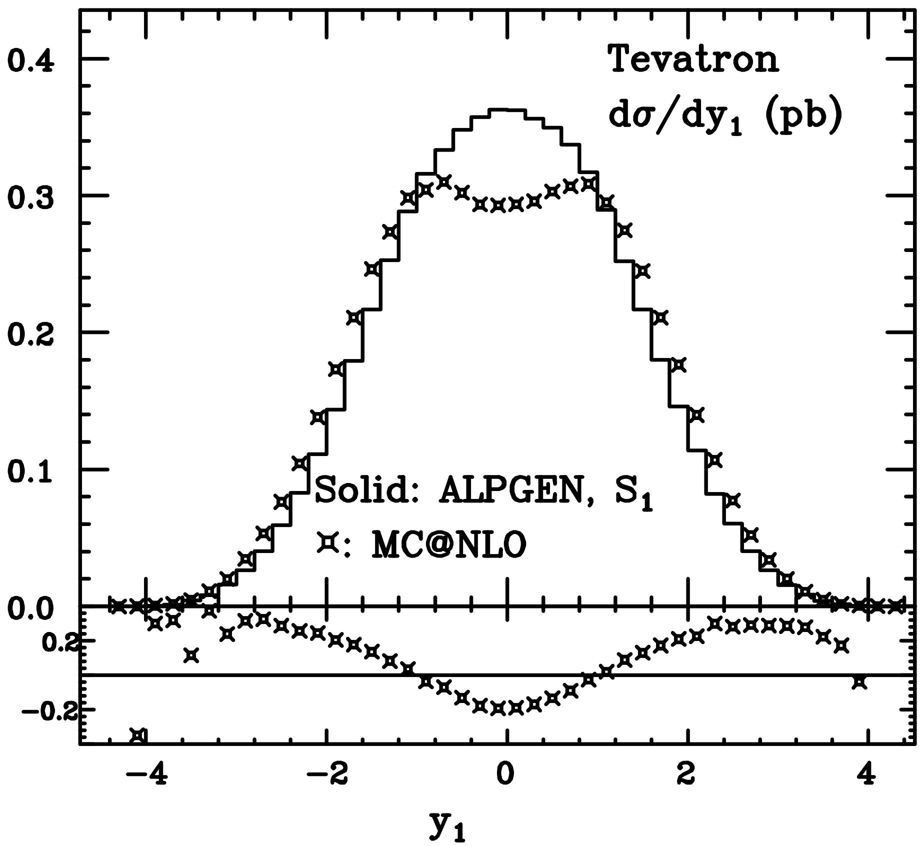
Truncated showers have been studied in $e^+e^- \to$ hadrons [Latunde-Dada, Gieseke and Webber] and is included in the HERWIG++ framework [Bähr, Gieseke, Gigg, Grellscheid, Hamilton, Plätzer, Richardson, Seymour and Tully, arXiv:0812.0529]

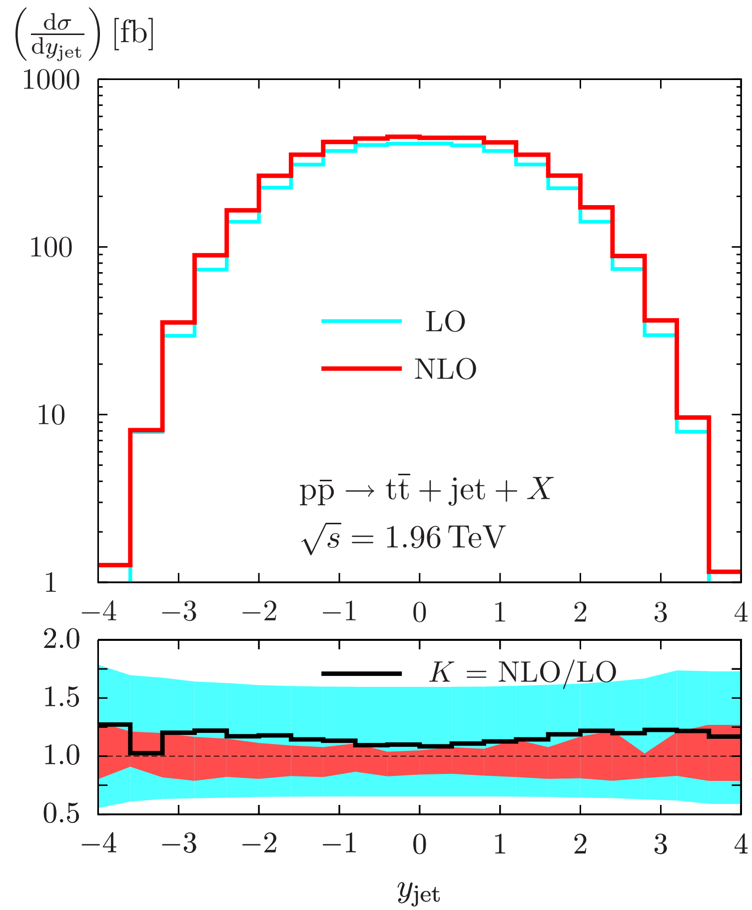
Z production: POWHEG + HERWIG vs MC@NLO



Small differences in the high- and low-p_T regions.

ALPGEN and NLO vs MC@NLO: $t\bar{t}$ + 1 jet



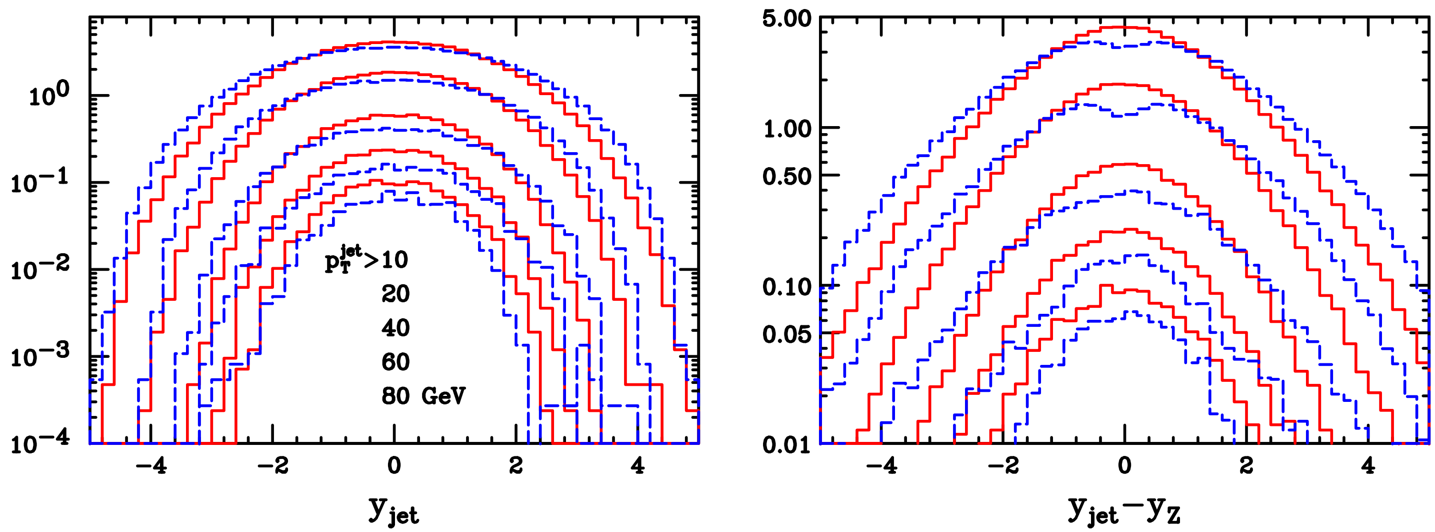


Rapidity y_1 of the leading jet (highest p_T).

POWHEG's distribution as in ALPGEN: no dip present. The size of discrepancy can be attributed to different treatment of higher-order terms.

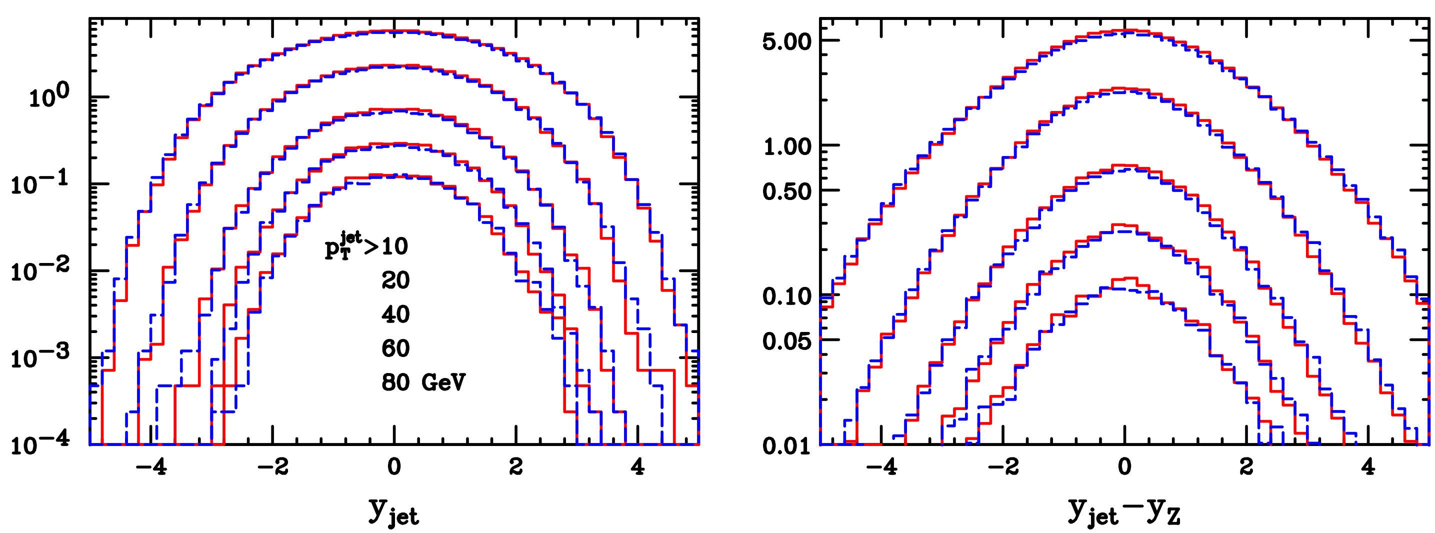
$pp \to t\bar{t}$ + jet at NLO [Dittmaier, Uwer, Weinzierl, arXiv:0810.0452] shows no dip too.

Rapidity distribution of hardest jet at Tevatron



POWHEG+HERWIG

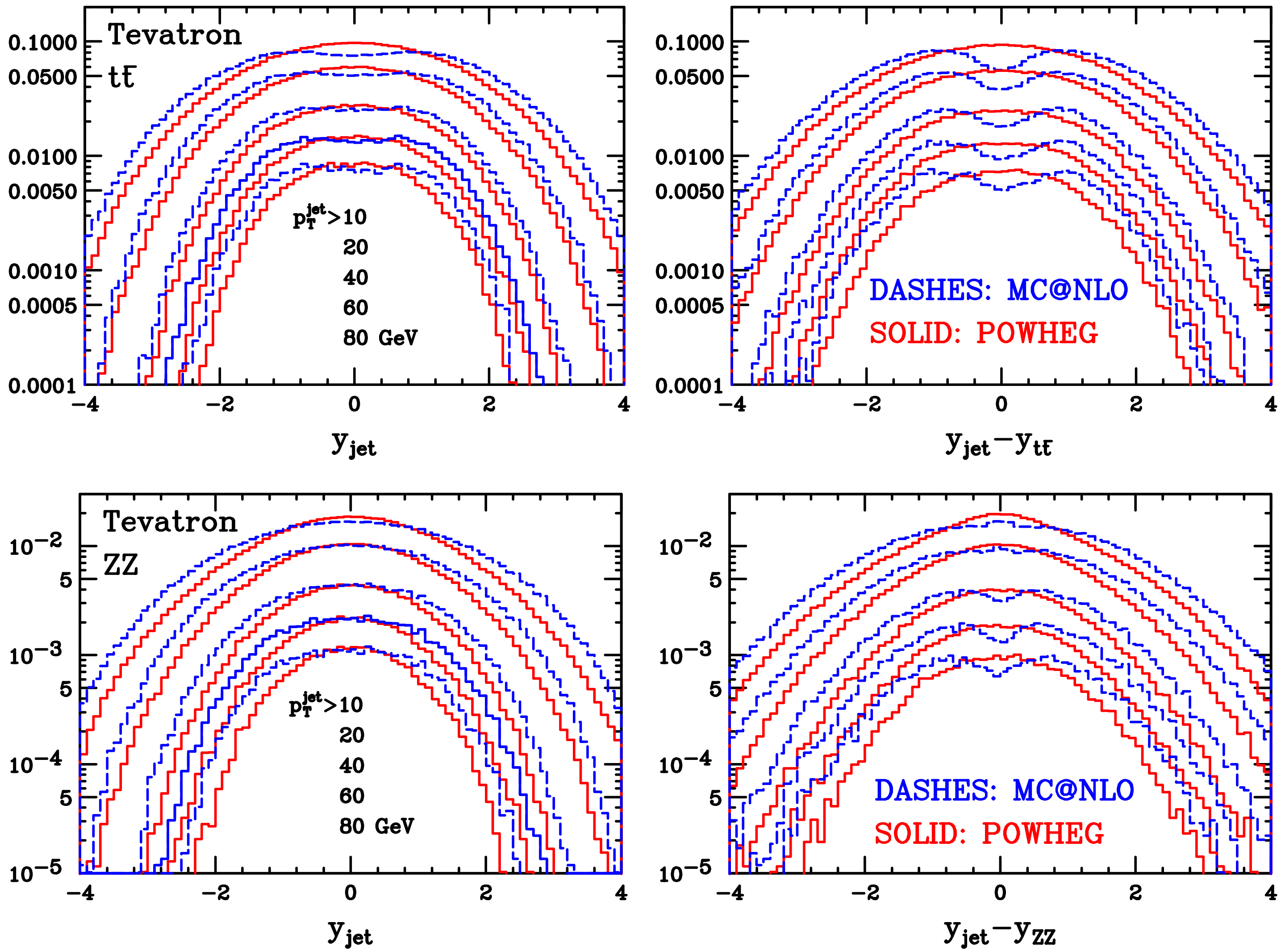
MC@NLO



POWHEG+PYTHIA

PYTHIA

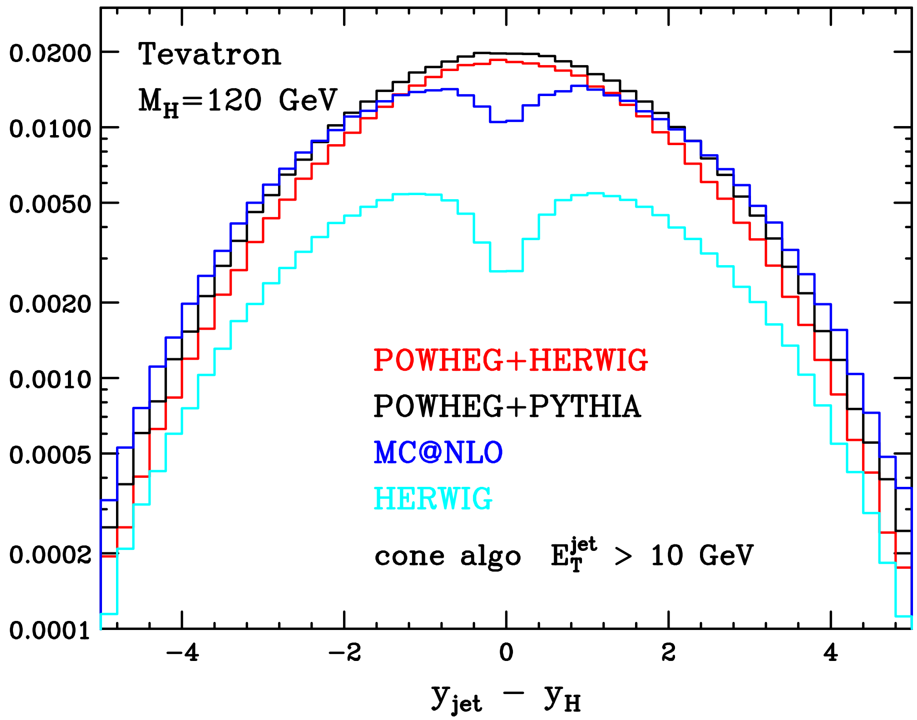
Rapidity distribution of hardest jet at Tevatron

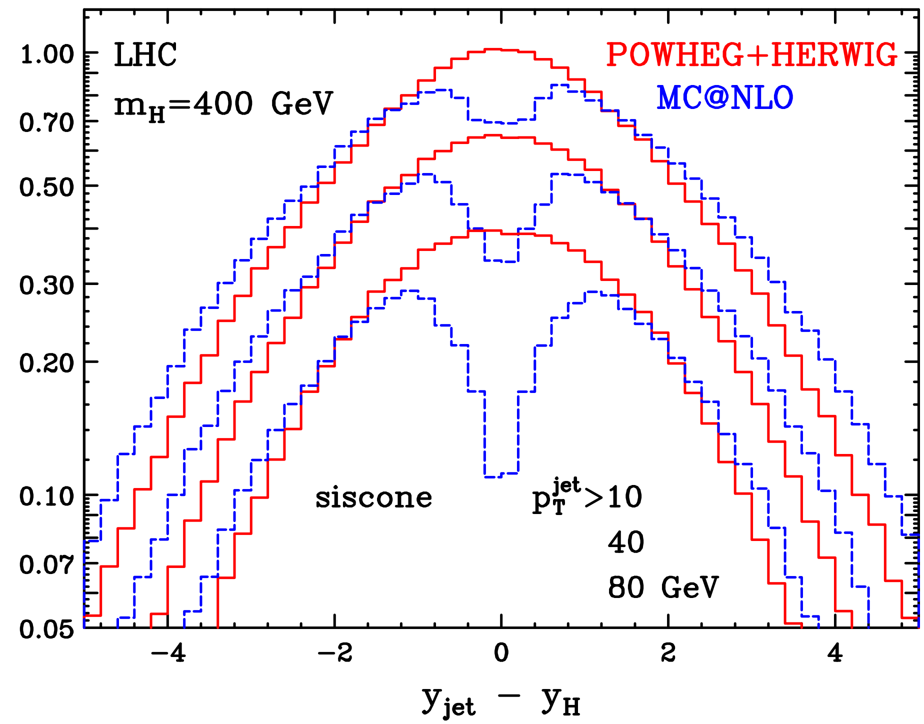


POWHEG+HERWIG
MC@NLO

POWHEG+HERWIG
MC@NLO

Rapidity distribution at Tevatron and LHC



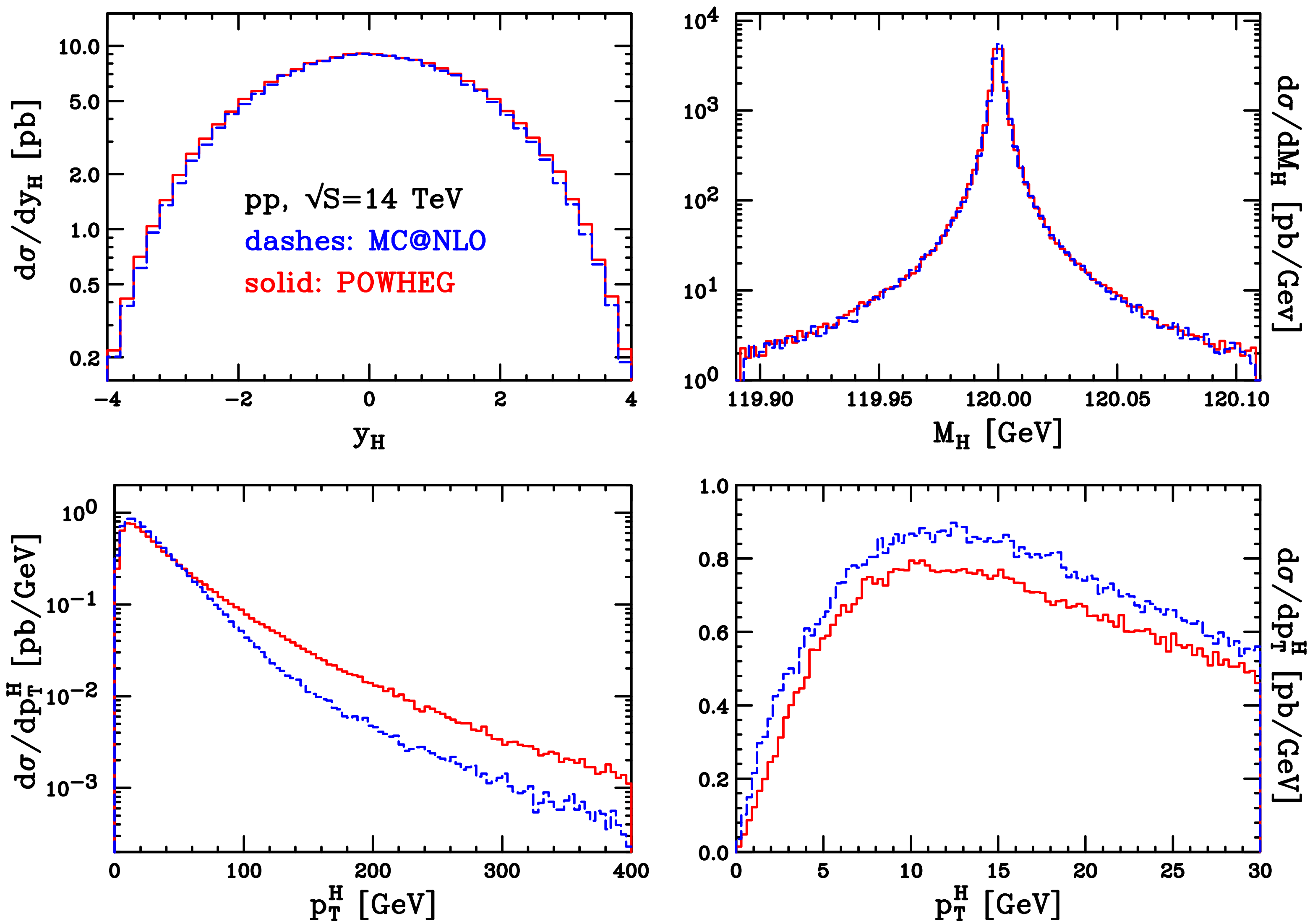


Dip inherited from the even-deeper dip of HERWIG. MC@NLO fills partially the dip.

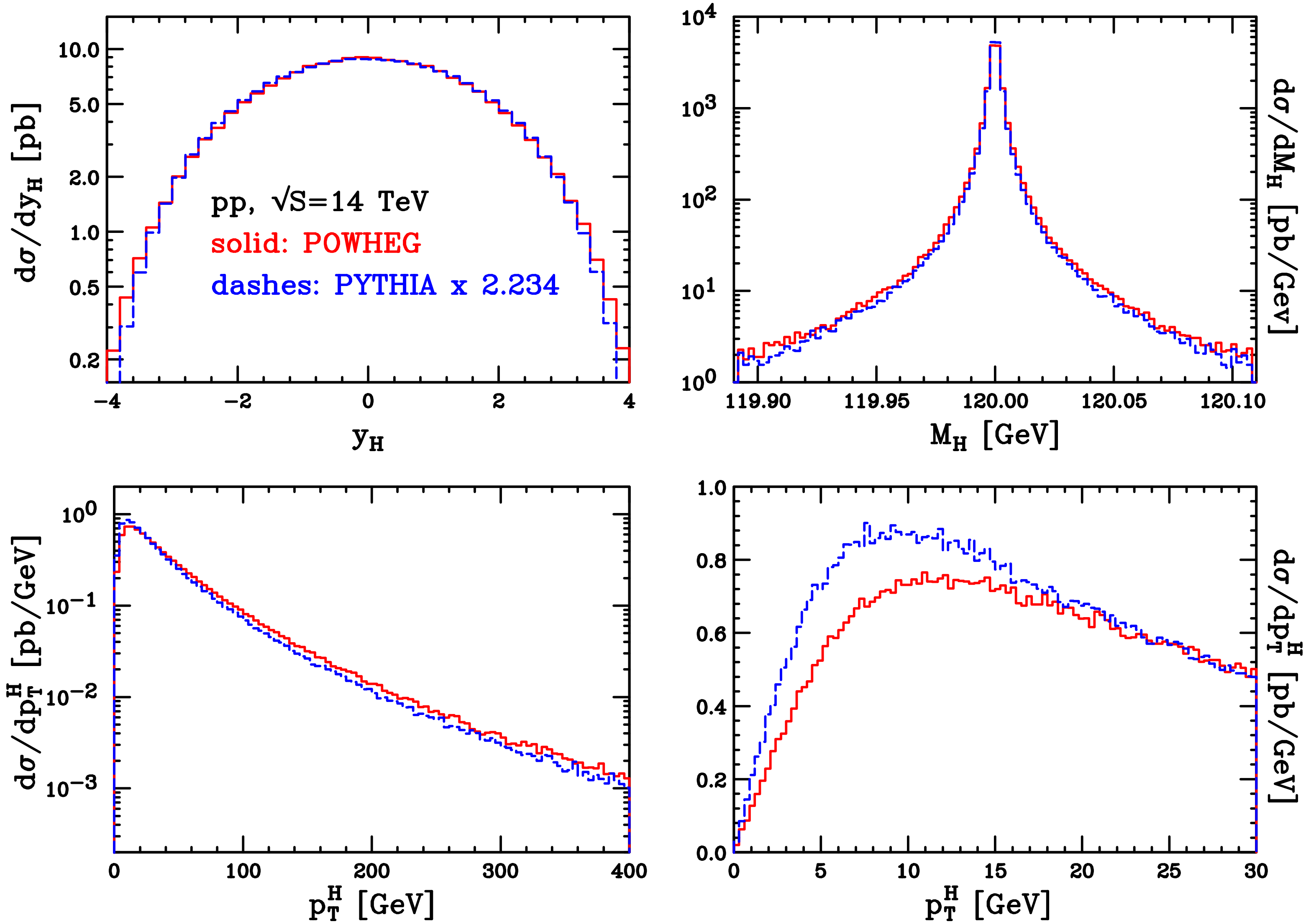
A shower Monte Carlo is accurate in the radiation of the hardest jet only in the collinear regions.

The dip in the MC@NLO result is compatible with an effect beyond NLO.

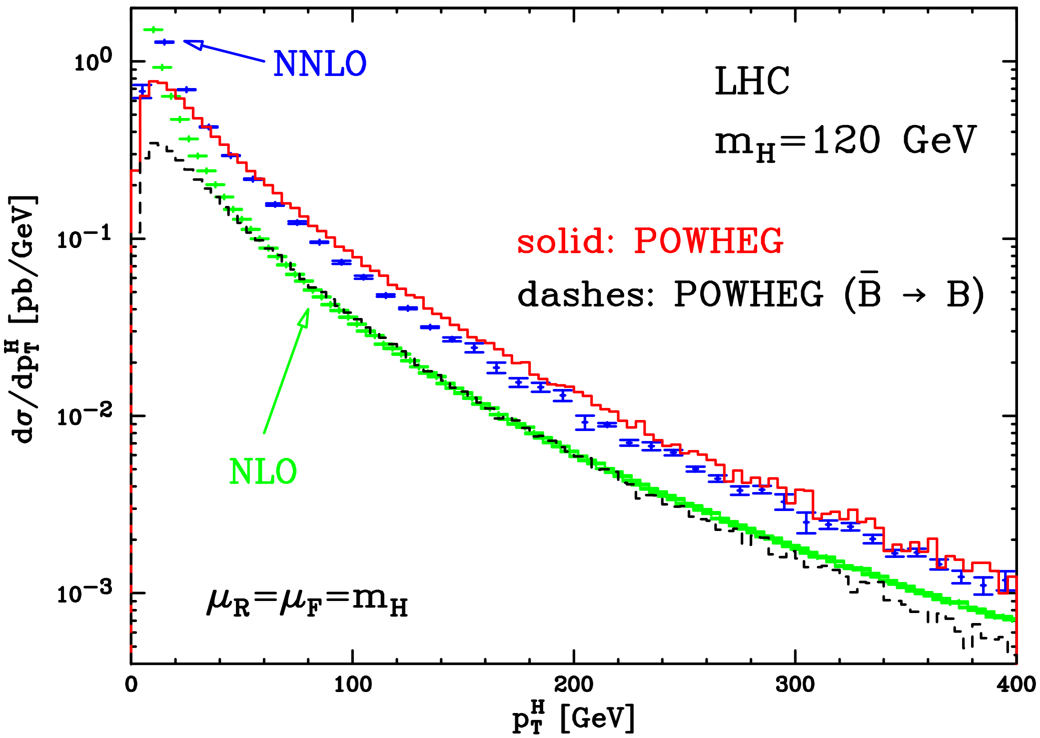
Higgs boson production at the LHC

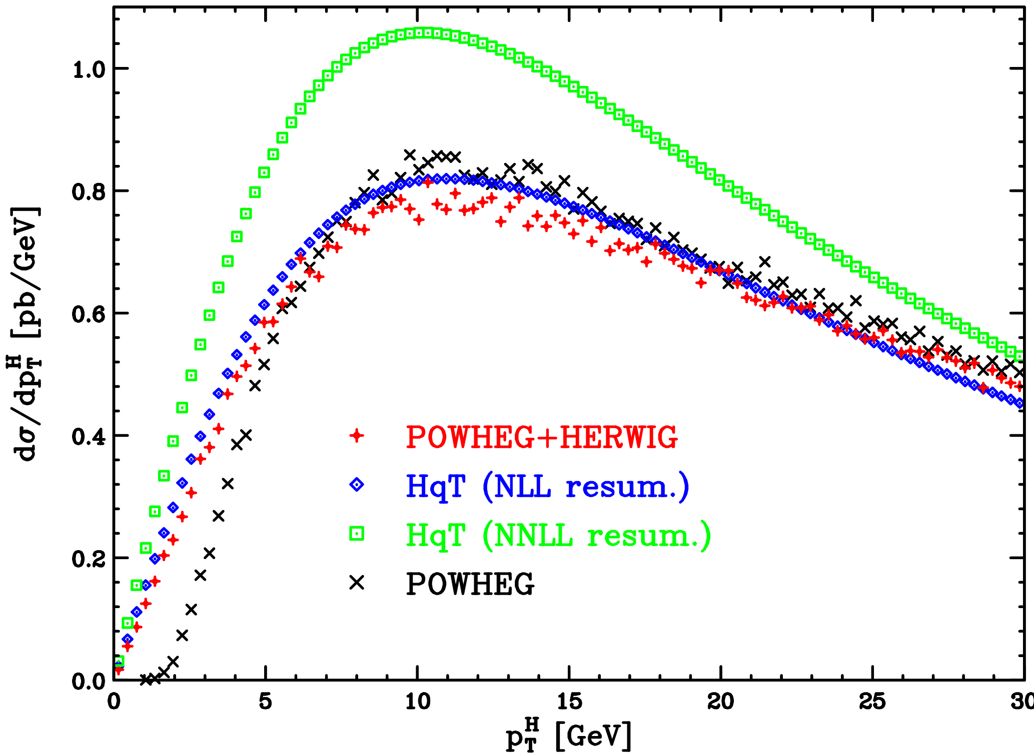


Higgs boson production at the LHC



Higgs boson production at the LHC





$$\overline{B}(\mathbf{\Phi}_n) = B(\mathbf{\Phi}_n) + V(\mathbf{\Phi}_n) + \int d\Phi_r \left[R(\mathbf{\Phi}_n, \Phi_r) - C(\mathbf{\Phi}_n, \Phi_r)\right]$$

$$d\sigma = \overline{B}(\mathbf{\Phi}_n)\, d\mathbf{\Phi}_n \left\{ \Delta(\mathbf{\Phi}_n, p_T^{min}) + \Delta(\mathbf{\Phi}_n, p_T)\, \frac{R(\mathbf{\Phi}_n, \Phi_r)}{B(\mathbf{\Phi}_n)} d\Phi_r \right\}$$

$$\approx \frac{\overline{B}(\mathbf{\Phi}_n)}{B(\mathbf{\Phi}_n)}\, R(\mathbf{\Phi}_{n+1})\, d\mathbf{\Phi}_{n+1} = \left\{ 1 + \mathcal{O}(\alpha_s) \right\} R(\mathbf{\Phi}_{n+1})\, d\mathbf{\Phi}_{n+1}$$

From NLO to POWHEG

POWHEG is a **method**, NOT (only) a set of programs!

POWHEG is fully general and can be applied to any NLO subtraction framework.

We have provided any user with all the formulae and ingredients to implement an existing NLO calculation in the POWHEG formalism [Frixione, Nason and Oleari, arXiv:0709.2092 [hep-ph]].

We have looked in detail at POWHEG in two subtraction schemes:

- the Frixione, Kunszt and Signer scheme
- the Catani and Seymour scheme.

We have discussed, in a pedagogical way, two examples:

- $e^+e^- \to q\bar{q}$
- $q\bar{q} \to V$

The `fortran` implementation of the POWHEG code for these two processes (and all the others) can be found at

`http://moby.mib.infn.it/~nason/POWHEG`

Strategy and conclusions

✓ Shower Monte Carlo programs to do the final shower already exist

✓ Most of them implement a p_T veto

✓ Most of them comply with a standard interface to hard processes, the so called Les Houches Interface (LHI)

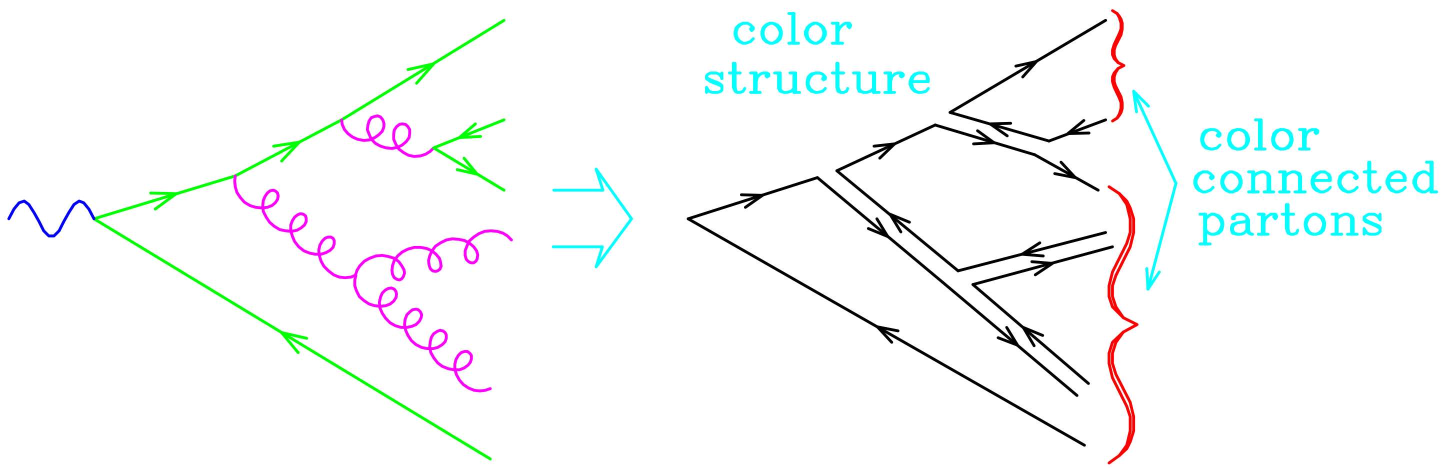
SO...

- construct a POWHEG for a NLO process. Output on LHI
- if needed, construct a generator capable to add truncated showers to events from the LHI. Output again on LHI
- use standard Shower Monte Carlo to perform the p_T-vetoed final shower from the event on LHI.

Backup slides

Color and hadronization

Shower Monte Carlo programs assign color labels to partons. Only color connections are recorded (in large N_c limit). The initial color is assigned according to hard cross section.



Color assignments are used in the hadronization model.

Most popular models: Lund string model, cluster model.

In all models, color singlet structures are formed out of color connected partons, and are decayed into hadrons, preserving energy and momentum.

Summarizing

- In high-energy collider physics not many questions can be answered without a Shower Monte Carlo (SMC).
- The name shower comes from the fact that we dress a hard event with QCD radiation.
- After a latency period, many physicists are now looking at shower Monte Carlo models again, under different perspective: Catani, Krauss, Kühn & Webber; Mangano, Moretti, Piccinini, Pittau, Polosa & Treccani; Frixione & Webber; Kramer, Mrenna, Nagy & Soper; Giele, Kosower & Skands; Bauer & Schwartz; Schumann & Krauss; Dinsdale, Ternick & Weinzierl; . . .
- Shower algorithms summarize most of our knowledge in perturbative QCD: infrared cancellations, Altarelli-Parisi equations, soft coherence, Sudakov form factors. All have a simple interpretation in terms of shower algorithms.

A word of warning

"The Monte Carlo simulation has become the major mean of visualization of not only detector performance but also of physics phenomena. So far so good. But it often happens that the physics simulations provided by the Monte Carlo generators carry the authority of data itself. They look like data and feel like data, and if one is not careful they are accepted as if they were data."

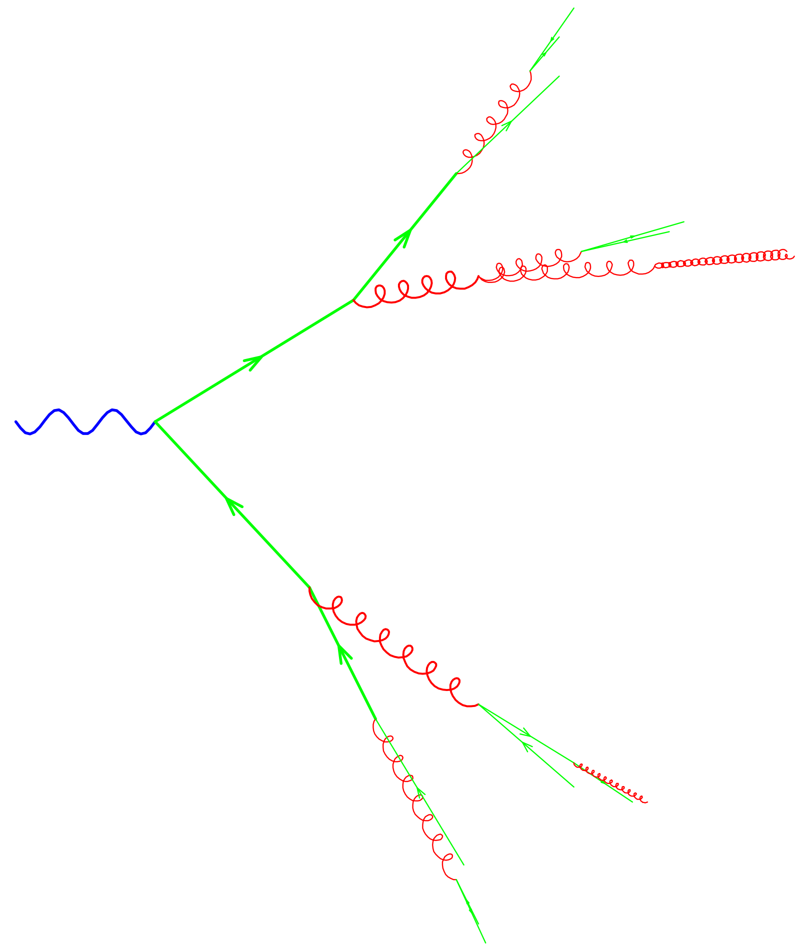
J.D. Bjorken

Talk given at the 75th anniversary celebration of the Max-Planck Institute of Physics, Munich, Germany, December 10th, 1992, as quoted in Beam Line, Winter 1992, Vol. 22, No. 4. Reference taken from Sjöstrand.

Typical dominant configuration at very high Q^2

$\gamma^* \longrightarrow$ hadrons

- Besides $q \rightarrow qg$, also $g \rightarrow gg$, $g \rightarrow q\bar{q}$ come into play.
- In the typical configurations, intermediate angles are of order of geometric average of upstream and downstream angles.
- Each angle is $\mathcal{O}(\alpha_s)$ smaller than its upstream angle, and $\mathcal{O}(\alpha_s)$ bigger than its downstream angle.
- As relative momenta become smaller, α_s becomes bigger, and this picture breaks down.



First branching

The probability of the first branching is independent of subsequent branchings because of Kinoshita-Lee-Nauenberg cancellation. It is given by

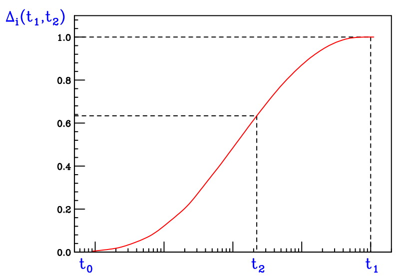
$$dP_{\text{first}} = \Delta_i(t,t')\,\frac{\alpha_S(t')}{2\pi}\,\frac{dt'}{t'}\,P_{i,jk}(z)\,dz\,\frac{d\varphi}{2\pi}$$

Upon integrating in z and φ, and summing over jk, we have

$$dP_{\text{first}} = \Delta_i(t,t')\,\frac{\alpha_S(t')}{2\pi}\,\frac{dt'}{t'}\int\sum_{(jk)}P_{i,jk}(z)\,dz\,\frac{d\varphi}{2\pi} = d\Delta_i(t,t')$$

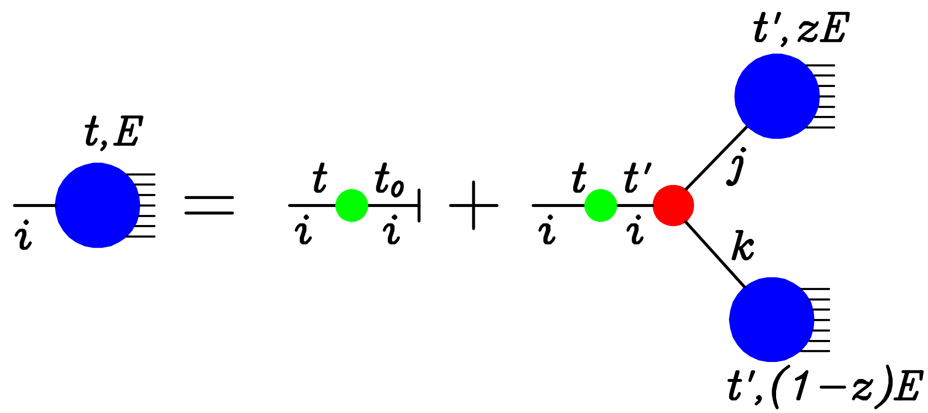
i.e. the distribution is uniform in the Sudakov form factor.
The integral over the whole t' range, from the minimum value t_0 (IR cutoff) up to t, is given by

$$\int_{t_0}^{t} dP_{\text{first}} = \int_{t_0}^{t} d\Delta_i(t,t') = \Delta_i(t,t) - \Delta_i(t,t_0) = 1 - 0 = 1$$



as it should be for a correct probabilistic interpretation.

Final recipe II



- include a factor $\Delta_i(t_1, t_2)$ to each internal parton i, from hardness t_1 to hardness t_2.

$$\Delta_i(t_1, t_2) = \exp\left[-\sum_{(jk)} \int_{t_2}^{t_1} \frac{dt}{t} \frac{\alpha_S(t)}{2\pi} \int dz\, P_{i,jk}(z) \int \frac{d\varphi}{2\pi}\right]$$

  The weights $\Delta_i(t_1, t_2)$ are called Sudakov form factors. They resum all the dominant virtual corrections to the tree graph (in the collinear approximation). Notice also that the inclusion of real and virtual corrections gives a net result of 1 (cancellation of collinear singularities in inclusive quantities).

- include a factor $\Delta_i(t, t_0)$ on final lines (t_0 = IR cutoff)

Actual implementation of the shower algorithm

We start from a given value of the ordering variable t. We want to generate the value t' for the next emission, according to the probability

$$dP_{\text{first}} = \Delta_i(t,t') \frac{\alpha_S(t')}{2\pi} \frac{dt'}{t'} \int \sum_{(jk)} P_{i,jk}(z)\, dz \frac{d\varphi}{2\pi} = d\Delta_i(t,t')$$

Since this is an exact differential form, we proceed as in the case we want to generate a random variable x according to a distribution function $f(x)$, whose indefinite integral is known, starting from a uniform random variable r

$$dP = f(X)\, dX = 1\, dR \qquad \text{where} \qquad f(X)\, dX = dF(X)$$

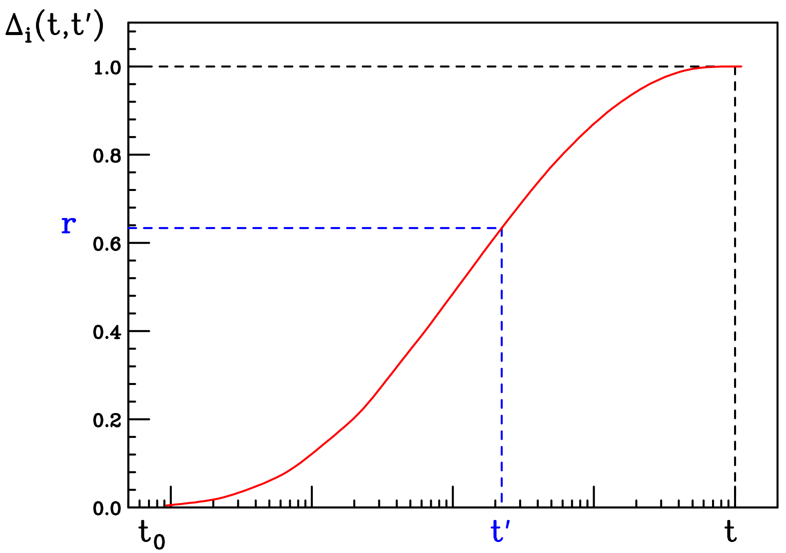
$$\int_{x_{\min}}^{x} f(X)\, dX = F(x) = \int_0^r 1\, dR = r \quad \Longrightarrow \quad x = F^{-1}(r)$$

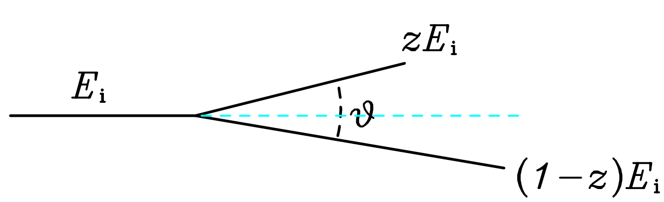
Actual implementation of the shower algorithm

✓ generate a hard process configuration with a probability proportional to its parton-level cross section. Parton densities are evaluated at the typical "high" scale Q of the process

✓ for each final-state colored parton, generate a shower

- set $t = Q^2$
- generate a uniform random number $0 < r < 1$
- solve the equation $\Delta_i(t, t') = r$ for t'
- if $t' < t_0$ stop here (final state line). Begin hadronization
- if $t' > t_0$, generate z, jk with probability $P_{i,jk}(z)$, and $0 < \varphi < 2\pi$ uniformly. Assign energies $E_j = zE_i$ and $E_k = (1-z)E_i$ to partons j and k. The angle θ between their momenta is fixed by t' and with φ their direction is completely specified
- restart shower from each of the two branched parton j and k, setting the ordering parameter $t = t'$.





Shower algorithm

✓ for each initial-state colored parton, generate a shower in a similar way, but using a "trick": the backward evolution (Sjöstrand)

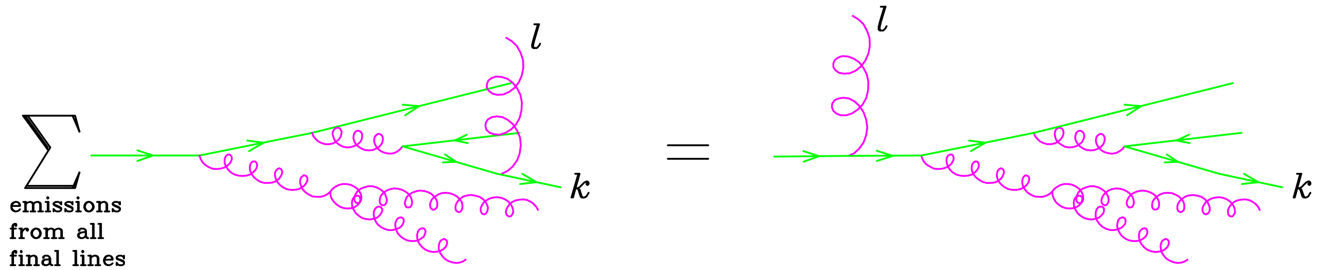
$$\frac{f_i^h(t', x)\,\Delta(t, t')}{f_i^h(t, x)} = r$$

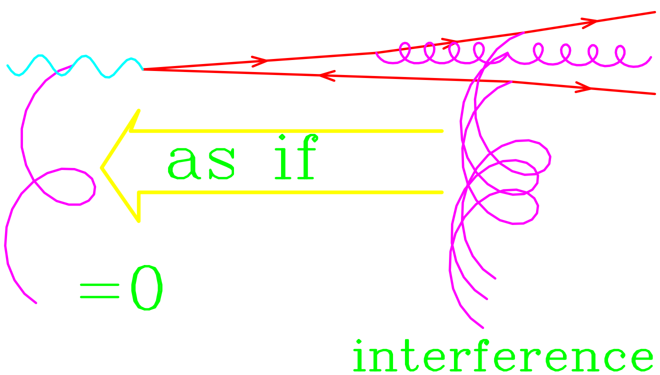
where f_i^h is the parton density for the colliding hadron h, where parton i carries a momentum fraction $x = E_i/E_h$

Some momentum reshuffling is needed in order to preserve local (at each vertex) and global momentum conservation

Color coherence

Soft gluons emitted at large angles from final-state partons add coherently





- angular ordering accounts for soft gluon interference.
- intensity for photon jets $= 0$
- intensity for gluon jets $= C_A$ instead of $2\,C_F + C_A$

In angular-ordered shower Monte Carlo, large-angle soft emission is generated first.

Hardest emission, i.e. highest $p_T = E\,z(1-z)\,\theta$, in general, happens later.

Some available codes

- COJETS Odorico (1984)
- ISAJET Paige+Protopopescu (1986)
- FIELDAJET Field (1986)
- JETSET Sjöstrand (1986)
- PYTHIA Bengtsson+Sjöstrand (1987), Sjöstrand+Skands (2004)
- HERWIG Marchesini+Webber (1988),
  Marchesini+Webber+Abbiendi+Knowles+Seymour+Stanco (1992)
- ARIADNE Lönnblad (1992)
- SHERPA Gleisberg+Höche+Krauss+Schälicke+Schumann+Winter (2004)

New developments

- Interfacing Matrix Elements (ME) generators with Parton Showers : CKKW matching [Catani, Krauss, Küen, Webber], MLM matching [Mangano]
- Interfacing NLO calculations with Parton Showers: MC@NLO [Frixione, Webber], POWHEG [Nason]

Several other approaches have appeared

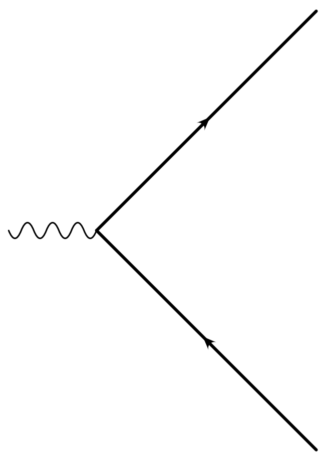
- $e^+e^- \to$ 3 partons [Kramer, Mrenna, Soper]
- Shower by antenna factorization [Giele, Kosower, Skands]
- Shower by Catani-Seymour dipole factorization [Schumann, Krauss]
- Shower with quantum interference [Nagy, Soper]
- Shower by Soft Collinear Effective Theory [Bauer, Schwartz]
- Shower from the dipole formalism [Dinsdale, Ternick, Weinzierl]

Up to now, complete results for hadron colliders only from MC@NLO and POWHEG.

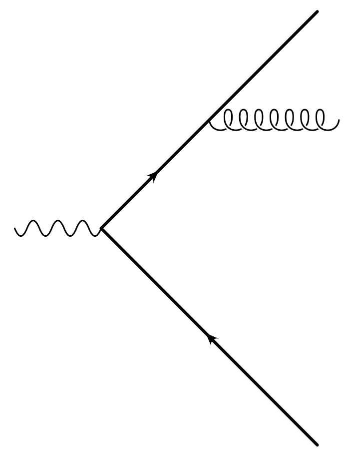
NLO + Parton Shower

The main problem in merging a NLO result and a Parton Shower is not to double-count radiation: the shower might produce some radiation already present at the NLO level.

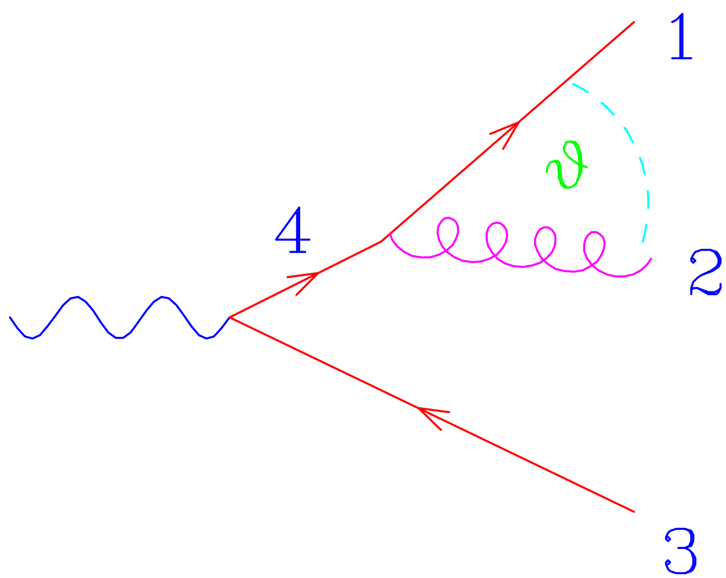
LO:



NLO:



Example of truncated shower: e^+e^-



- nearby partons: 1 and 2
- truncated shower: 1 and 2 pair, from θ up to a maximum angle. The truncated shower reintroduces coherent soft radiation from 1 and 2 at angles larger than θ (angular-ordered shower Monte Carlo programs generate those earlier).
- 1 and 2 shower from θ to cutoff
- 3 showers from maximum to cutoff

Truncated showers not yet implemented.

No evidence of effects from their absence in ZZ and e^+e^- production. Might be some effects in heavy-quark production.

NLO calculations

We can always parametrize the $(n+1)$-body phase space $\mathbf{\Phi}_{n+1}$ in terms of the Born phase space $\mathbf{\Phi}_n$ and three radiation variables Φ_r: $\mathbf{\Phi}_{n+1} = \{\mathbf{\Phi}_n, \Phi_r\}$

$$\langle O \rangle = \int O \, d\sigma = \int d\mathbf{\Phi}_n \, O(\mathbf{\Phi}_n) \, [B(\mathbf{\Phi}_n) + V_b(\mathbf{\Phi}_n)] + \int d\mathbf{\Phi}_n \, d\Phi_r \, O(\mathbf{\Phi}_n, \Phi_r) \, R(\mathbf{\Phi}_n, \Phi_r)$$

where V_b is the (divergent) virtual differential cross section. The virtual and real-radiation integrals are separate divergent. Their sum is finite (for any infra-red safe observable).
A typical subtraction method re-organize the integrals in the form

$$\begin{aligned} \langle O \rangle &= \int d\mathbf{\Phi}_n \, O(\mathbf{\Phi}_n) \left[B(\mathbf{\Phi}_n) + V_b(\mathbf{\Phi}_n) + \int d\Phi_r \, C(\mathbf{\Phi}_n, \Phi_r) \right] \\ &+ \int d\mathbf{\Phi}_n \, d\Phi_r \, \underbrace{\left[O(\mathbf{\Phi}_n, \Phi_r) \, R(\mathbf{\Phi}_n, \Phi_r) - O(\mathbf{\Phi}_n) \, C(\mathbf{\Phi}_n, \Phi_r) \right]}_{\text{finite}} \end{aligned}$$

Defining

$$V(\mathbf{\Phi}_n) = V_b(\mathbf{\Phi}_n) + \int d\Phi_r \, C(\mathbf{\Phi}_n, \Phi_r) \qquad \Longleftarrow \text{ finite}$$

we have

$$\langle O \rangle = \int d\mathbf{\Phi}_n \, O(\mathbf{\Phi}_n) \, [B(\mathbf{\Phi}_n) + V(\mathbf{\Phi}_n)] + \int d\mathbf{\Phi}_n \, d\Phi_r \, [O(\mathbf{\Phi}_n, \Phi_r) \, R(\mathbf{\Phi}_n, \Phi_r) - O(\mathbf{\Phi}_n) \, C(\mathbf{\Phi}_n, \Phi_r)]$$

NLO in SMC

Shower Monte Carlo (SMC) cross section for first emission ($d\Phi_r = dt\,dz\,d\varphi$)

$$\langle O \rangle = \int d\mathbf{\Phi}_n\, B(\mathbf{\Phi}_n) \left\{ O(\mathbf{\Phi}_n)\Delta_{t_0} + \int_{t_0} \frac{dt}{t}\, dz\, d\varphi\, O(\mathbf{\Phi}_n, \Phi_r)\, \Delta_t\, \frac{\alpha_s}{2\pi}\, P(z) \right\}$$

with

$$\Delta_t = \exp\left[- \int_t \frac{dt'}{t'}\, dz'\, d\varphi'\, \frac{\alpha_s}{2\pi}\, P(z') \right]$$

The expansion at order α_s gives the NLO_{SMC}

$$\langle O \rangle = \int d\mathbf{\Phi}_n\, B(\mathbf{\Phi}_n) \left\{ O(\mathbf{\Phi}_n) + \int_{t_0} \frac{dt}{t}\, dz\, d\varphi\, \left[O(\mathbf{\Phi}_n, \Phi_r) - O(\mathbf{\Phi}_n) \right] \frac{\alpha_s}{2\pi}\, P(z) \right\}$$

This is the inexact NLO correction implemented by the SMC

How do we reach exact NLO accuracy?

Towards NLO accuracy

$$\begin{aligned}\langle O\rangle &= \int d\boldsymbol{\Phi}_n\, O(\boldsymbol{\Phi}_n)\,[B(\boldsymbol{\Phi}_n)+V(\boldsymbol{\Phi}_n)] \\ &+ \int d\boldsymbol{\Phi}_n\, d\Phi_r\,[O(\boldsymbol{\Phi}_n,\Phi_r)\,R(\boldsymbol{\Phi}_n,\Phi_r) - O(\boldsymbol{\Phi}_n)\,C(\boldsymbol{\Phi}_n,\Phi_r)] \\ &= \int d\boldsymbol{\Phi}_n\, O(\boldsymbol{\Phi}_n)\left\{B(\boldsymbol{\Phi}_n)+V(\boldsymbol{\Phi}_n)+\int d\Phi_r\left[R(\boldsymbol{\Phi}_n,\Phi_r)-C(\boldsymbol{\Phi}_n,\Phi_r)\right]\right\} \\ &+ \int d\boldsymbol{\Phi}_n\, d\Phi_r\, R(\boldsymbol{\Phi}_n,\Phi_r)\,[O(\boldsymbol{\Phi}_n,\Phi_r) - O(\boldsymbol{\Phi}_n)]\end{aligned}$$

Define

$$\overline{B}(\boldsymbol{\Phi}_n) = B(\boldsymbol{\Phi}_n)+V(\boldsymbol{\Phi}_n)+\int d\Phi_r\left[R(\boldsymbol{\Phi}_n,\Phi_r)-C(\boldsymbol{\Phi}_n,\Phi_r)\right]$$

$$\langle O\rangle = \int d\boldsymbol{\Phi}_n\, O(\boldsymbol{\Phi}_n)\,\overline{B}(\boldsymbol{\Phi}_n) + \int d\boldsymbol{\Phi}_n\, d\Phi_r\, R(\boldsymbol{\Phi}_n,\Phi_r)\left[O(\boldsymbol{\Phi}_n,\Phi_r)-O(\boldsymbol{\Phi}_n)\right]$$

In $\mathrm{NLO_{SMC}}$, it was

$$\langle O\rangle = \int d\boldsymbol{\Phi}_n\, O(\boldsymbol{\Phi}_n)\, B(\boldsymbol{\Phi}_n) + \int d\boldsymbol{\Phi}_n\, d\Phi_r\, B(\boldsymbol{\Phi}_n)\,\frac{\alpha_s}{2\pi}\,P(z)\,\frac{1}{t}\left[O(\boldsymbol{\Phi}_n,\Phi_r)-O(\boldsymbol{\Phi}_n)\right]$$

POWHEG

$$\mathrm{NLO_{SMC}} \leftrightarrow \mathrm{NLO}: \qquad B(\mathbf{\Phi}_n) \leftrightarrow \overline{B}(\mathbf{\Phi}_n) \qquad B(\mathbf{\Phi}_n)\frac{\alpha_s}{2\pi}P(z)\frac{1}{t} \leftrightarrow R(\mathbf{\Phi}_n, \Phi_r)$$

All-order emission probability in SMC

$$\langle O \rangle = \int d\mathbf{\Phi}_n \, B(\mathbf{\Phi}_n) \left\{ O(\mathbf{\Phi}_n)\,\Delta_{t_0} + \int_{t_0} d\Phi_r \, O(\mathbf{\Phi}_n, \Phi_r)\,\Delta_t \, \frac{\alpha_s}{2\pi} P(z) \, \frac{1}{t} \right\}$$

with

$$\Delta_t = \exp\left[- \int d\Phi_r' \, \frac{\alpha_s}{2\pi} P(z') \, \frac{1}{t'} \, \theta(t' - t) \right]$$

All order emission probability in POWHEG

$$\langle O \rangle = \int d\mathbf{\Phi}_n \, \overline{B}(\mathbf{\Phi}_n) \left\{ O(\mathbf{\Phi}_n)\,\Delta_{t_0} + \int d\Phi_r \, O(\mathbf{\Phi}_n, \Phi_r)\,\Delta_t \, \frac{R(\mathbf{\Phi}_n, \Phi_r)}{B(\mathbf{\Phi}_n)} \right\}$$

$$\Delta_t = \exp\left[- \int d\Phi_r' \, \frac{R(\mathbf{\Phi}_n, \Phi_r')}{B(\mathbf{\Phi}_n)} \, \theta(t' - t) \right]$$

with $t = k_T(\mathbf{\Phi}_n, \Phi_r)$ and $\overline{B}(\mathbf{\Phi}_n) = B(\mathbf{\Phi}_n) + V(\mathbf{\Phi}_n) + \int d\Phi_r \left[R(\mathbf{\Phi}_n, \Phi_r) - C(\mathbf{\Phi}_n, \Phi_r) \right]$

POSITIVE if $\overline{B}$ is positive (i.e. NLO $<$ LO).

Accuracy of the Sudakov form factor

POWHEG's Sudakov form factor has the form (with $c \approx 1$)

$$\Delta_t = \exp\left[-\int_t^{Q^2} \frac{dk_T^2}{k_T^2} \frac{\alpha_s(c\, k_T^2)}{\pi} \left\{A \log\frac{E^2}{k_T^2} + B\right\}\right]$$

The next-to-leading log (NLL) Sudakov form factor has the form

$$\Delta_t^{\mathrm{NLL}} = \exp\left[-\int_t^{Q^2} \frac{dk_T^2}{k_T^2} \frac{\alpha_s(k_T^2)}{\pi} \left\{\left(A_1 + A_2 \frac{\alpha_s(k_T^2)}{\pi}\right) \log\frac{E^2}{k_T^2} + B\right\}\right]$$

provided the color structure of the process is sufficiently simple ($\leqslant 3$ colored legs). Can use this to fix c in POWHEG's Sudakov form factor as suggested in Catani, Webber, Marchesini, (1991). HERWIG uses this.

For colored legs $\geqslant 4$, exponentiation only holds at leading-log (LL) or LL + NLL in the large-N_c limit (i.e. planar color structure of Feynman diagrams)

POWHEG's Sudakov form factor is always LL accurate. NLL accurate for $\leqslant 3$ colored legs, NLL accurate in leading N_c in all cases.

Mathematical tricks

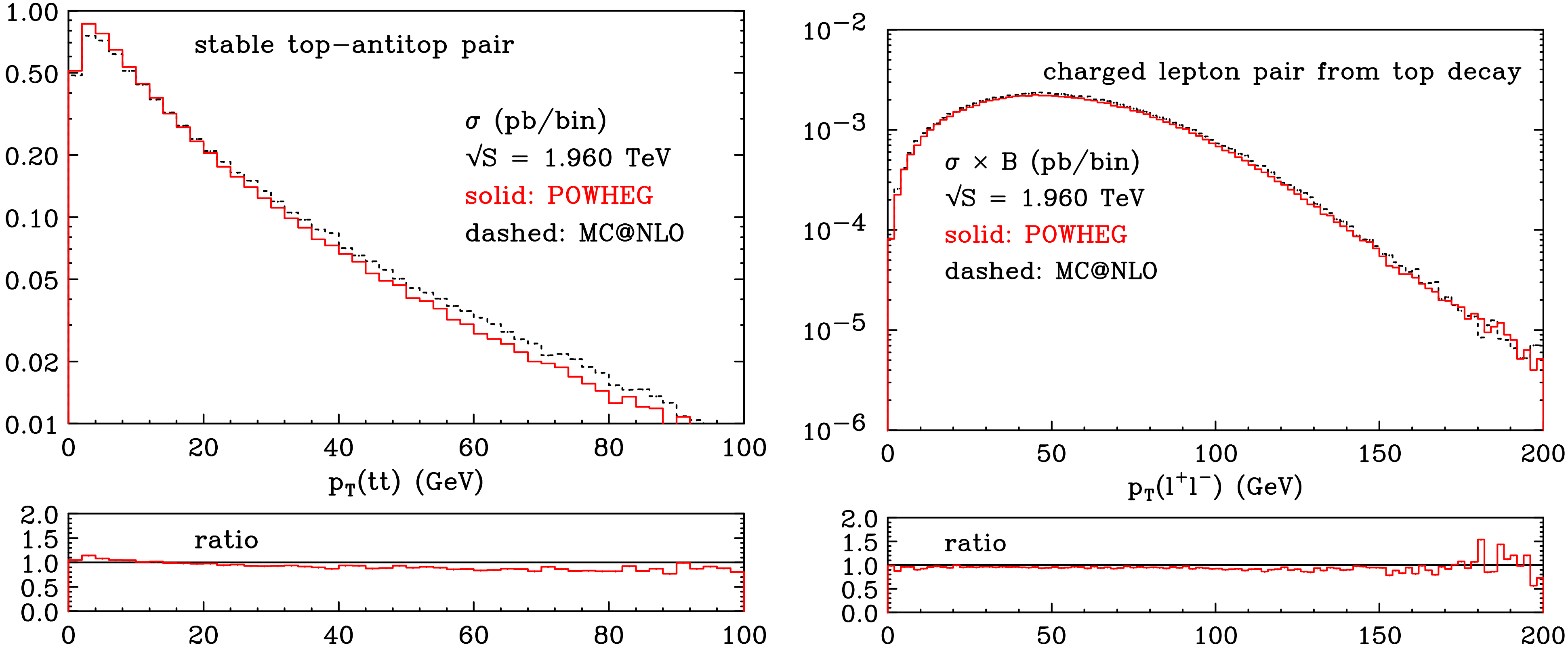
✓ To generate the underlying Born variables ($\mathbf{\Phi}_n$), distributed according to $\overline{B}(\mathbf{\Phi}_n)$, one uses programs like BASES/SPRING or MINT, that, after a single integration, can generate points distributed according to the integrand function.

✓ Use the veto technique and the highest-p_T bid procedure, to generate the radiation variables, distributed according to $d\Delta_i(t, t')$.

These tricks are well known to Monte Carlo experts.

We have collected a few of them in the appendixes of our paper [Frixione, Nason and Oleari, arXiv:0709.2092 [hep-ph]].

$t\bar{t}$ production



Good agreement for all observables considered. There are sizable differences that can be ascribed to different treatment of higher terms. But more investigation needed (different scale choices, no truncated shower, different hard/soft radiation emission,. . .).

ALPGEN vs MC@NLO: $t\bar{t}$ + 1 jet

ALPGEN can generate samples of $t\bar{t} + n$ jets. Can be compared to NLO + Parton Shower [Mangano, Moretti, Piccinini & Treccani, hep-ph/0611129]

✓ advantage: better high jet multiplicity (exact Matrix Element)

✗ disadvantage: worse normalization (no NLO)

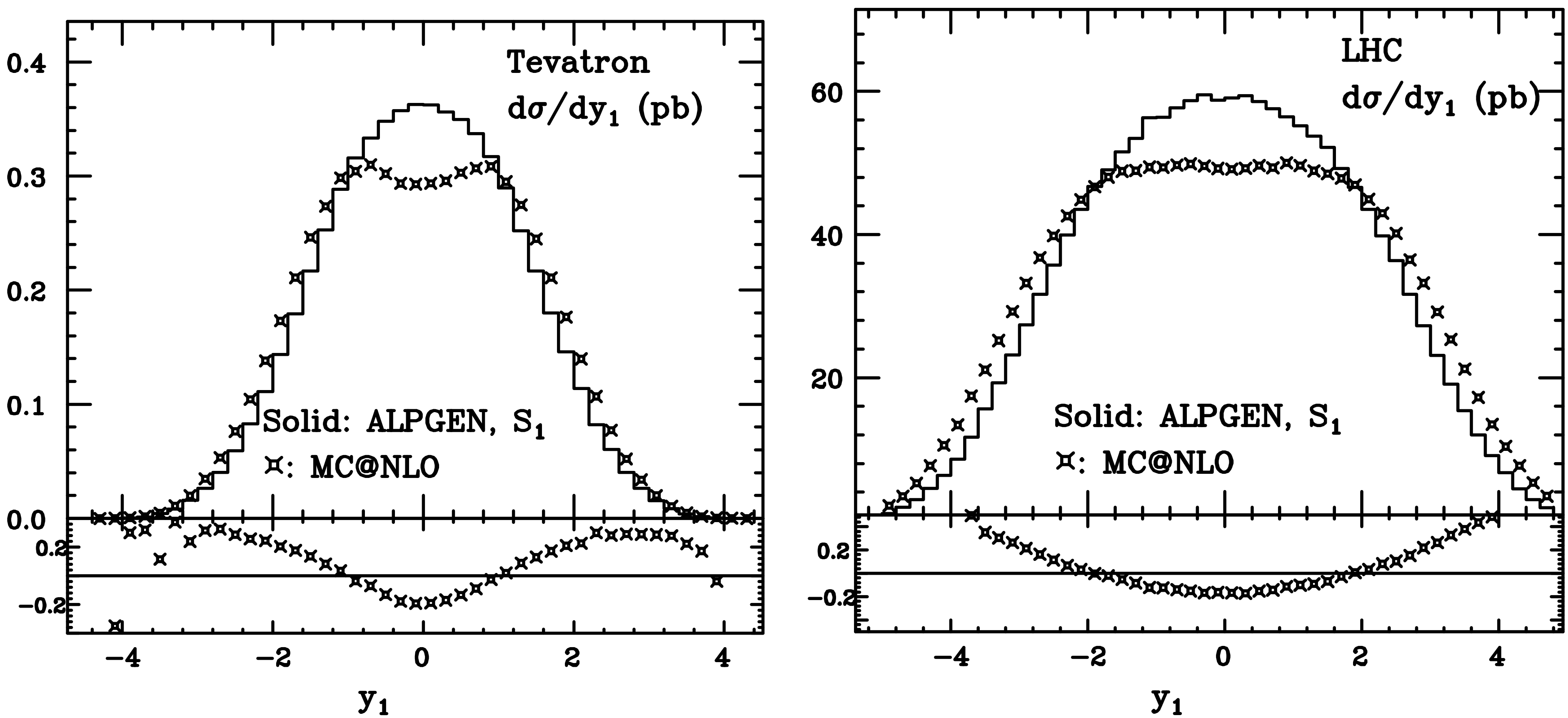
ALPGEN

- Generation: $P^T_{\text{min}} = 30$ GeV, $\Delta R = 0.7$
- Matching: $E^T_{\text{min}} = 30$ GeV, $\Delta R = 0.7$

Jet definitions

- Tevatron: $E^T_{\text{min}} = 15$ GeV, $\Delta R = 0.4$, K factor = 1.45
- LHC: $E^T_{\text{min}} = 20$ GeV, $\Delta R = 0.5$, K factor = 1.57

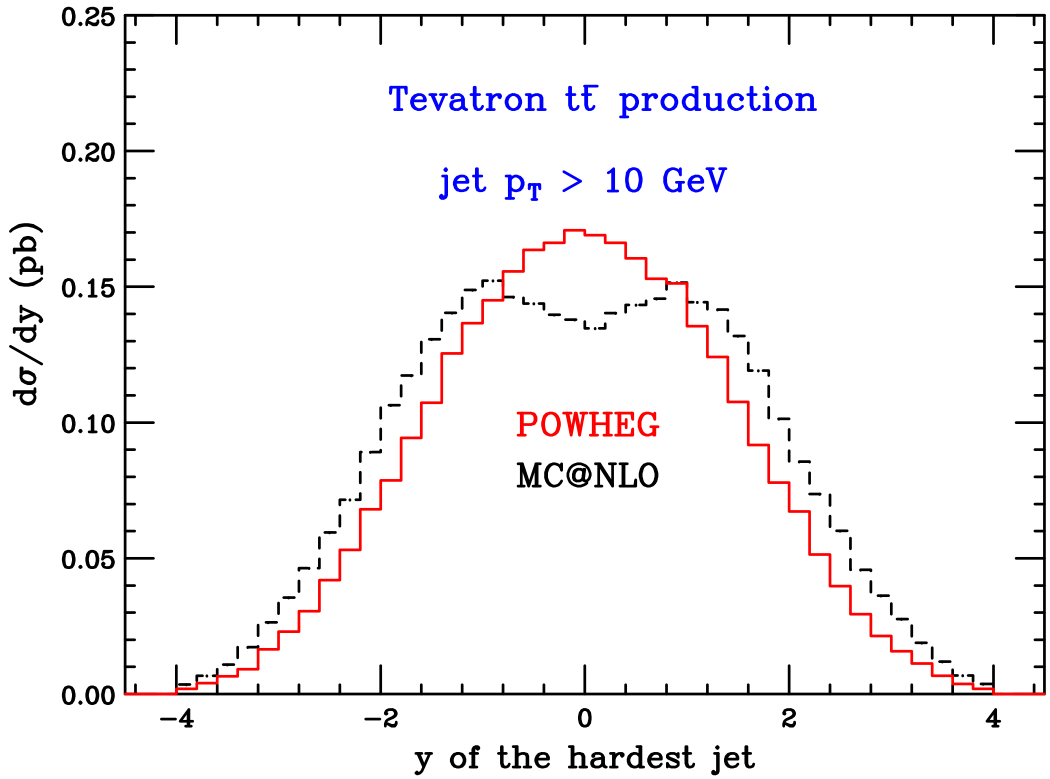
ALPGEN vs MC@NLO: $t\bar{t}$ + 1 jet

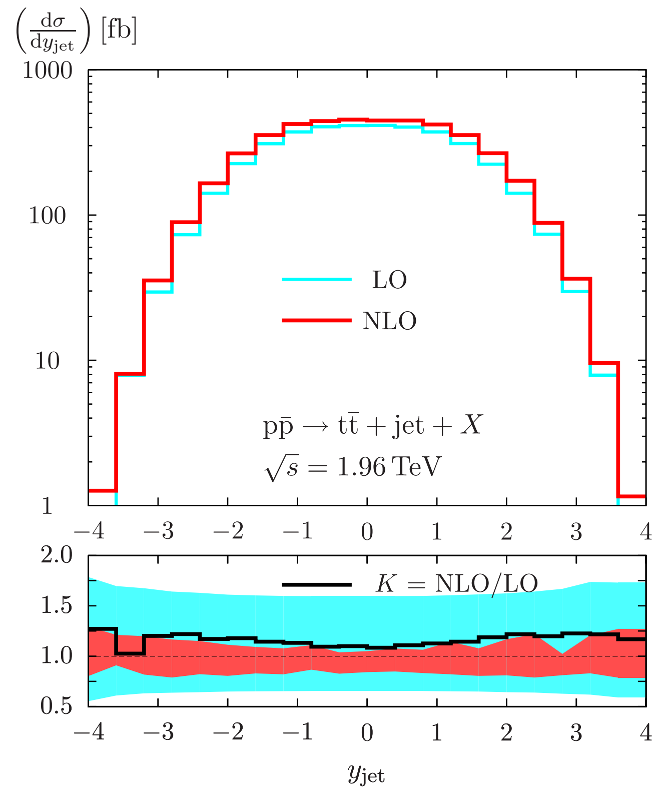


Rapidity y_1 of the leading jet (highest p_T).

Different shapes both at Tevatron and at the LHC

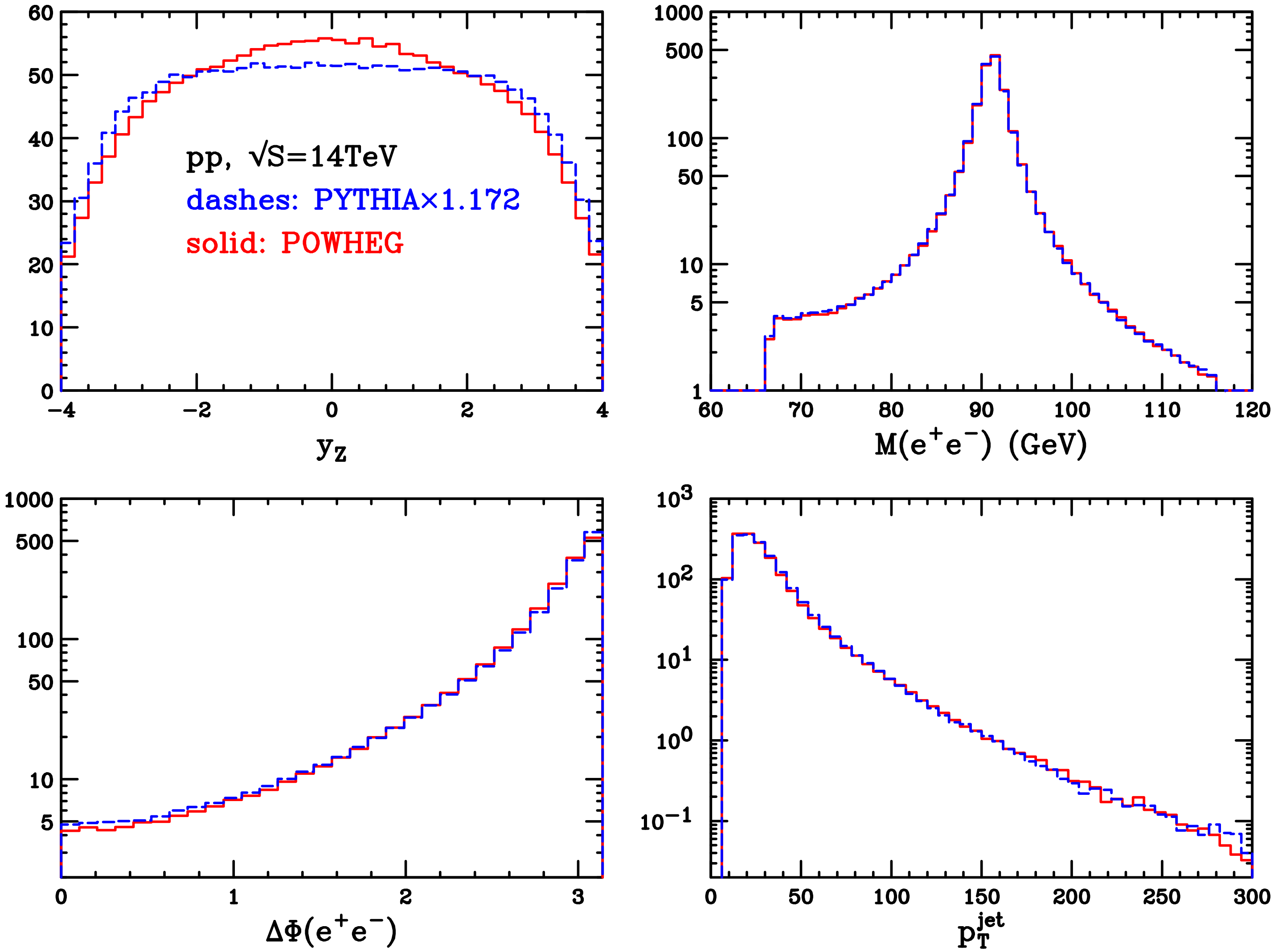
POWHEG: rapidity of the leading jet





POWHEG's distribution as in ALPGEN: no dip present. The size of discrepancy can be attributed to different treatment of higher-order terms. Is this "feature" really there? The new $pp \to t\bar{t}$ + jet at NLO [Dittmaier, Uwer, Weinzierl, hep-ph/0703120] shows no dip too.

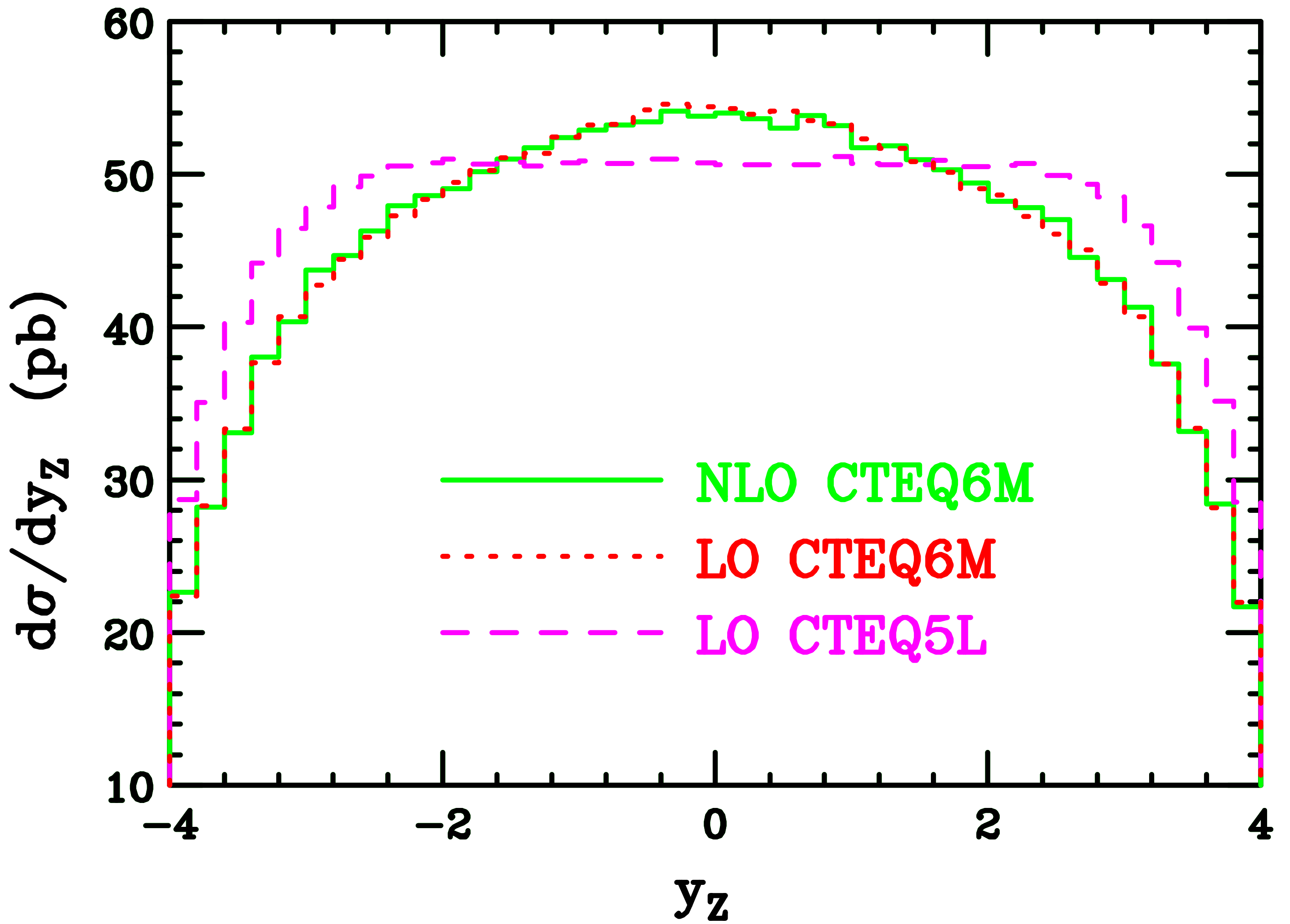
Z production: POWHEG + PYTHIA vs PYTHIA



For vector-boson production, PYTHIA generates radiation in a way similar to POWHEG.

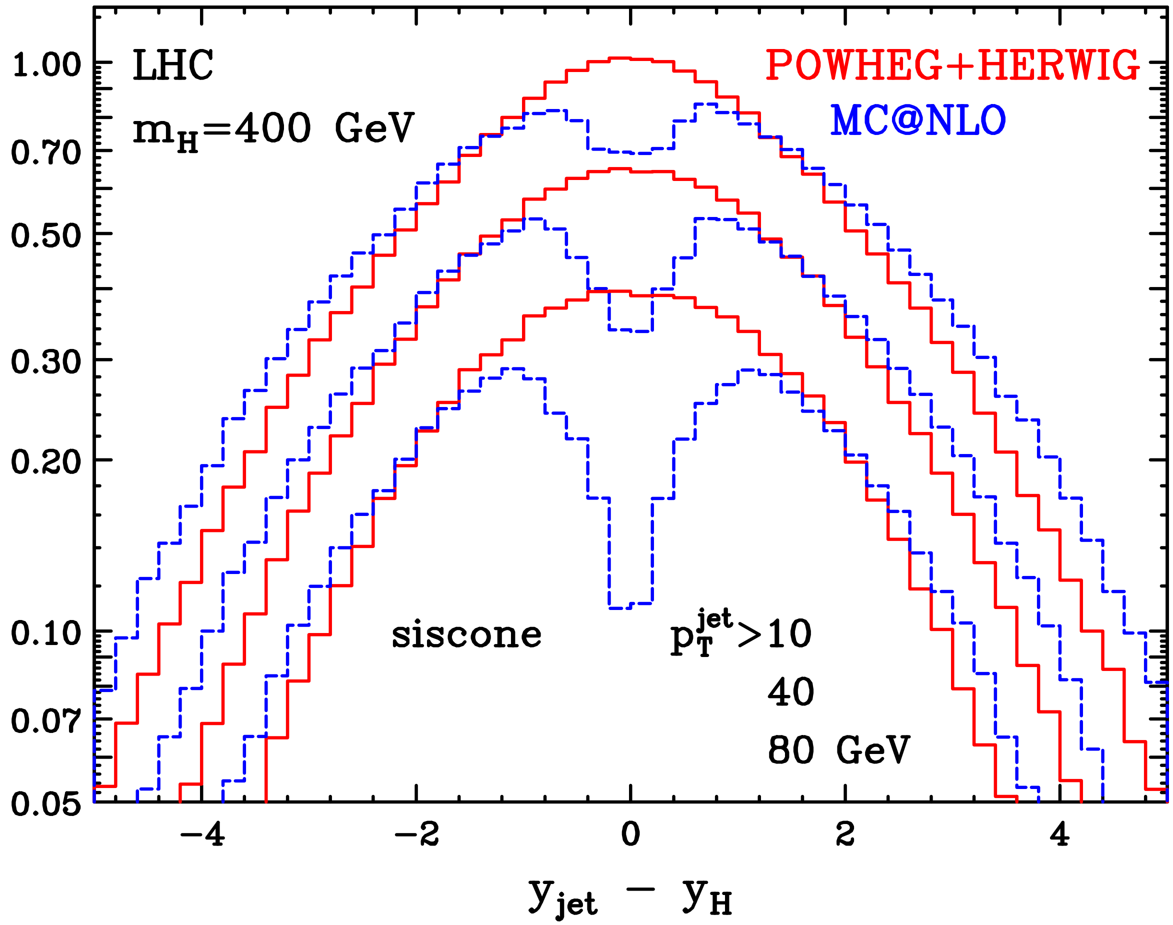
Differences ascribed to the use of LO parton densities.

Z production: POWHEG + PYTHIA vs PYTHIA



Plots normalized to the NLO total cross section.

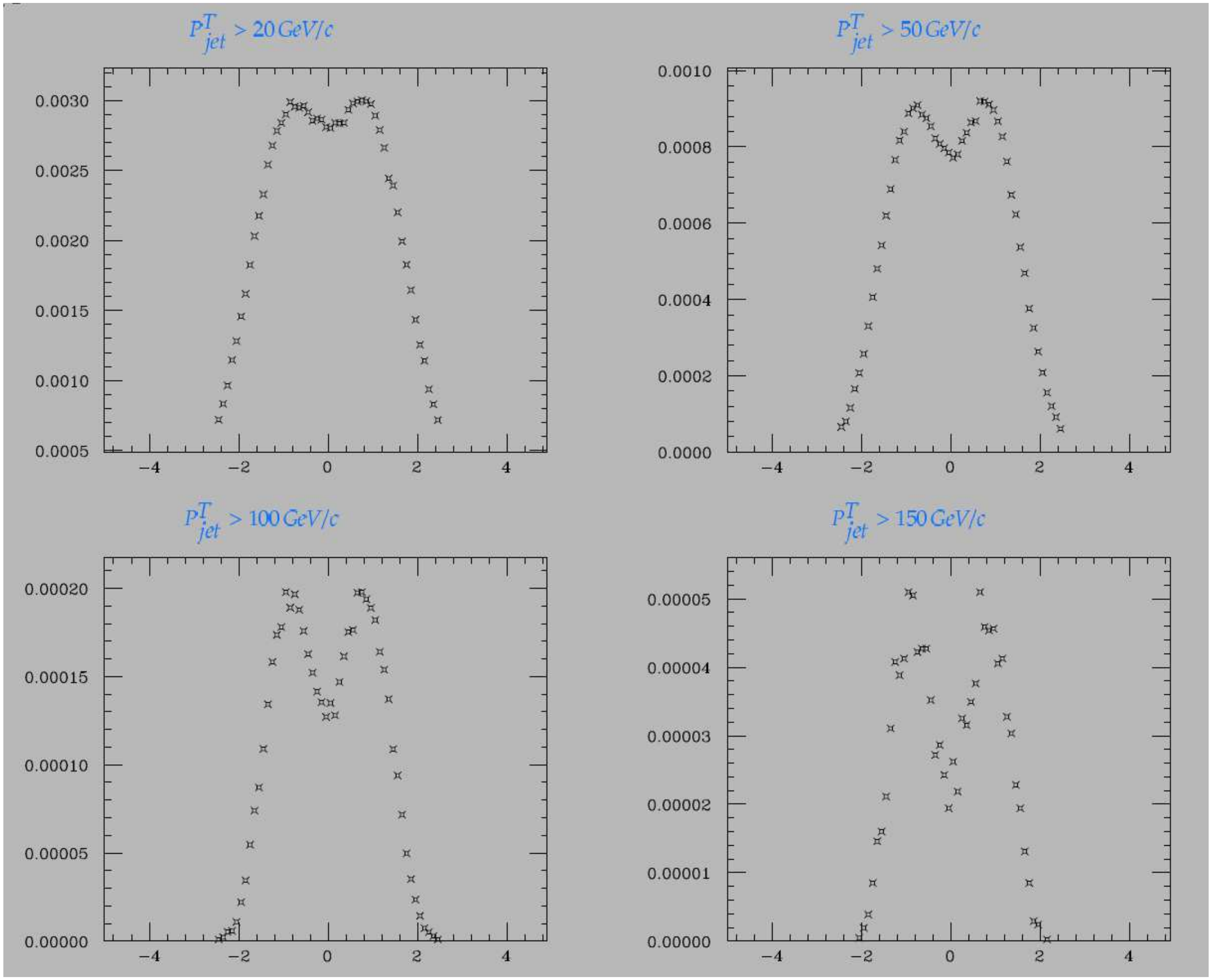
Higgs boson production at the LHC



POWHEG + HERWIG vs MC@NLO

$m_H = 400$ GeV

HERWIG: rapidity of the leading jet



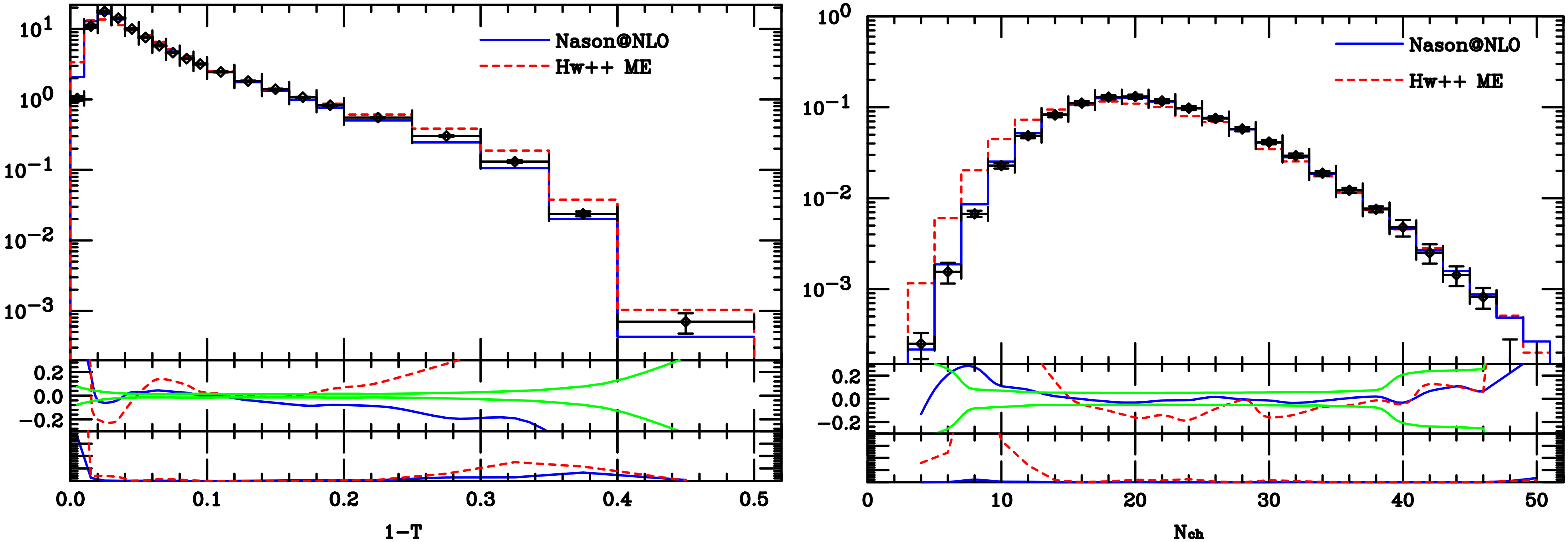
POsitive-Weight Hardest Emission Generator

[Nason, hep-ph/0409146]

is a **method**, NOT (only) a set of programs

- ✓ generates events with positive weights
- ✓ can be interfaced to any Parton Shower Monte Carlo, if the vetoed shower is implemented, according to the Les Houches Interface.
  It is independent from parton-shower programs. POWHEG can be interfaced with both PYTHIA and HERWIG, or with your favorite showering program,
- ✓ As far as the hardest emission is concerned, POWHEG guarantees:
  - NLO accuracy of (integrated) shape variables
  - Collinear, double-log, soft (large-N_c limit) accuracy of the Sudakov (in fact, corrections that exponentiates are obviously OK)
- ✓ As far as subsequent (less hard) emissions, the output has the accuracy of the SMC one is using.

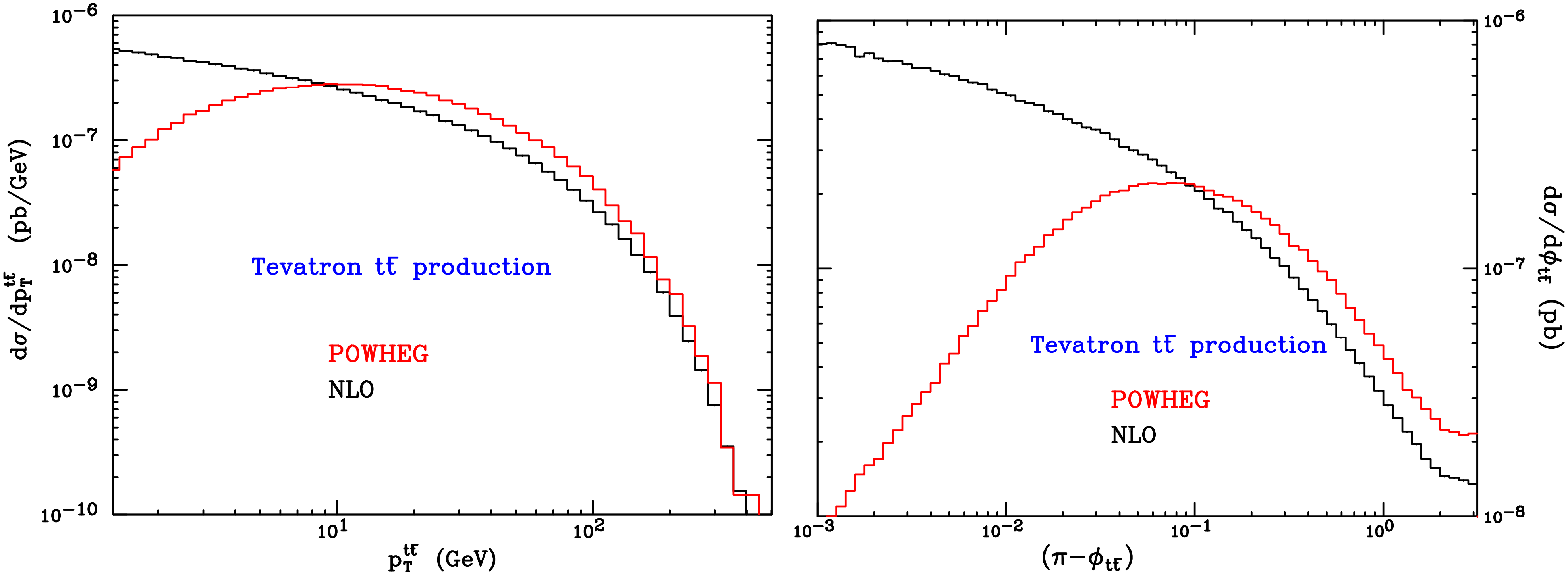
$e^+e^- \longrightarrow$ hadrons



[Latunde-Dada, Gieseke and Webber, hep-ph/0612281]

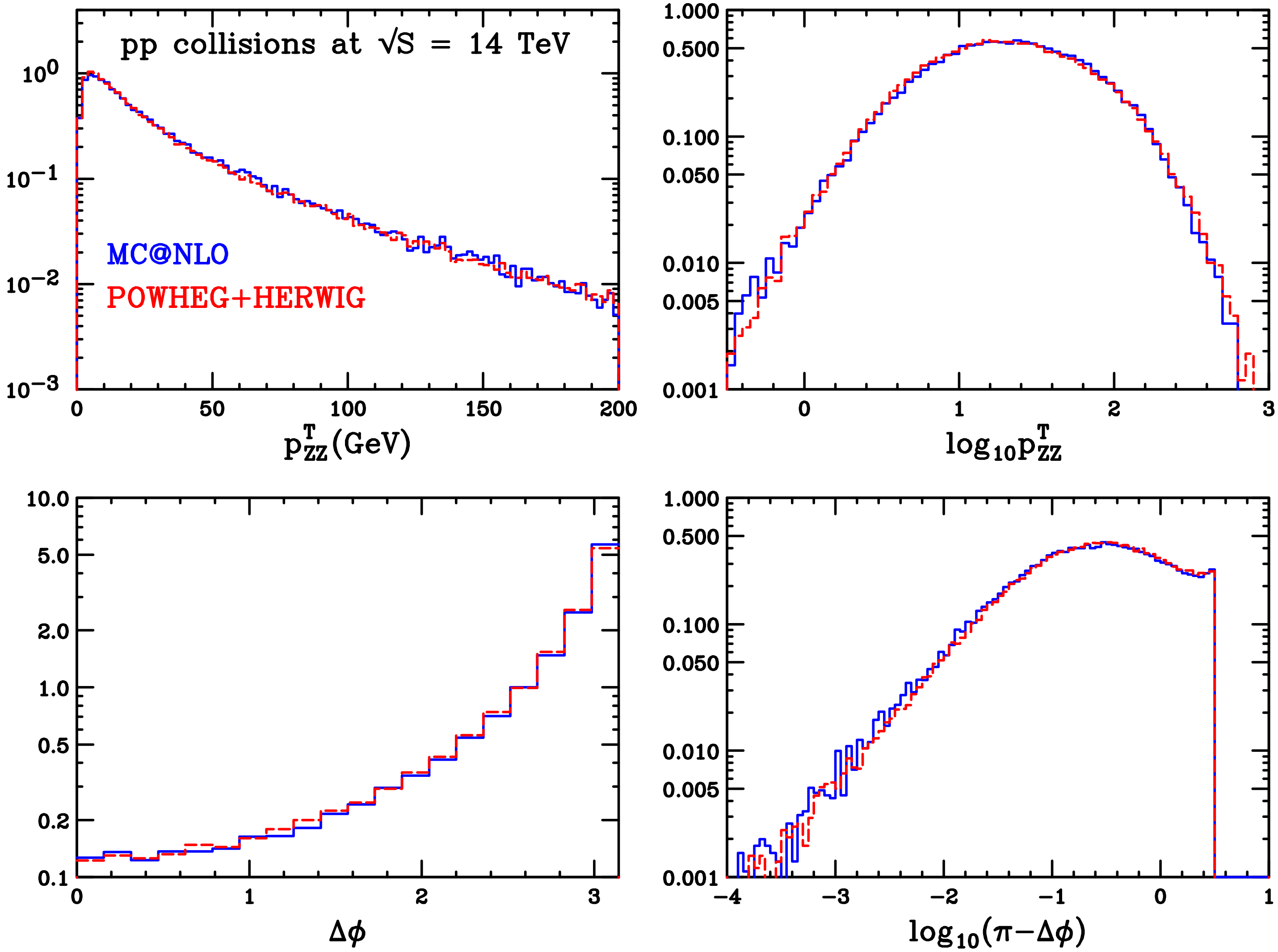
Fit to e^+e^- data: better agreement than in the standard matrix-element correction approach.

$t\bar{t}$ production: POWHEG vs. NLO



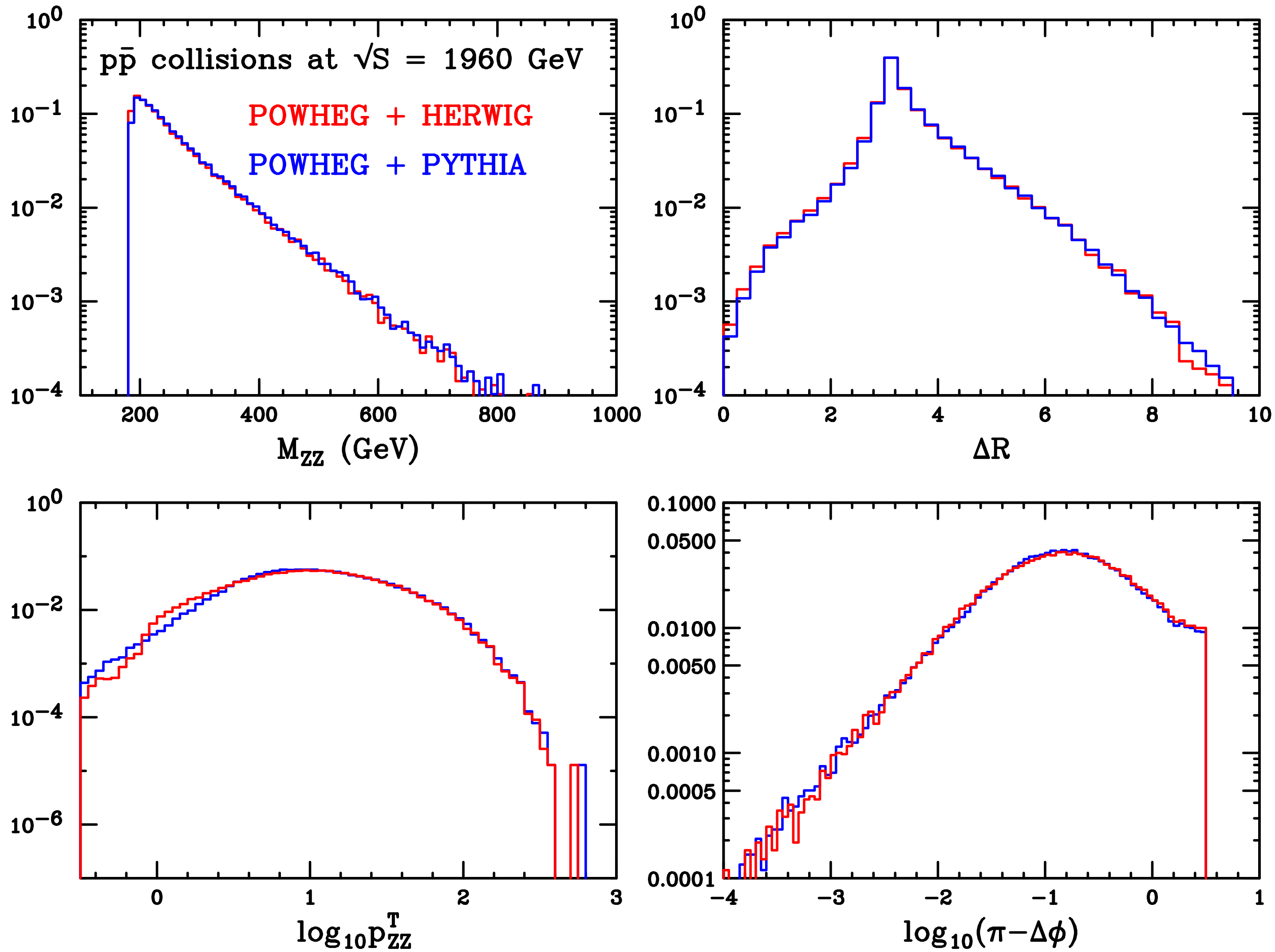
- when $p_T^{t\bar{t}} \to 0$, POWHEG treats correctly the resummation of soft/collinear radiation
- when $p_T^{t\bar{t}}$ becomes large, POWHEG approaches the NLO result
- when $\Phi_{t\bar{t}} \to 0$, the emitted radiation becomes hard and POWHEG goes to the NLO result.

ZZ production: POWHEG + HERWIG vs MC@NLO



No significant difference with MC@NLO [Nason and Ridolfi, hep-ph/0606275]

POWHEG + HERWIG vs POWHEG + PYTHIA



Agreement between POWHEG + HERWIG and POWHEG + PYTHIA

[Nason and Ridolfi, hep-ph/0606275]

W/Z production