4-Momentum Generation

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Learning To Discover, Paris 2022

Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

CLUSTER OF EXCELLENCE
QUANTUM UNIVERSE

4-Momentum?

- Classical momentum:

$$\mathbf{p} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = \mathbf{v}m = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} m$$

- Not Lorentz invariant (e.i. dependent on reference frame)
  - No concern for everyday applications
  - Problematic in relativistic cases, like high energy physics

4-Momentum?

- Extend 3-vectors to 4-vectors
- 3D-coordinates → 4D-spacetime coordinates

$$\mathbf{r} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} \rightarrow \mathbf{R} = \begin{bmatrix} ct \\ r_1 \\ r_2 \\ r_3 \end{bmatrix}$$

Speed of Light × Time

4-Momentum?

- Extend 3-vectors to 4-vectors
- 4-velocity **U**:

$$\mathbf{U} = \frac{d}{dt}\mathbf{R} = \frac{d}{dt}\begin{bmatrix} ct \\ r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} c \\ u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

4-Momentum?

- Extend 3-vectors to 4-vectors
- 4-momentum **P**:

$E = mc^2$

$$\mathbf{P} = \gamma m_0 \mathbf{U} = \gamma m_0 \begin{bmatrix} c \\ u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} cm \\ p_1 \\ p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} E/c \\ p_1 \\ p_2 \\ p_3 \end{bmatrix}$$

Relativistic mass $\gamma m_0 = m$

Lorentz factor $\gamma = \dfrac{1}{\sqrt{1 - u^2/c^2}}$

4-Momentum?

- Extend 3-vectors to 4-vectors
- 4-momentum **P**:

Energy

$$\mathbf{P} = \mathbf{P}m = m \begin{bmatrix} 1 \\ u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} m \\ p_1 \\ p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} E \\ p_1 \\ p_2 \\ p_3 \end{bmatrix}$$

- Particle physics: c = 1
- Lorentz invariant

4-Momentum in HEP

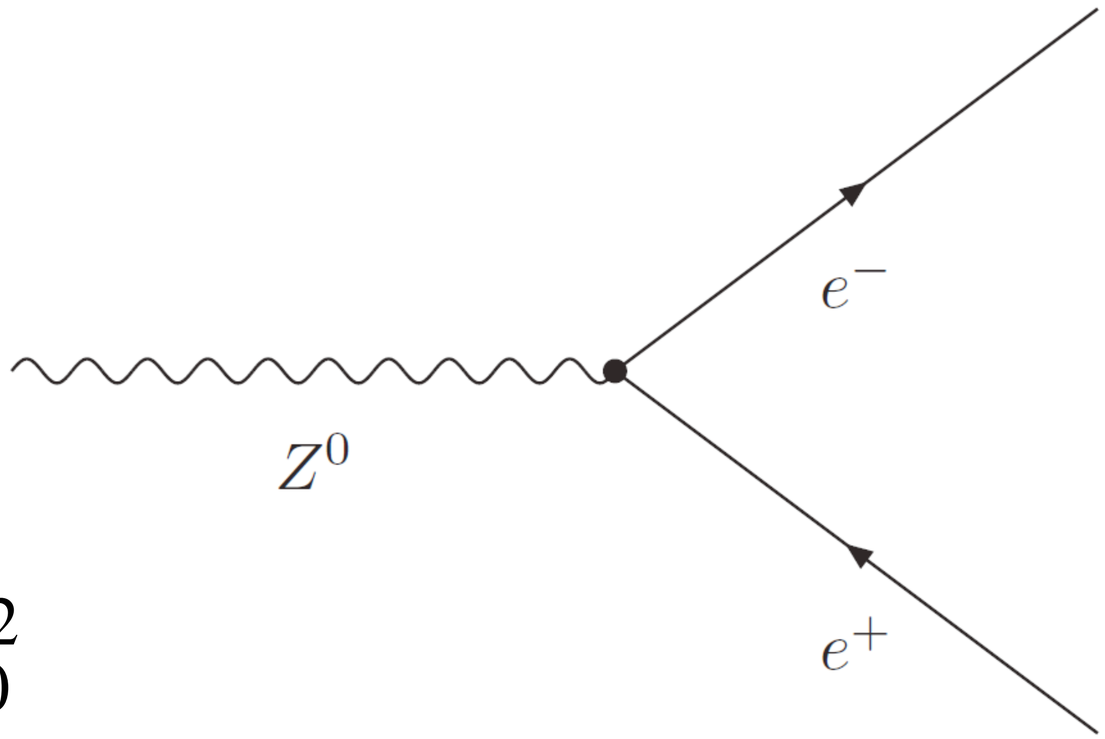
- 4-momentum **P**:
  - Is conserved in decays:

$$\mathbf{P}_Z = \mathbf{P}_{e-} + \mathbf{P}_{e+}$$

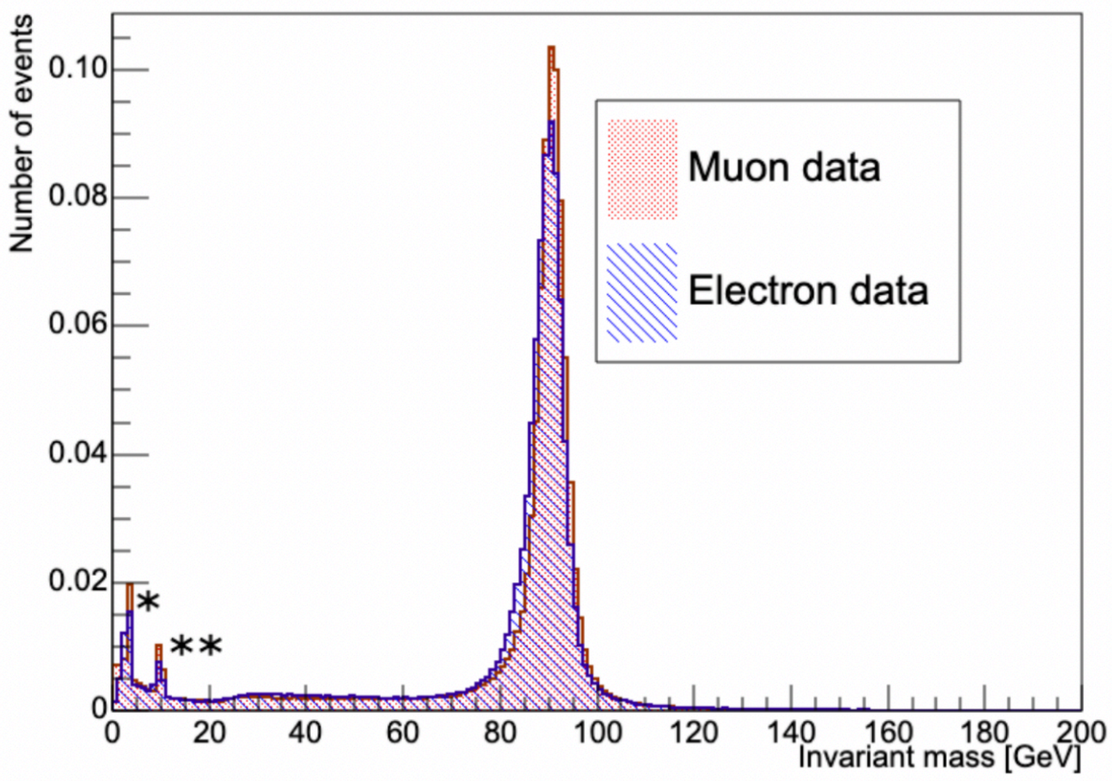
  - Invariant mass:

$$\mathbf{P}^2 = \gamma^2 m_0^2 \left(c^2 - u^2\right) = m_0^2$$

$$m_{0,Z}^2 = \mathbf{P}_Z^2 = (\mathbf{P}_{e-} + \mathbf{P}_{e+})^2$$



4-Momentum in HEP



Taken form "Accuracy and Precision of the Z Boson Mass Measurement with the ATLAS Detector"

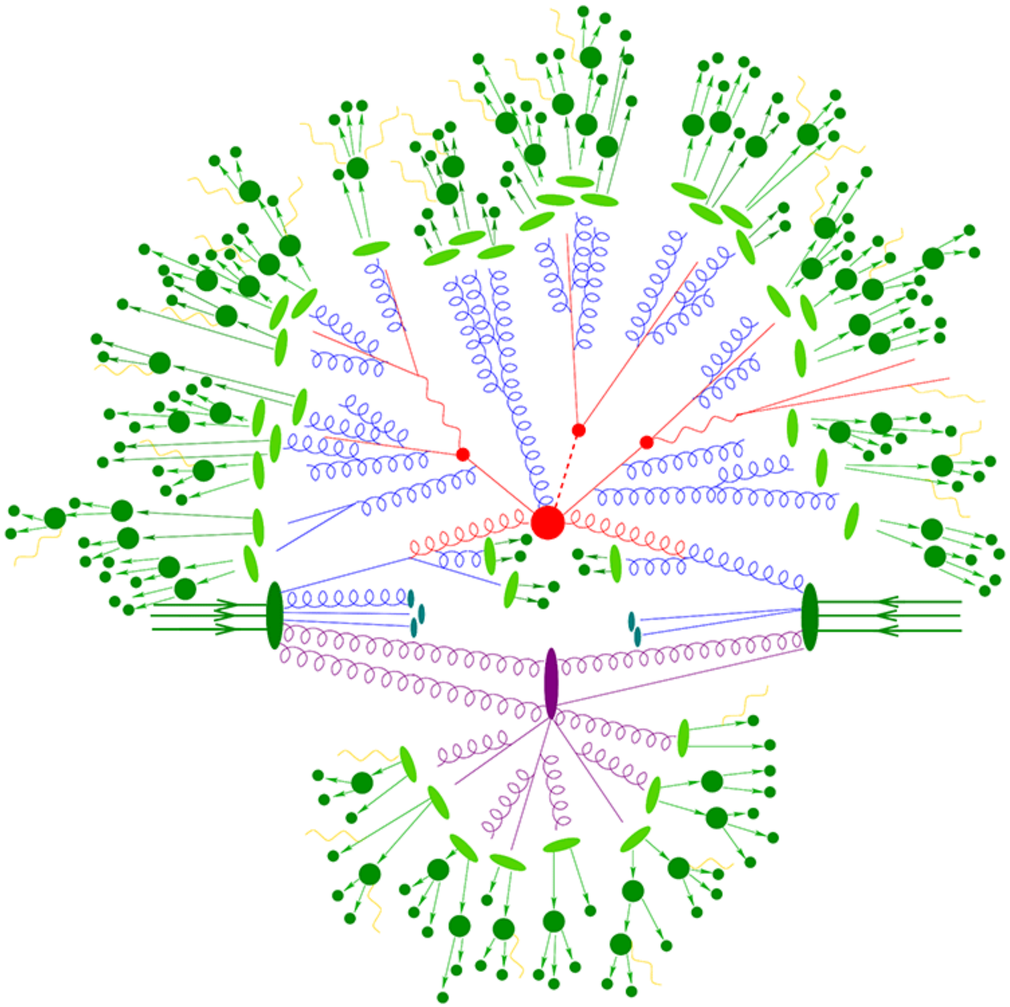
- Invariant mass:

$$\mathbf{P}^2 = \gamma^2 m_0^2 \, (c^2 - u^2) = m_0^2$$

$$m_{0,Z}^2 = \mathbf{P}_Z^2 = (\mathbf{P}_{e-} + \mathbf{P}_{e+})^2$$

- However not single value
- Peak with measurable width for final state particles (on-shell)
- More variable for intermediate (off-shell) particles

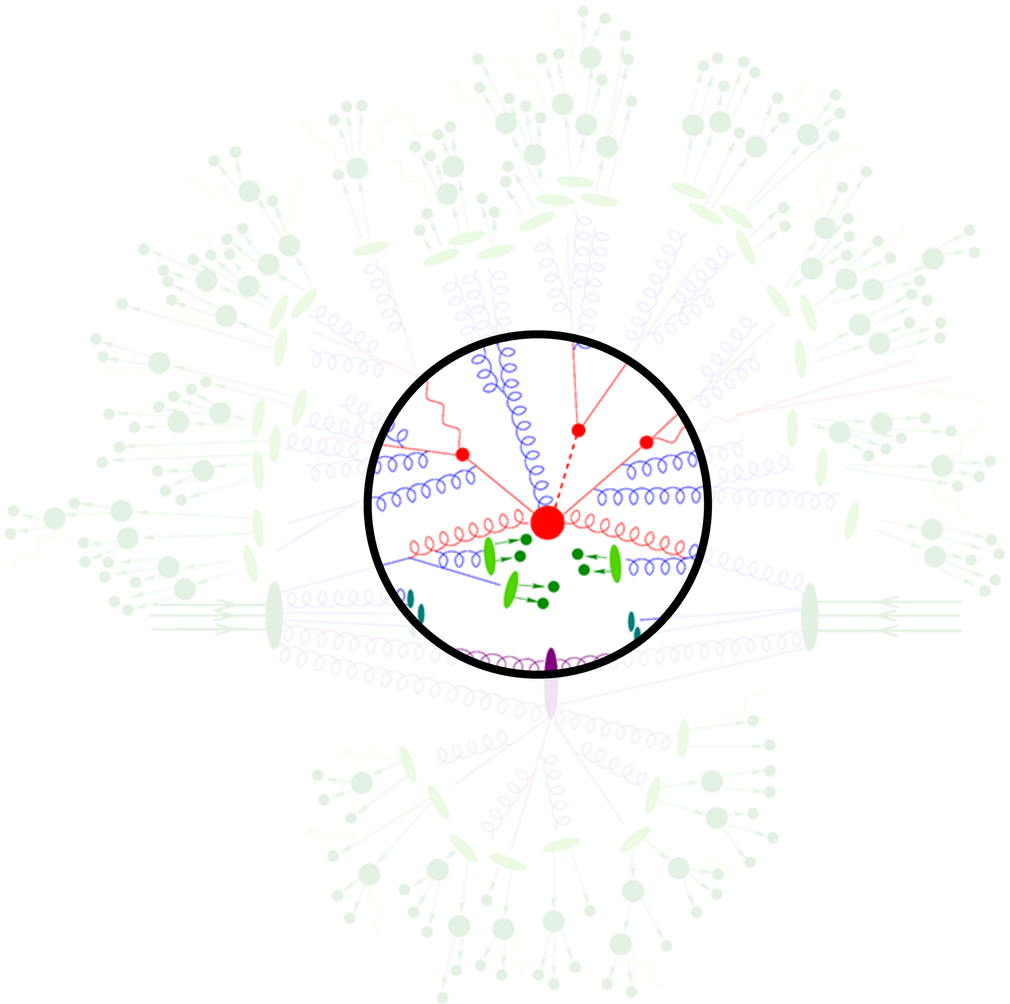
4-Momentum in Generation



- Particle collision simulation:

Taken from https://www.physik.uzh.ch/en/researcharea/tpp.html

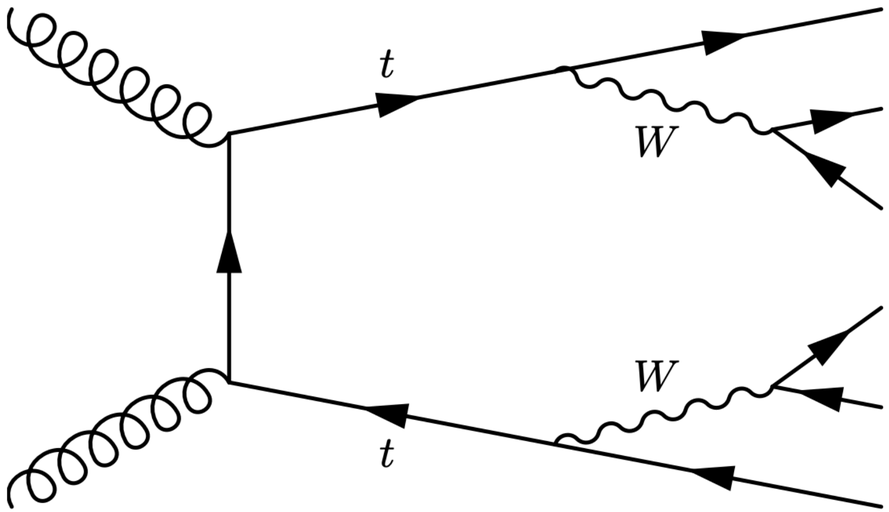
4-Momentum in Generation



- Particle collision simulation:
  - Parton level:
    - Low, fixed number of objects
    - Sharp structures

Parton Level Event Generation

$$pp \to t\bar{t} \to (bW^-)\,(\bar{b}W^+) \to (bq_1\bar{q}_1')\,(\bar{b}q_2\bar{q}_2')$$



- Top quark pair production
- Tops decay promptly
- 6 particles in ‘final state’
  - Each particle 4-momentum
  - 24 dimensions
  - However: masses known

  → 24 - 6 = 18 dimensions

Butter et al.: **How to GAN LHC Events:**
SciPost Phys. 7, 075 (2019), 1907.03764

Parton Level Event Generation

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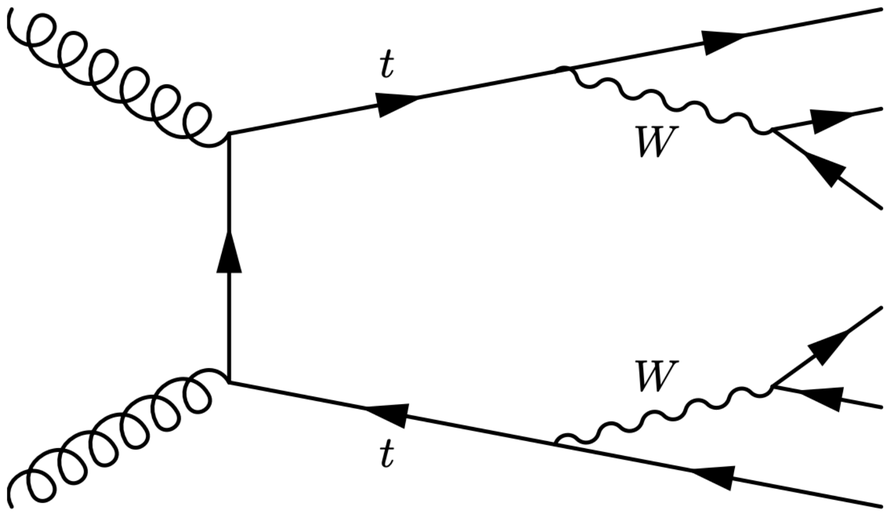


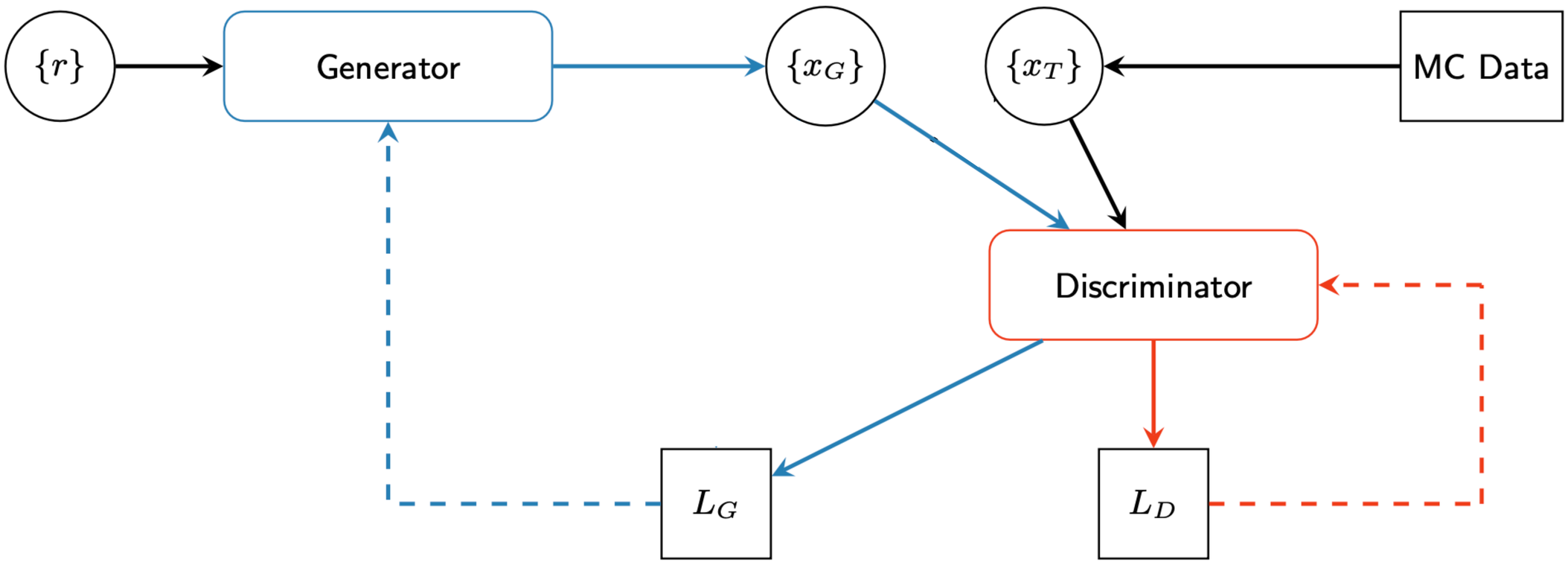
- Particle number and type fixed
- Ordered list of floats

$$\begin{bmatrix} p_{1,particle1} \\ p_{2,particle1} \\ p_{3,particle1} \\ \vdots \\ p_{3,particle6} \end{bmatrix}$$

Butter et al.: **How to GAN LHC Events:**
SciPost Phys. 7, 075 (2019), 1907.03764

Parton Level Event Generation

- Ordered list: appropriate for GAN architecture



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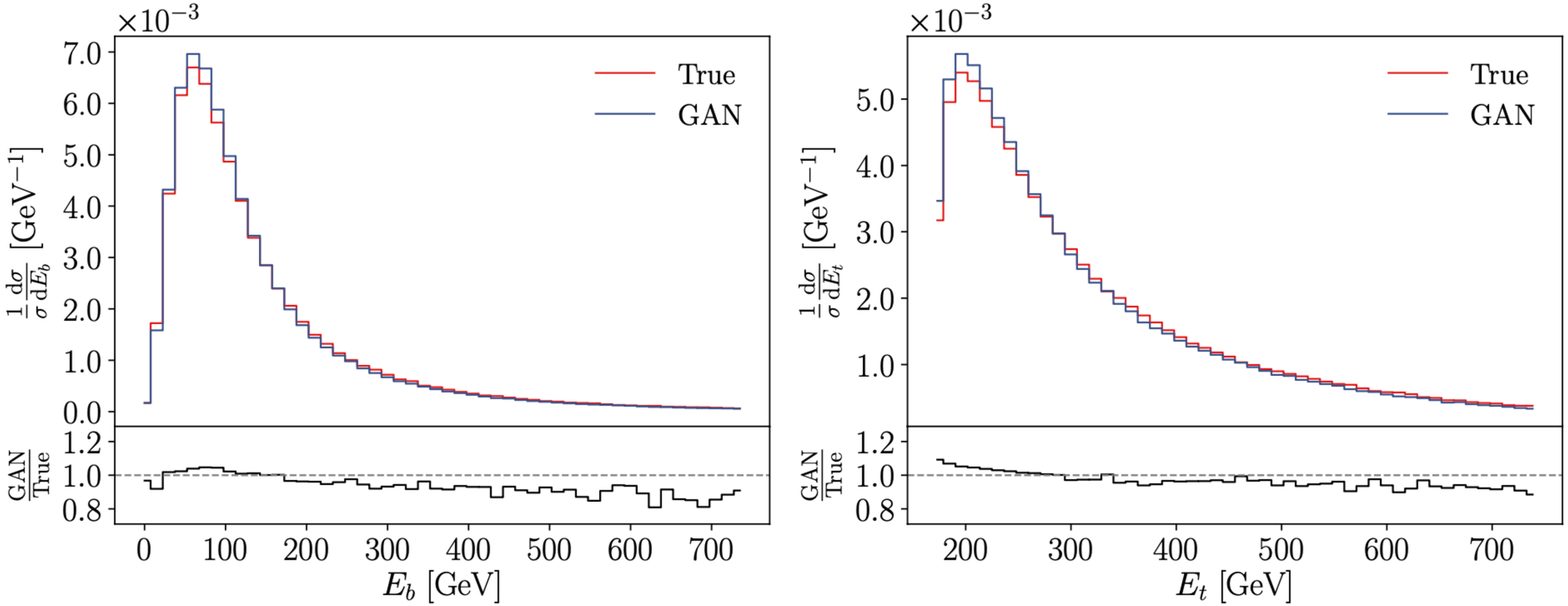
Parton Level Event Generation

- Works well for individual distributions



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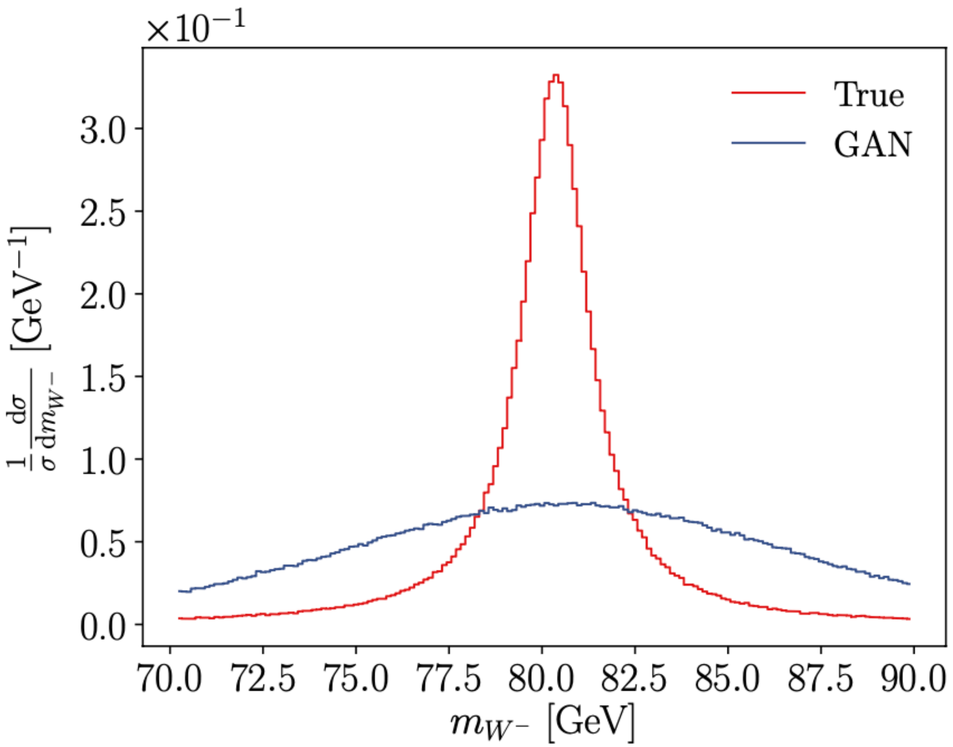
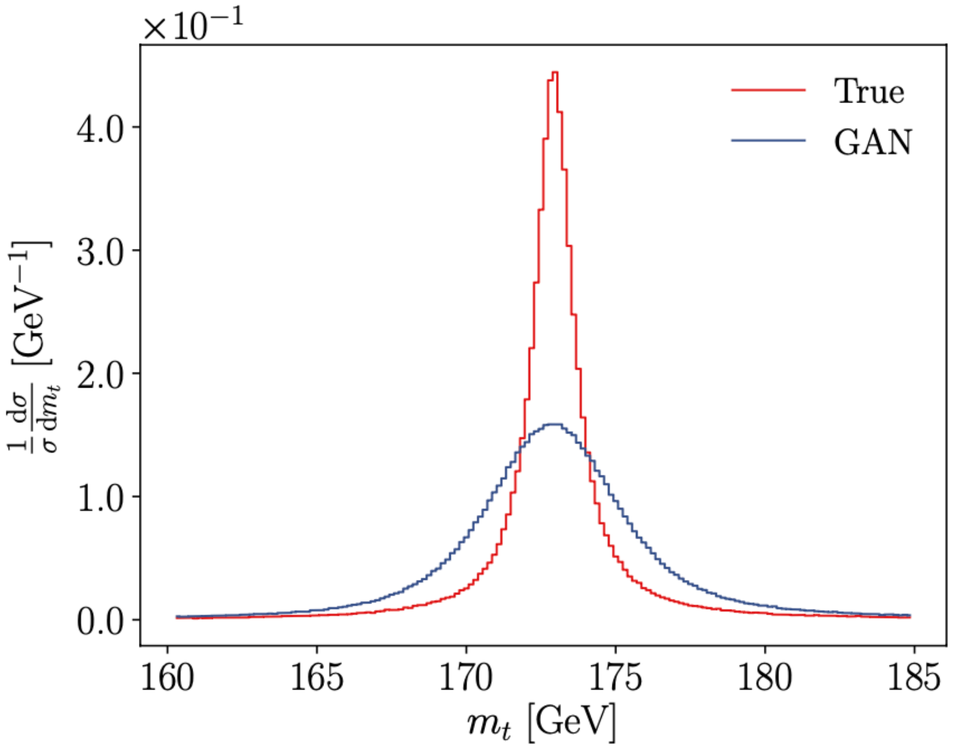
Parton Level Event Generation

- Difficulties with complex correlations like mass peaks





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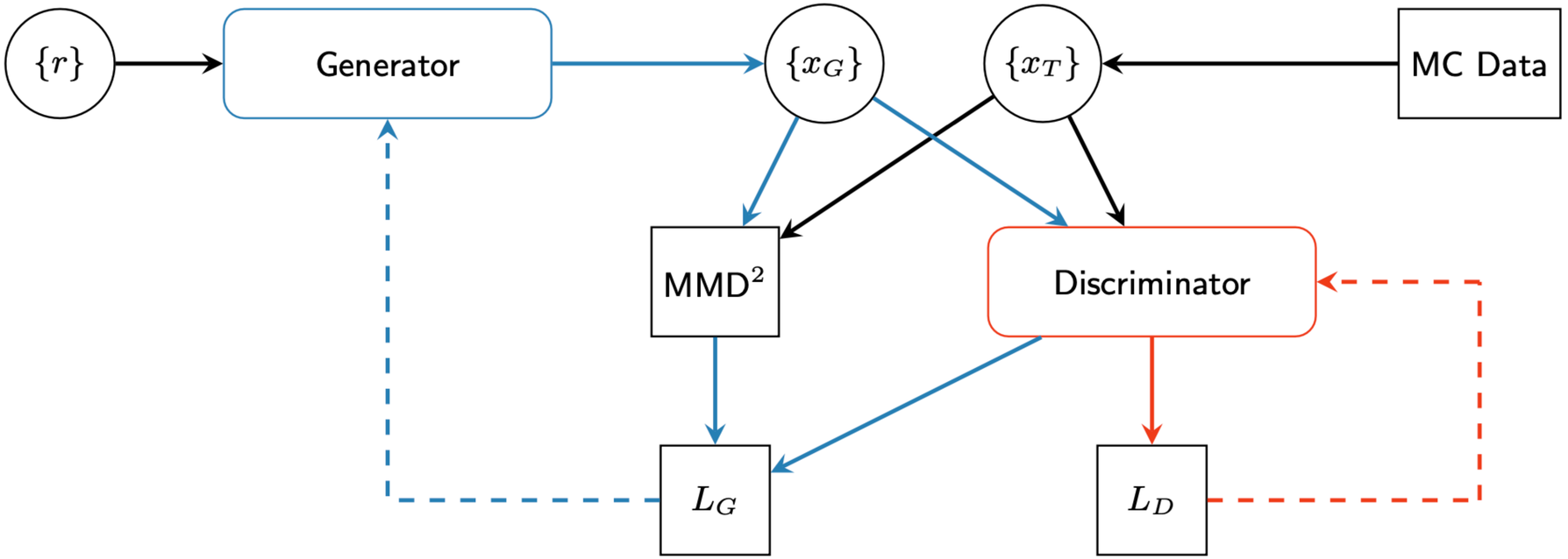
Parton Level Event Generation

- Approach: additional Maximum Mean Discrepancy (MMD) loss term in the invariant mass

$$\mathrm{MMD}^2(P_T, P_G) = \langle k(x,x') \rangle_{x,x' \sim P_T} + \langle k(y,y') \rangle_{y,y' \sim P_G} - 2\langle k(x,y) \rangle_{x \sim P_T, y \sim P_G}$$



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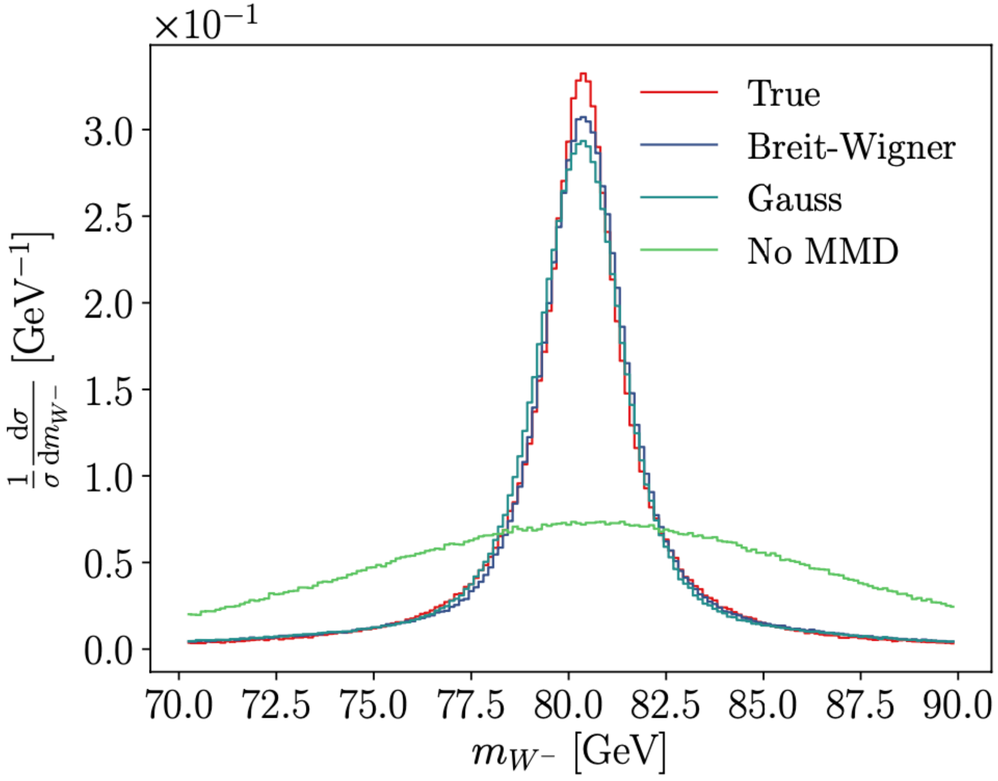
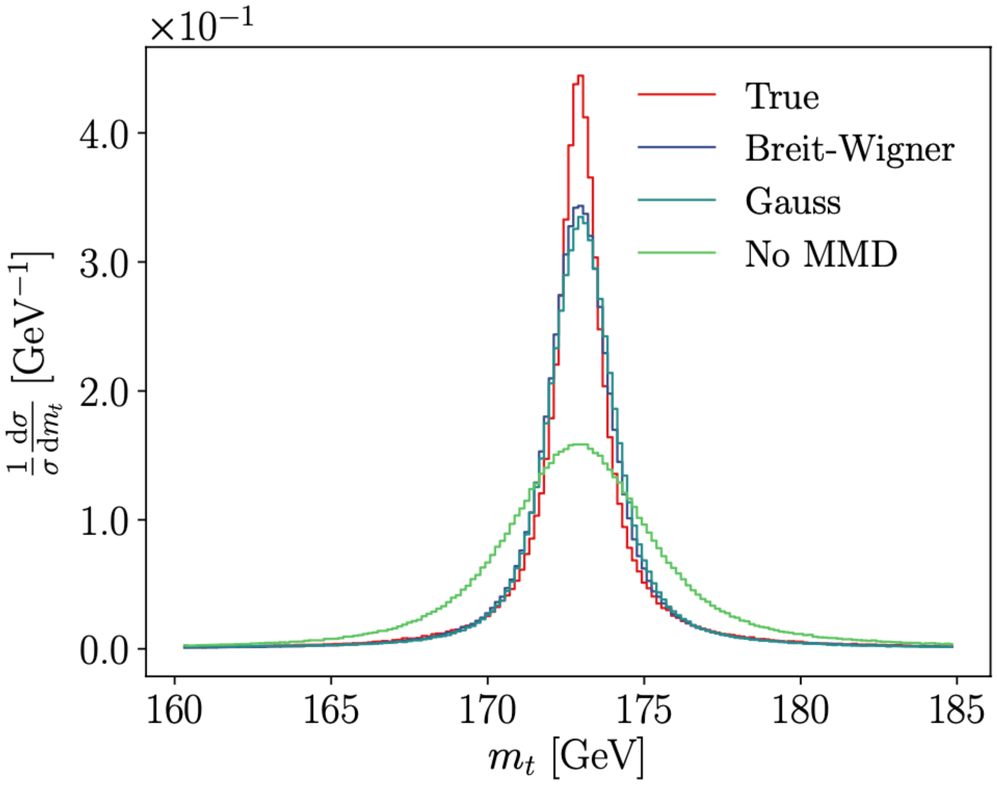
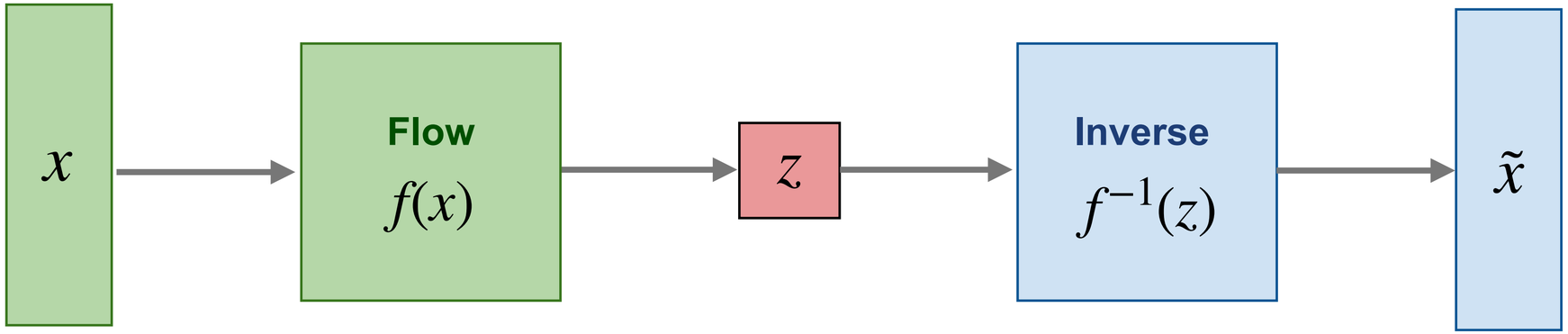




Butter et al.: **How to GAN LHC Events:** SciPost Phys. 7, 075 (2019), 1907.03764

Flow Generators

- Matrix Element Simulation:
  - Low dimensional (comparatively) data space
  - Also well suited for normalising flows
  - Trains more easily



Flow Generators

- New process:

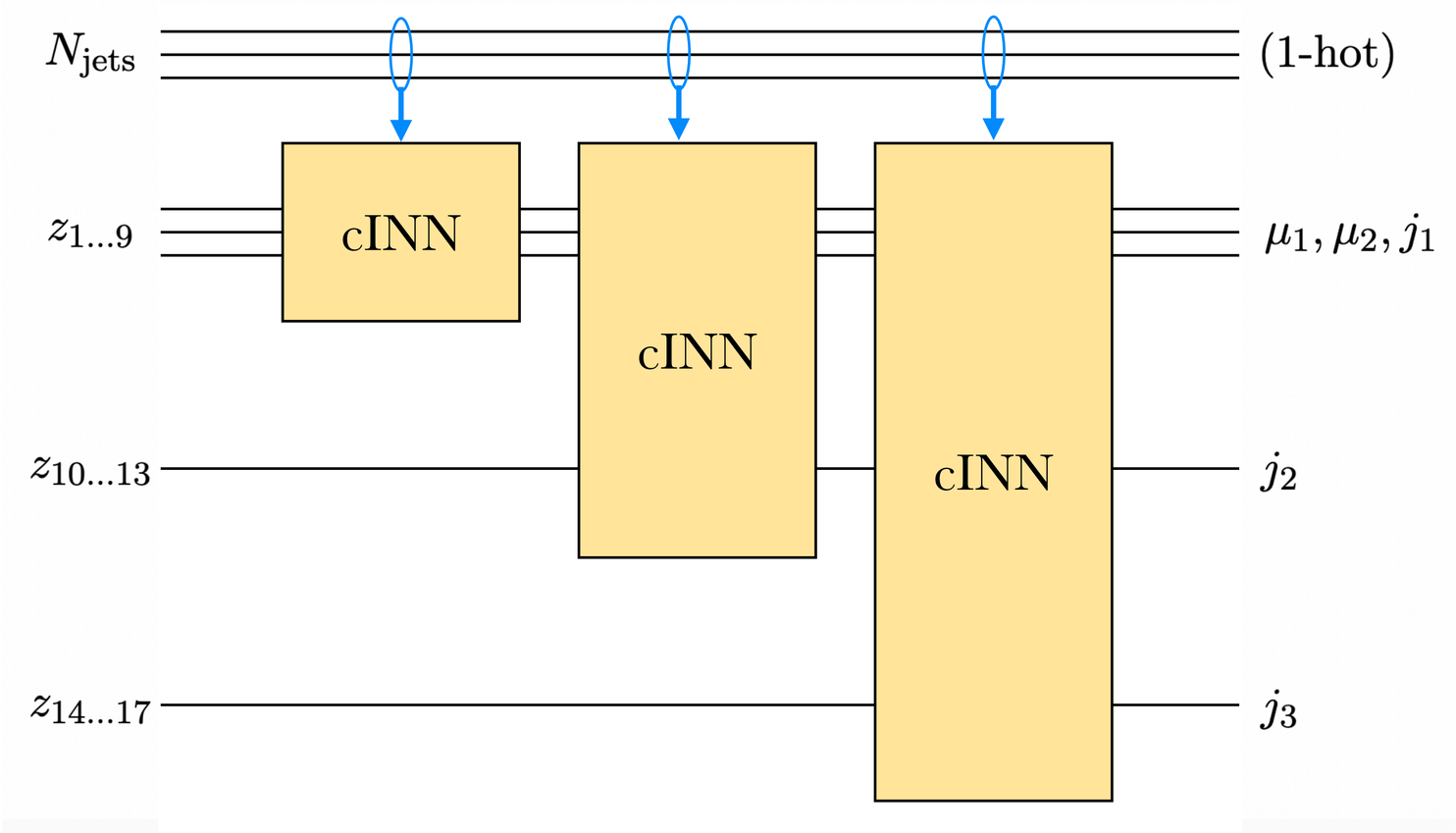
$$pp \to Z_{\mu\mu} + \{1, 2, 3\} \text{ jets}$$

- Sharp Z-mass peak
- Variable number of jets
  - 1, 2, 3 jets
  - 9, 13, 17 dimensions

Butter et al.: **Generative Networks for Precision Enthusiasts** (2021), 2110.13632

Flow Generators

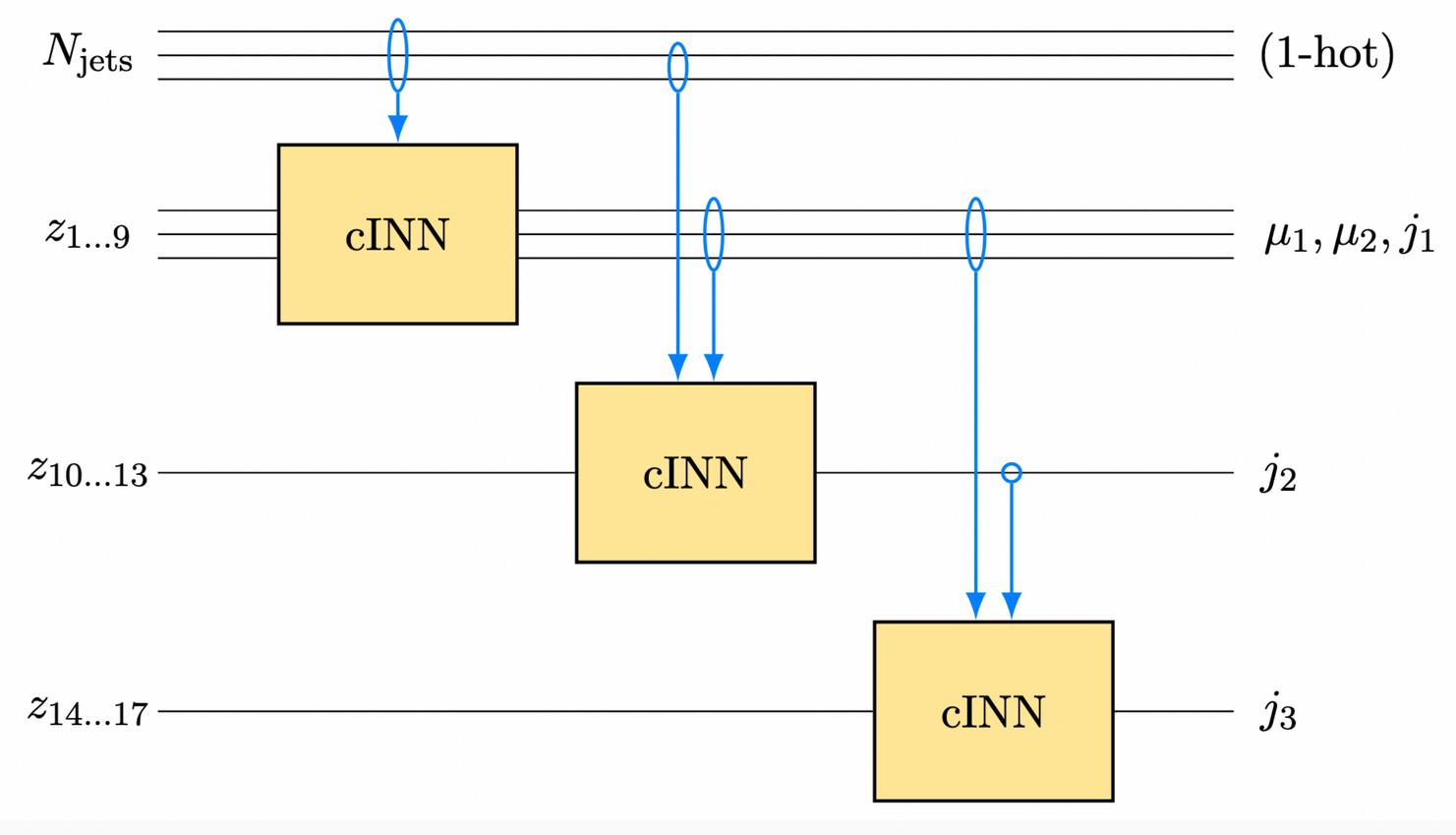
- Variable number of jets
  - 1, 2, 3 jets
  - 9, 13, 17 dimensions
- Still orderer list
- Different network for each number of jets
  - Redundant parts



Butter et al.: **Generative Networks for Precision Enthusiasts** (2021), 2110.13632

Flow Generators

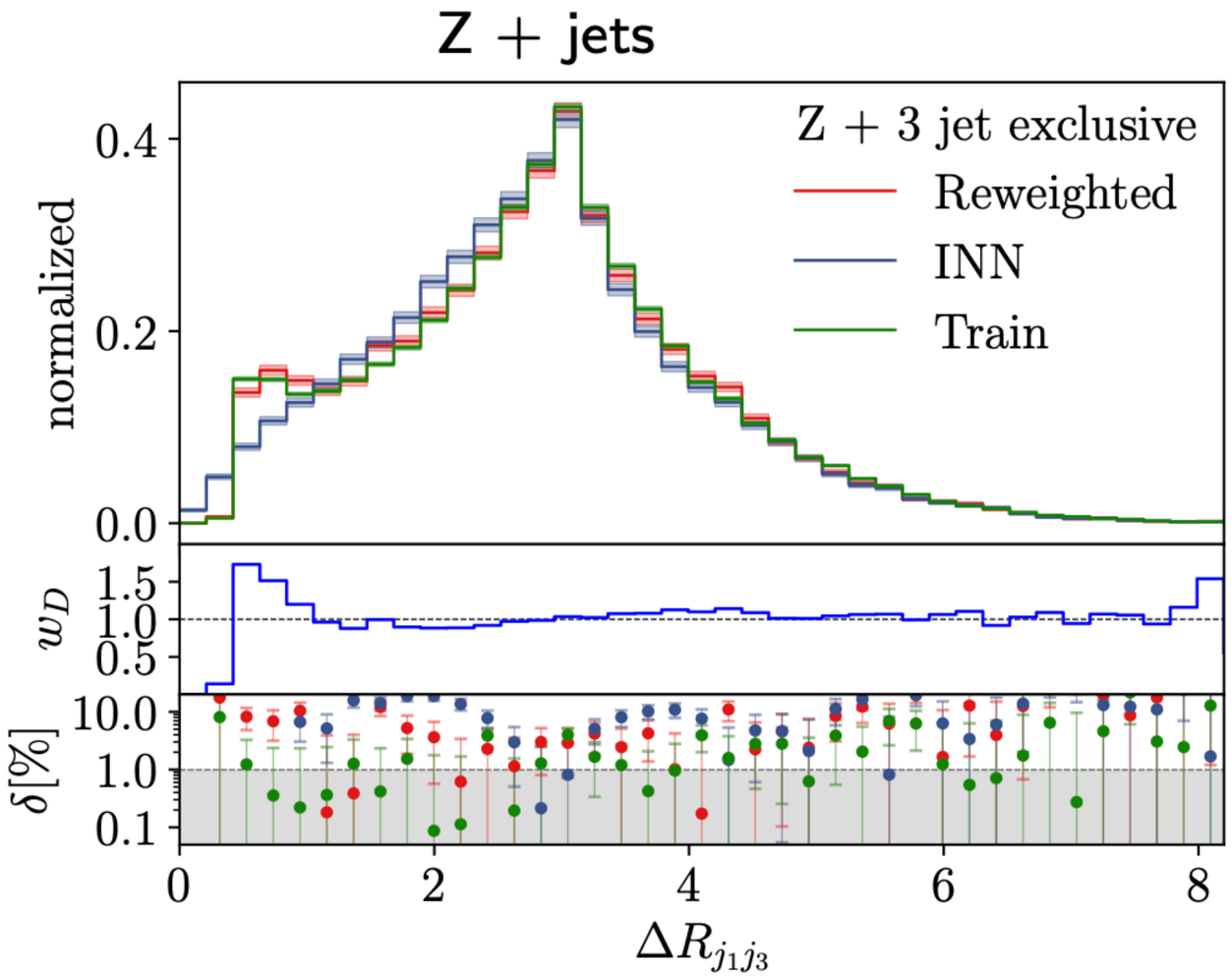
- Variable number of jets
  - 1, 2, 3 jets
  - 9, 13, 17 dimensions
- Still orderer list
- Different network for each number of jets
  - Redundant parts
- Series of networks conditional on previous



Butter et al.: **Generative Networks for Precision Enthusiasts** (2021), 2110.13632

Flow Generators

- Topological Problems:
  - Mapping single peak gaussian to two peak structure
- Train classifier on real vs fake samples



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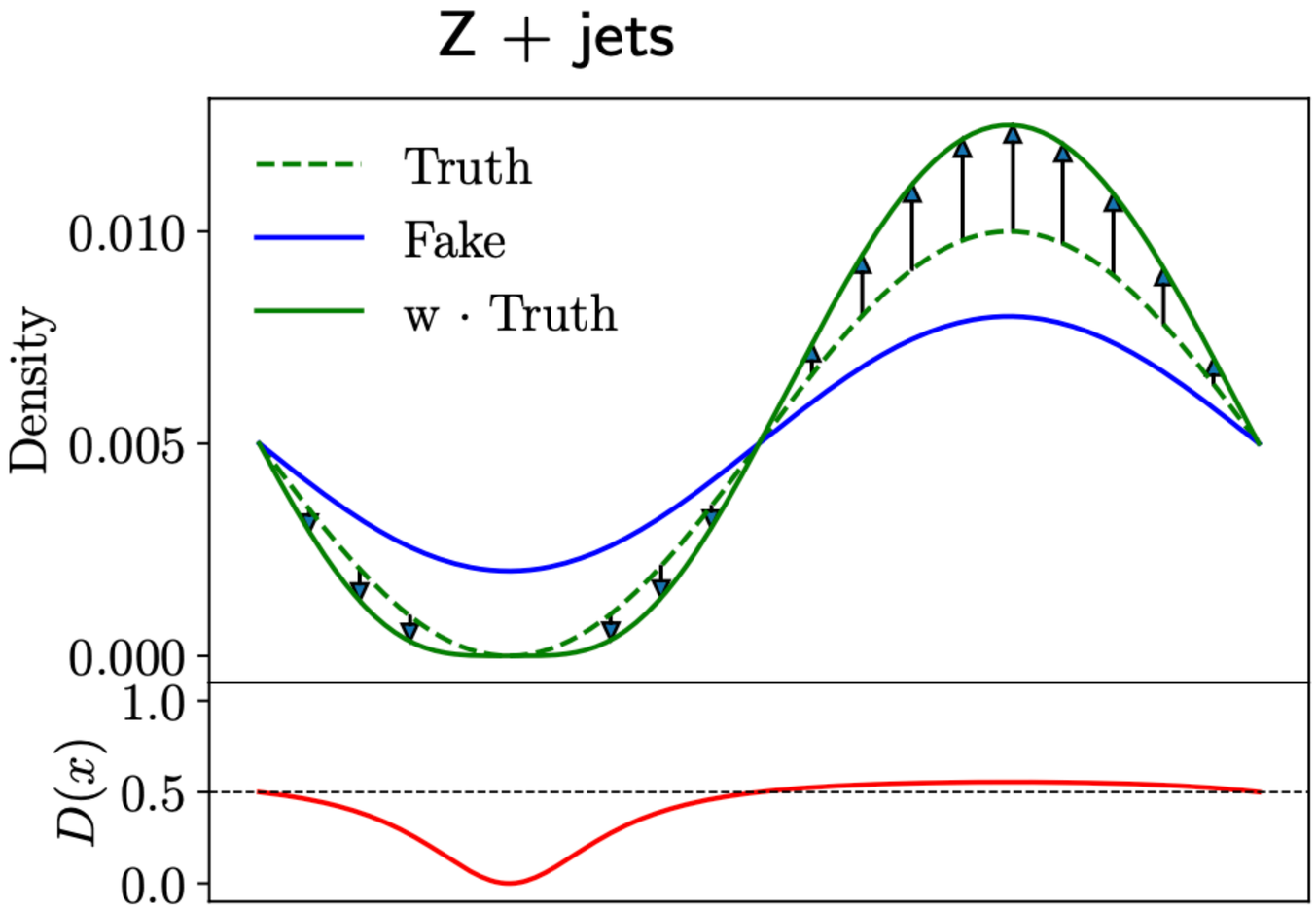
Flow Generators

- Topological Problems:
  - Mapping single peak gaussian to two peak structure
- Train classifier on real vs fake samples
- Use classifier weight during training



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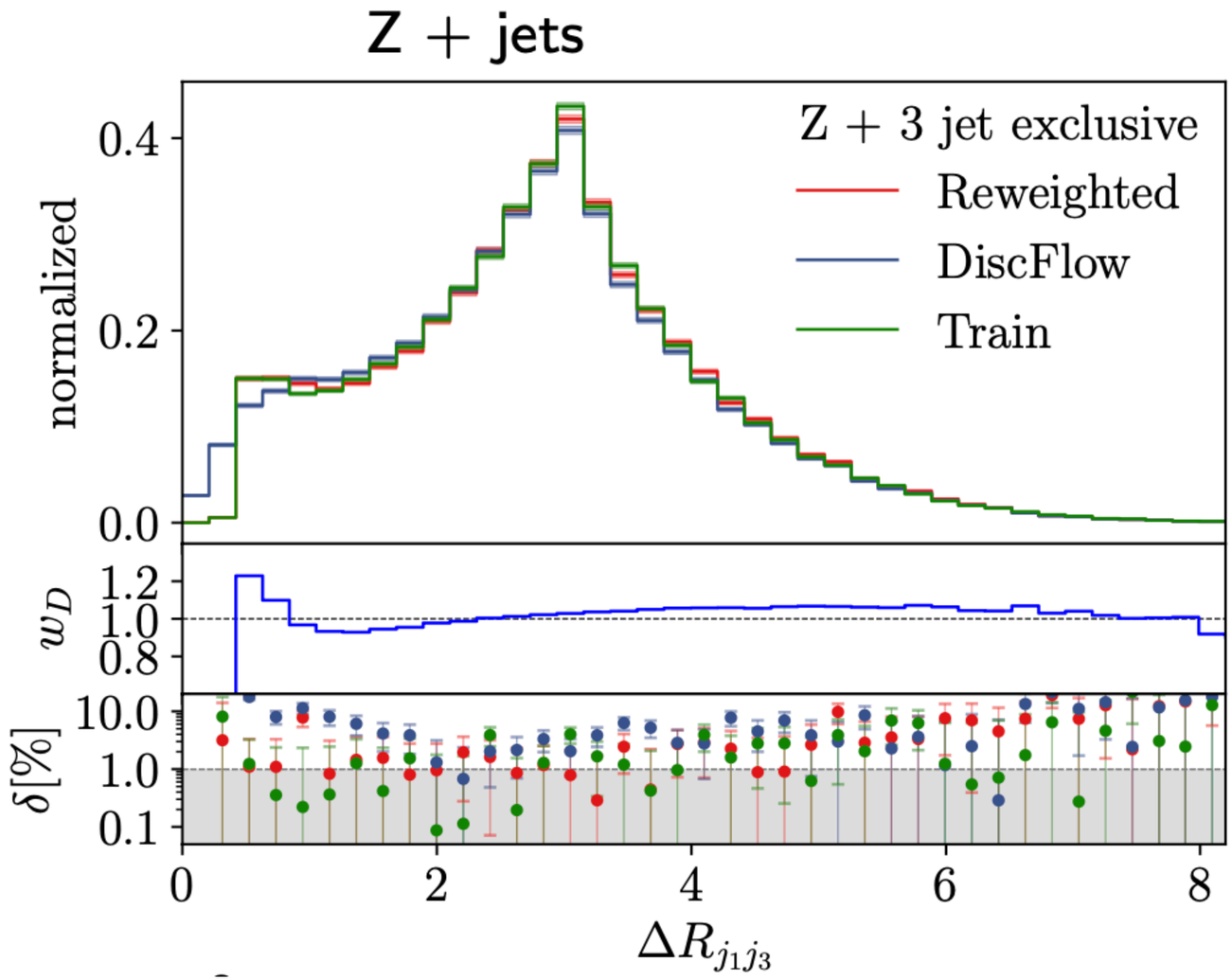
Flow Generators

- Topological Problems:
  - Mapping single peak gaussian to two peak structure
- Train classifier on real vs fake samples
- Use classifier weight during training
- Significant Improvement



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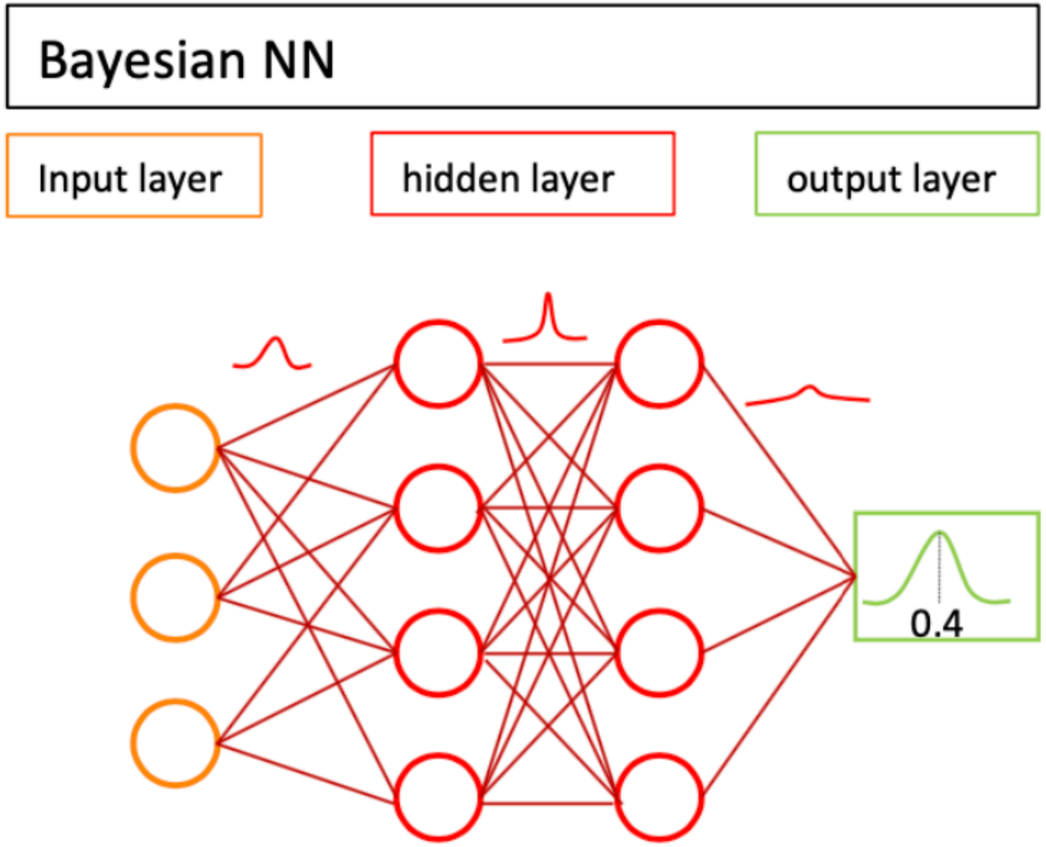
Uncertainties

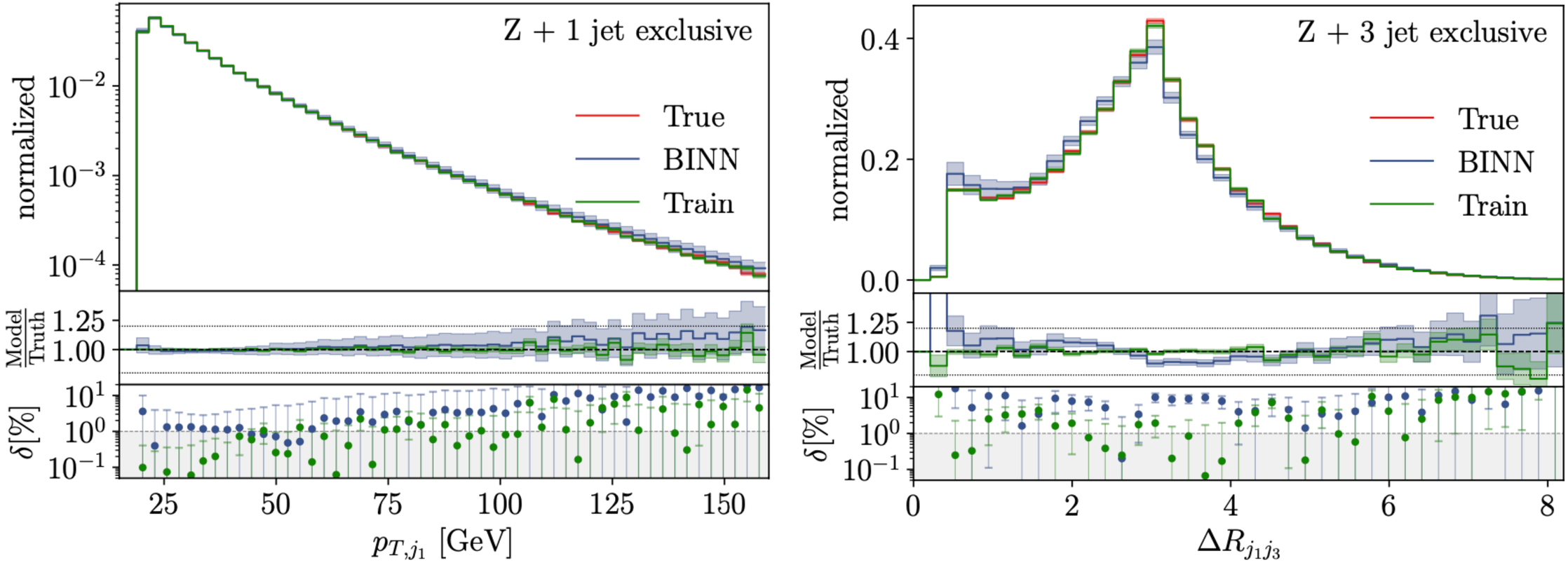


- Bayesian Neural Network
- Replace network weights with gaussian distributions
- Additional loss to regularise gaussian distributions
- Viable option for Flows/INNs
- Can we estimate generative uncertainties?

$$\mathcal{L} = \mathcal{L}_{INN} + KL_{prior}$$

Butter et al.: **Generative Networks for Precision Enthusiasts** (2021), 2110.13632

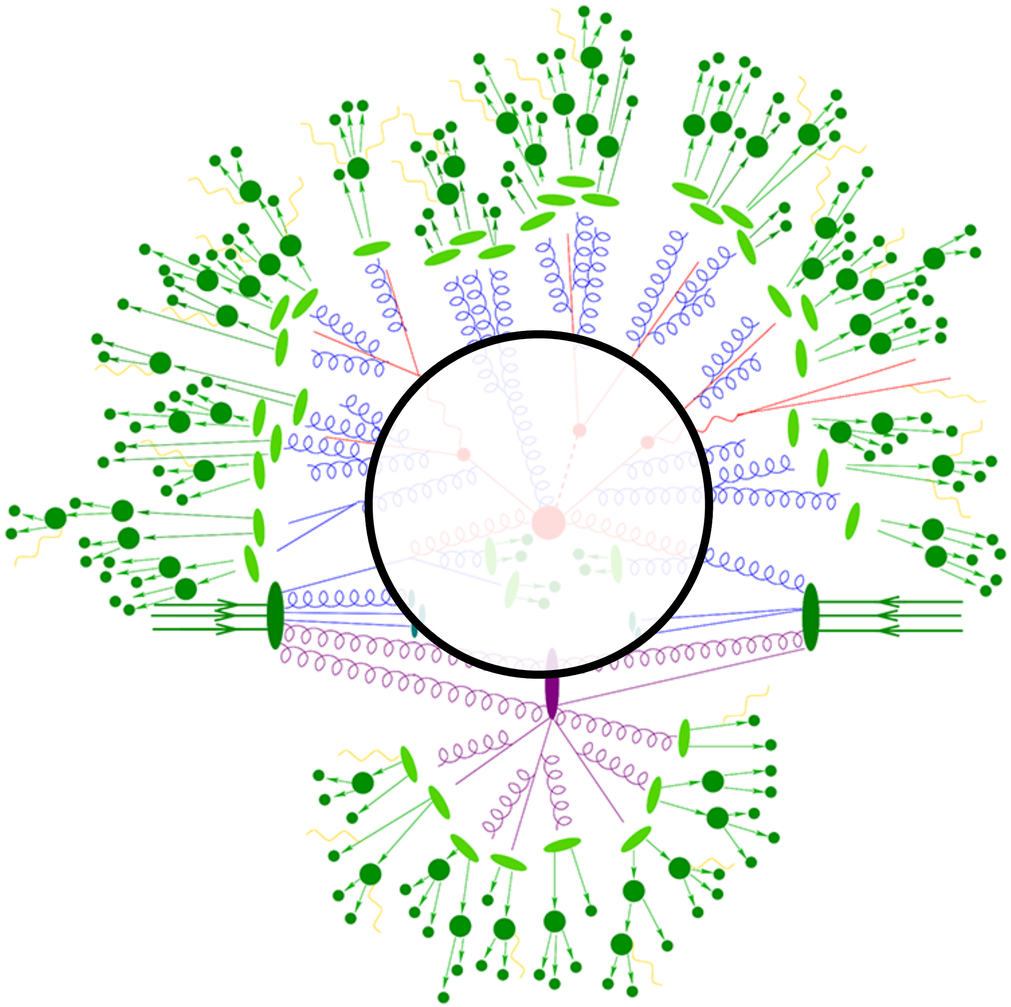
Uncertainties



- BINN uncertainty captures convergence of the network
- Does NOT capture how where the network fails

Butter et al.: **Generative Networks for Precision Enthusiasts** (2021), 2110.13632

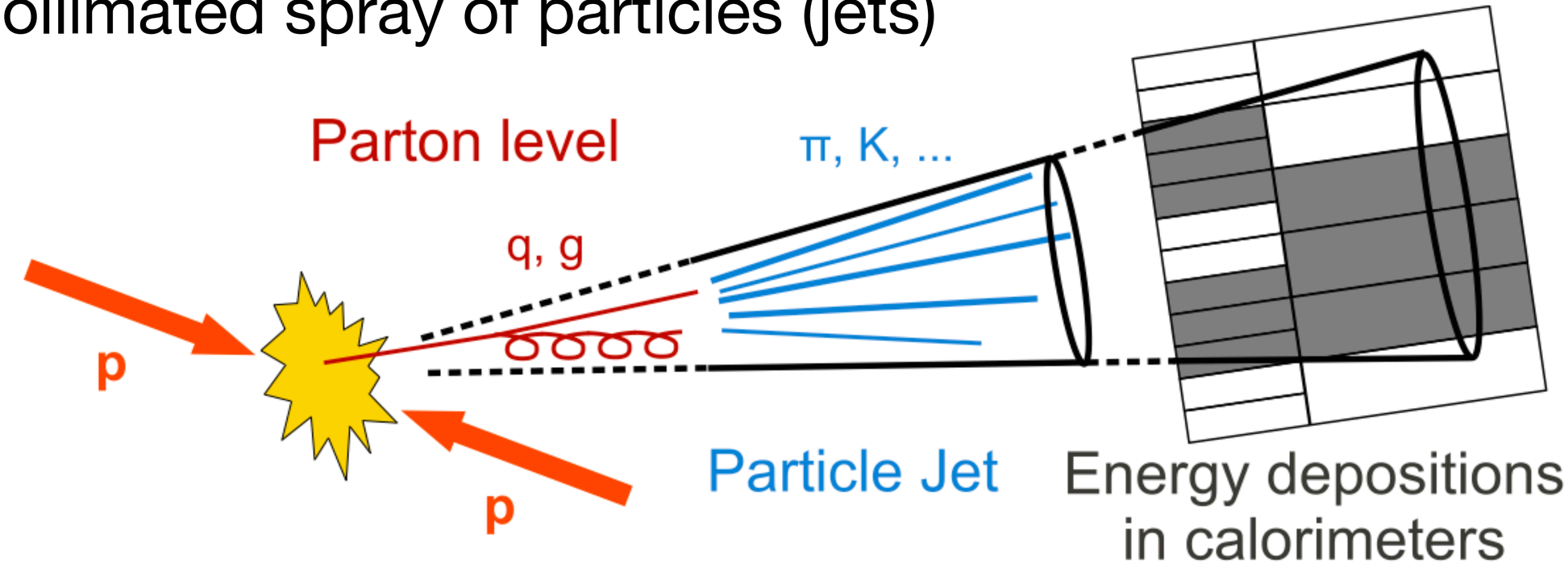
4-Momentum in Generation



- Particle collision simulation:
  - Parton level:
    - Low, fixed number of objects
    - Sharp structures
  - Hadronisation:
    - High, variable number of objects

Hadronisation

- Some particles (quarks, gluons) radiate/split of particles
  - Collimated spray of particles (jets)

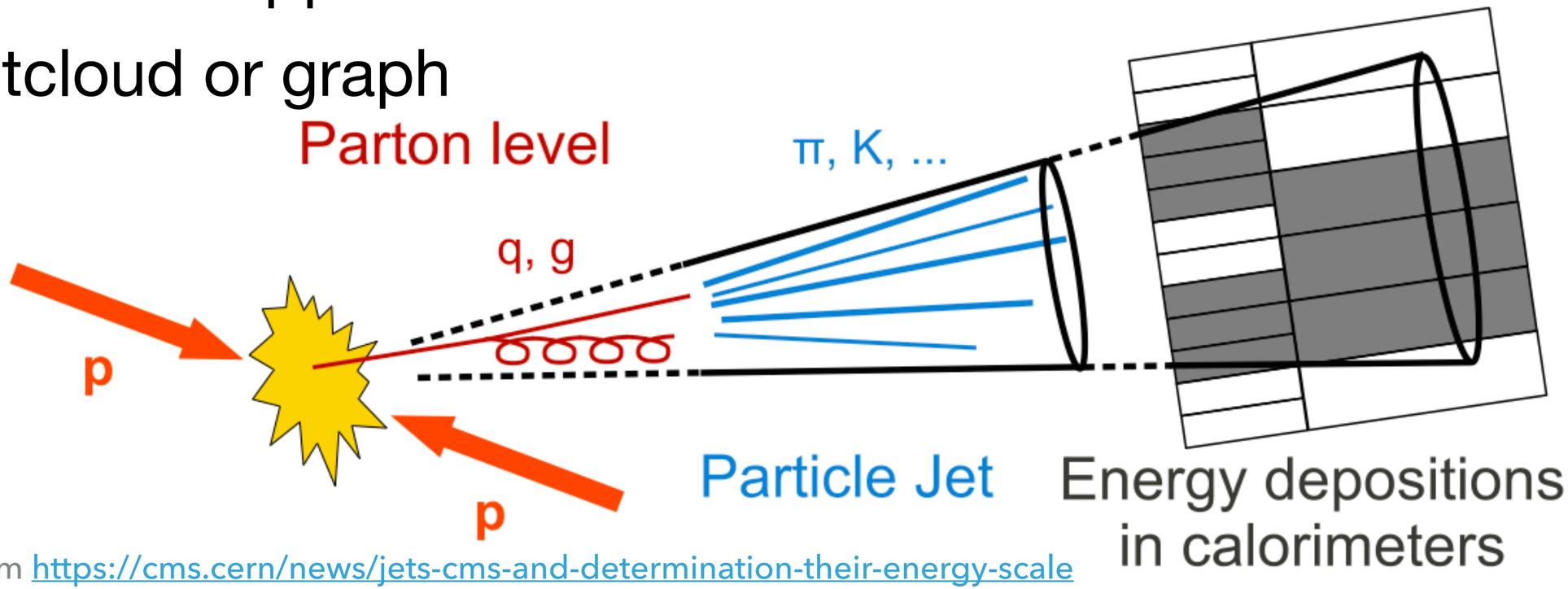


Taken form https://cms.cern/news/jets-cms-and-determination-their-energy-scale

- Jet constituents expressed as 4-momenta

Hadronisation

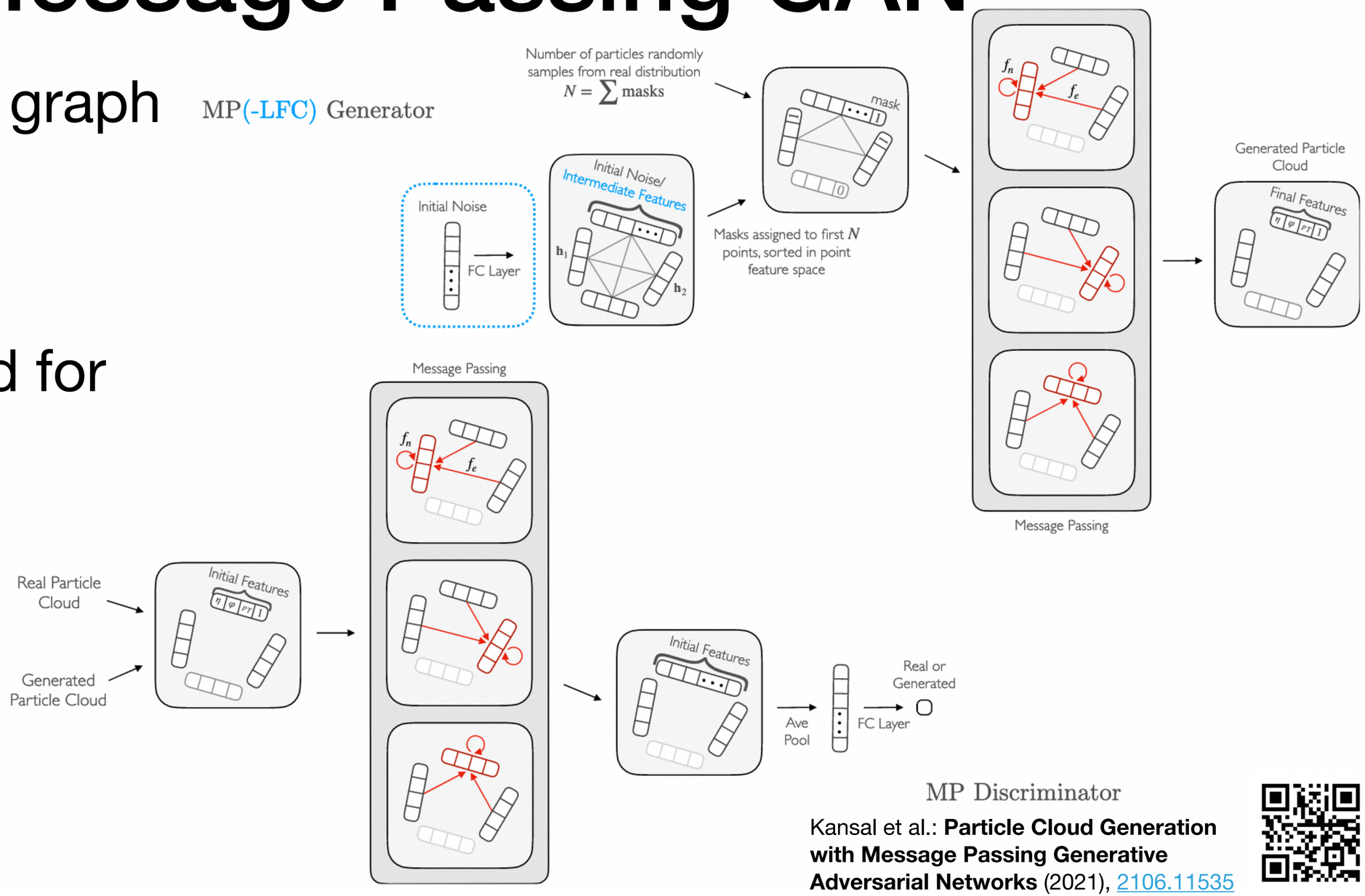
- Jet simulation:
  - Type of particles not determined
  - Number of particle not determined
  - Ordered list approach not viable
  - Pointcloud or graph



Taken form https://cms.cern/news/jets-cms-and-determination-their-energy-scale

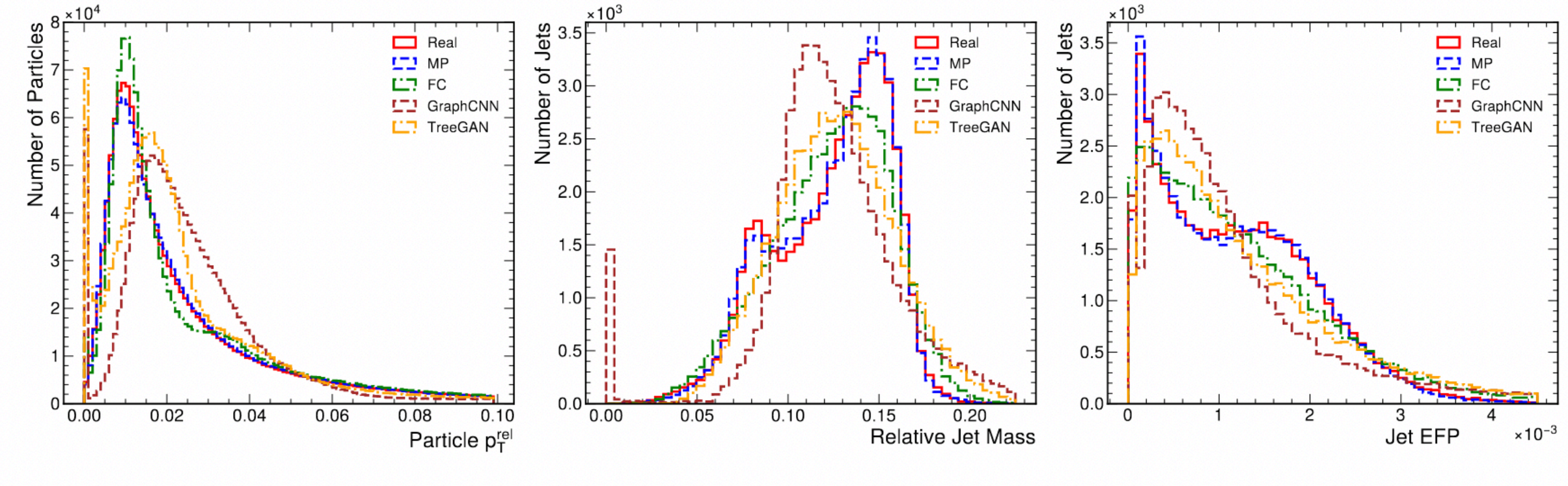
Message Passing GAN

- Fully connected graph
- 30 constituents
- Zero padding
- Padding masked for MP/pooling
- Permutation invariant data



Kansal et al.: **Particle Cloud Generation with Message Passing Generative Adversarial Networks** (2021), 2106.11535

Message Passing GAN



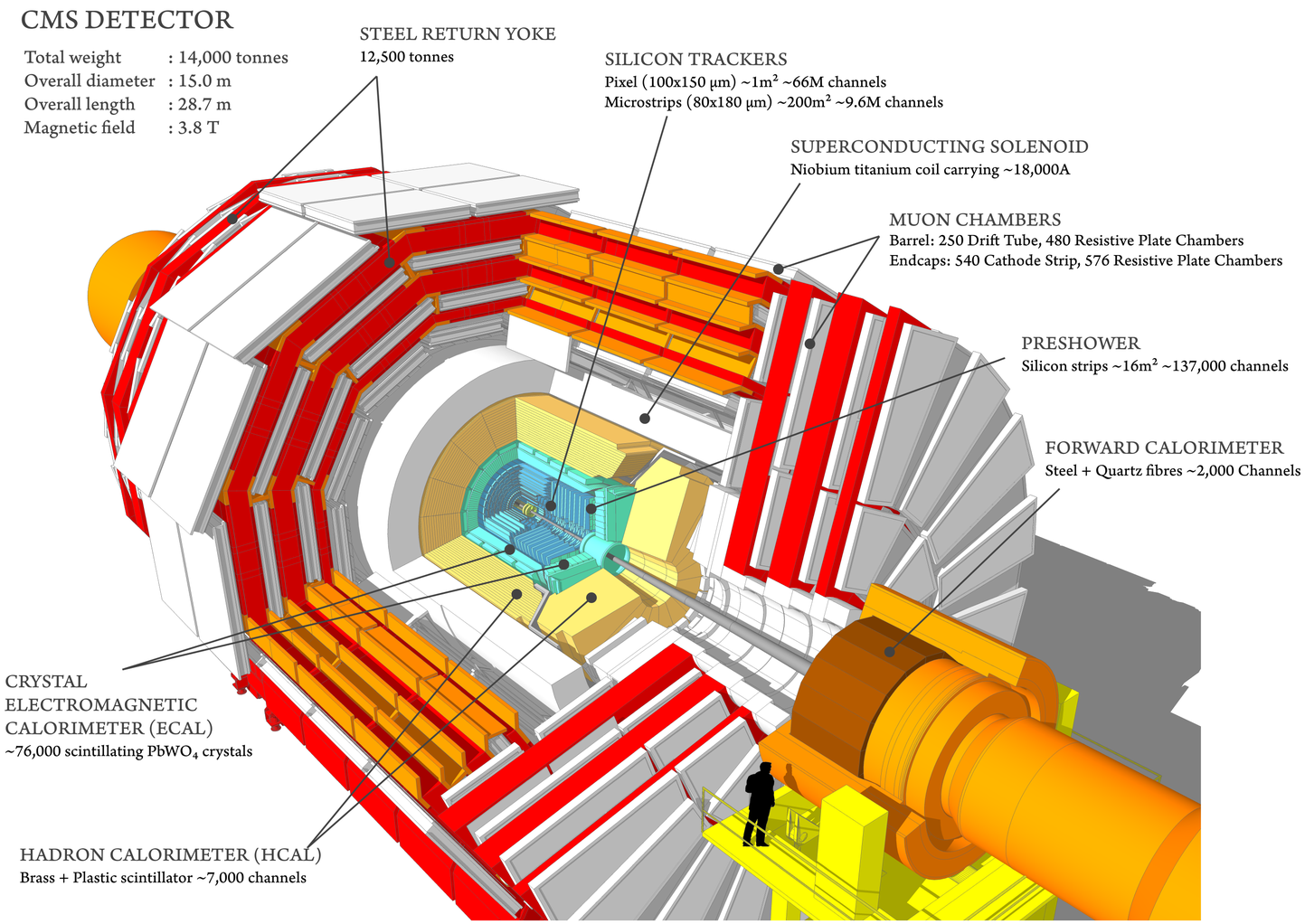
- Message Passing GAN: blue MP line
- High accuracy in explicit and derived features

Butter et al.: **Generative Networks for Precision Enthusiasts** (2021), 2110.13632

Summary

- Data structure with large hidden complexity
  - Encodes significant physics information
  - Difficult to model with generative models
- Impressive results achieved so far
  - Significant ongoing effort to further improve setups
  - Especially for large numbers of particles

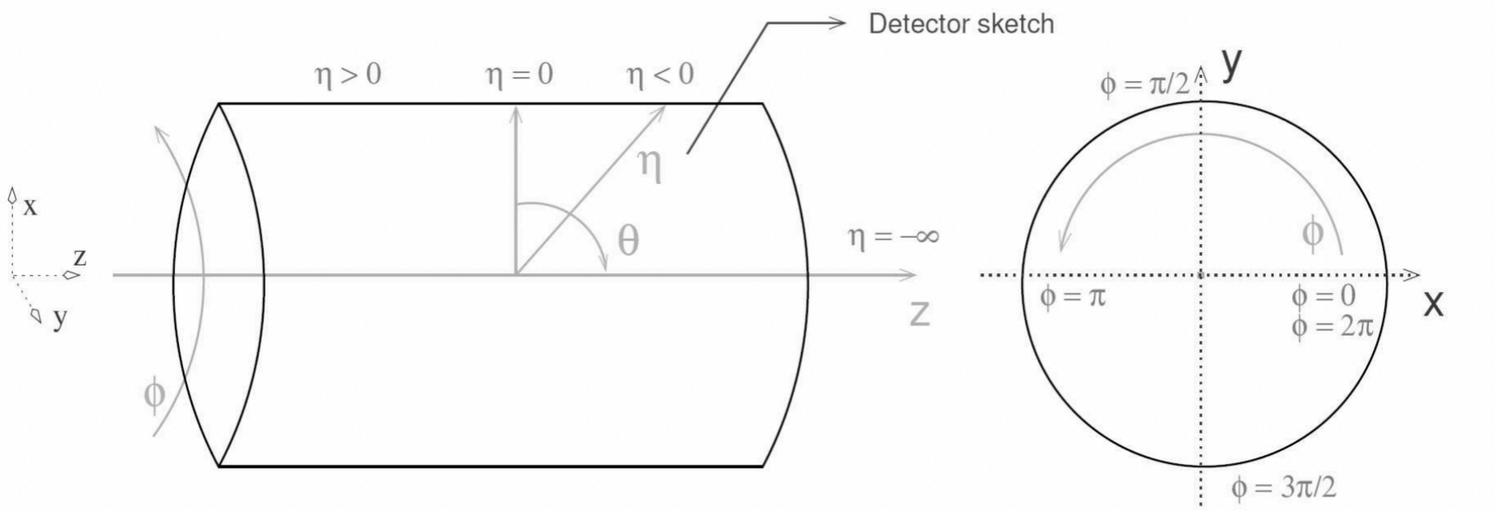
Backup



Taken form https://cms.cern/detector

- Cylindrical symmetry in detector setup
  - 4 momentum can be translated into cylindrical coordinates

Backup

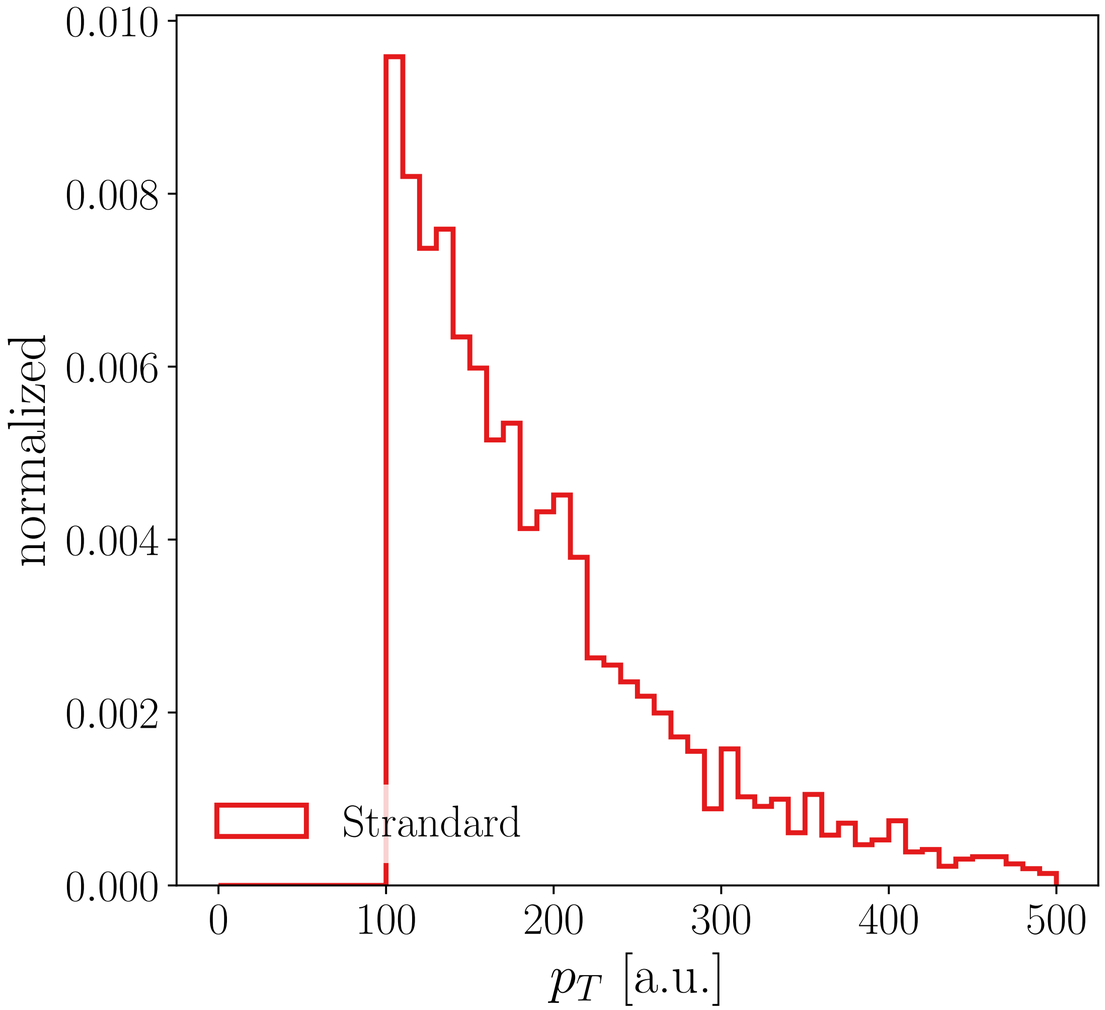


Taken form https://pos.sissa.it/050/055/pdf

- Cylindrical symmetry in detector setup
  - 4 momentum can be translated into cylindrical coordinates

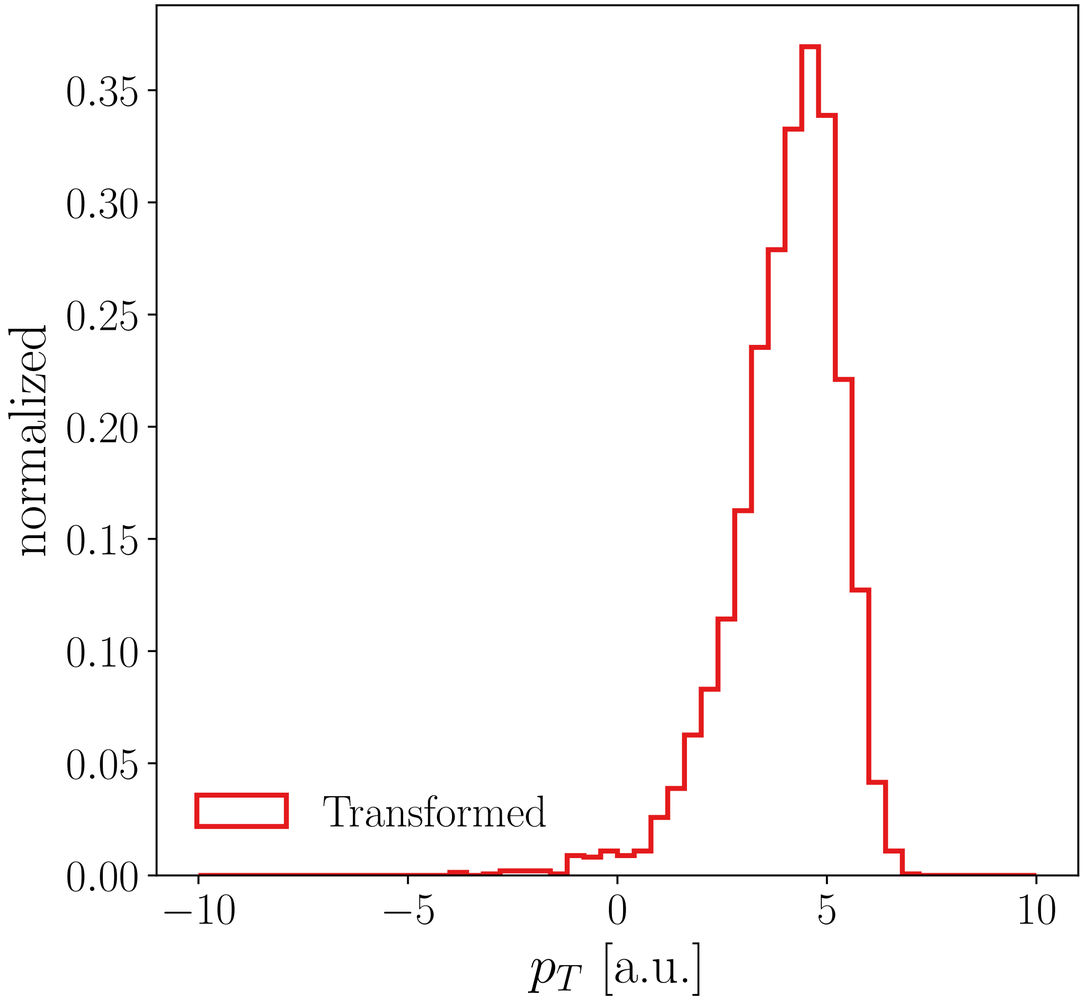
$$\begin{bmatrix} E \\ p_1 \\ p_2 \\ p_3 \end{bmatrix} \rightarrow \begin{bmatrix} m \\ p_T \\ \eta \\ \phi \end{bmatrix}$$

Backup



- Transversal momentum p_T
- Often has lower cutoff
  - Removes noise
  - Workable for GAN
  - Difficult for flows

Backup



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➡ Log transform