GRAN SASSO SCIENCE INSTITUTE
SCHOOL OF ADVANCED STUDIES
Scuola Universitaria Superiore

LECTURE 1

COSMIC RAY ACCELERATION

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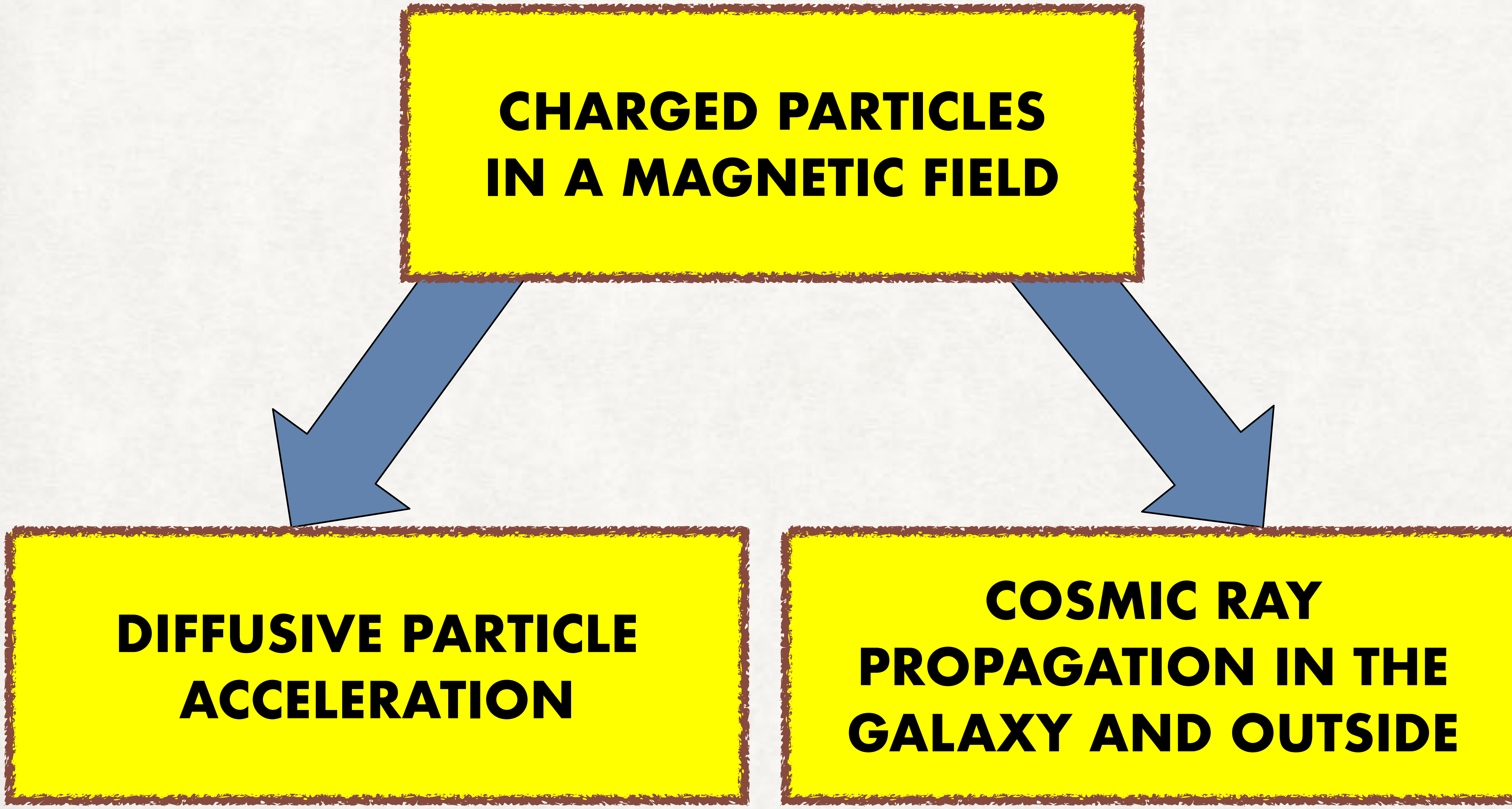
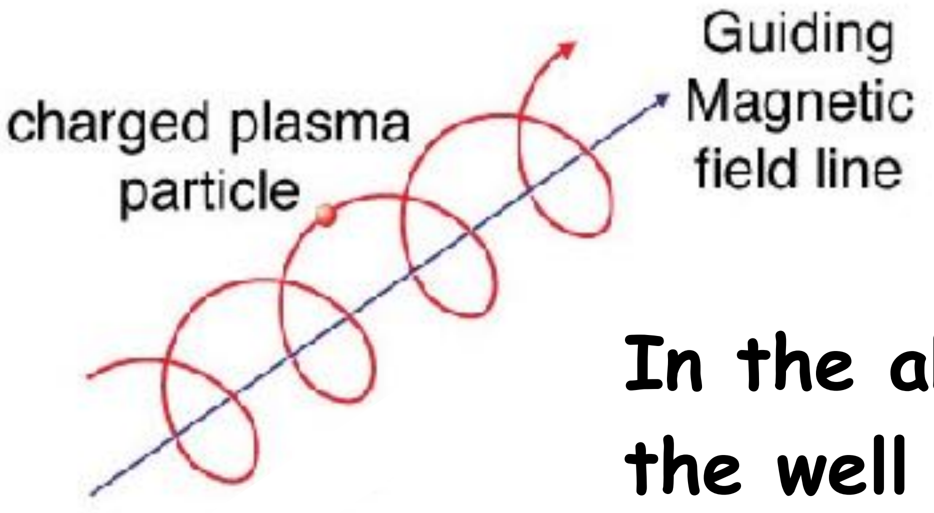
OUTLINE OF THE LECTURE

- *Principles of CR transport*
- *Second Order Fermi Acceleration*
- *Diffusive Shock Acceleration (DSA): test particle theory*
- *DSA: modern aspects*
    - *dynamical reaction of accelerated particles*
    - *B-field amplification due to accelerated particles*
    - *Maximum energy*
    - *postcursor physics and effect on spectra*

COSMIC RAY TRANSPORT



CHARGED PARTICLES IN A REGULAR B FIELD



$$\frac{d\vec{p}}{dt} = q\left[\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right]$$

In the absence of an electric field one obtains the well known solution:

$$p_z = \text{Constant}$$

$$v_x = V_0 \cos[\Omega\, t]$$

$$v_y = V_0 \sin[\Omega\, t]$$

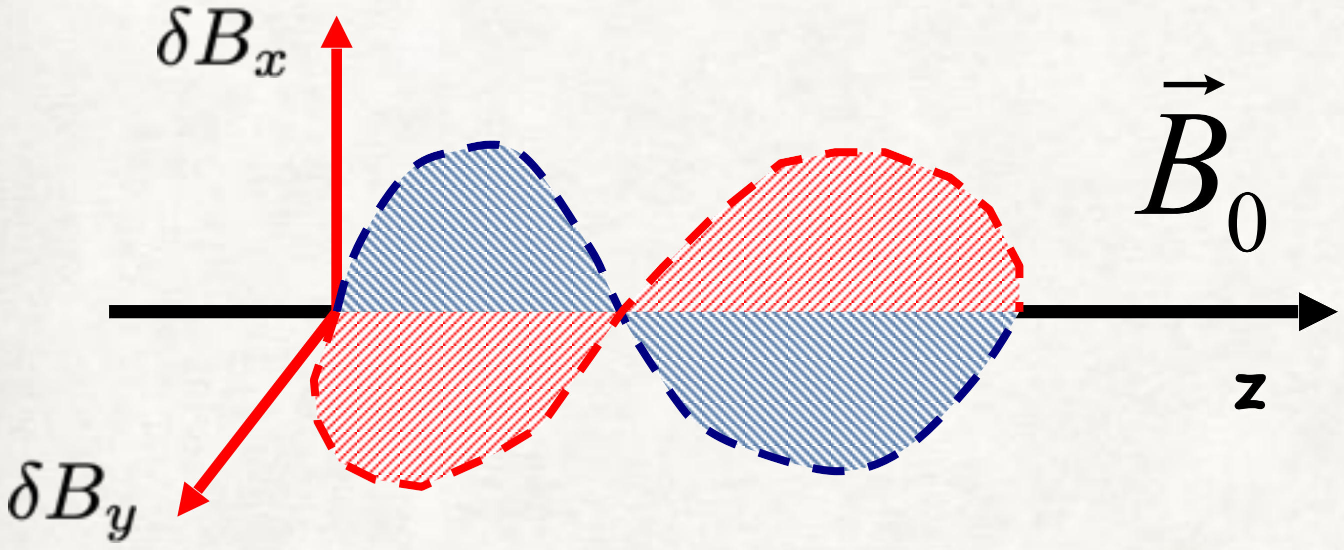
LARMOR FREQUENCY

$$\Omega = \frac{q\, B_0}{m\, c\, \gamma}$$

A FEW THINGS TO KEEP IN MIND

- THE MAGNETIC FIELD DOES NOT CHANGE PARTICLE ENERGY -> NO ACCELERATION BY B FIELDS
- A RELATIVISTIC PARTICLE MOVES IN THE Z DIRECTION ON AVERAGE AT C/3

MOTION OF A PARTICLE IN A WAVY FIELD



Let us consider an Alfven wave propagating in the z direction:

$$\delta B << B_0 \qquad \delta\vec{B} \perp \vec{B}_0$$

We can neglect (for now) the electric field associated with the wave, or in other words we can sit in the reference frame of the wave:

$$\frac{d\vec{p}}{dt} = q\,\frac{\vec{v}}{c} \times (\vec{B}_0 + \delta\vec{B})$$

THIS CHANGES ONLY THE X AND Y COMPONENTS OF THE MOMENTUM

THIS TERM CHANGES ONLY THE DIRECTION OF $P_Z=P\mu$

Remember that the wave typically moves with the Alfven speed:

$$v_a = \frac{B}{(4\pi\rho)^{1/2}} = 2 \times 10^6 B_\mu n_1^{-1/2} \; cm/s$$

Alfven waves have frequencies << ion gyration frequency $\Omega_p = qB/m_p c$

It is therefore clear that for a relativistic particle these waves, in first approximation, look like static waves.

The equation of motion can be written as:

$$\frac{d\vec{p}}{dt} = \frac{q}{c}\vec{v} \times (\vec{B}_0 + \vec{\delta} B)$$

If to split the momentum in parallel and perpendicular, the perpendicular component cannot change in modulus, while the parallel momentum is described by

$$\frac{dp \parallel}{dt} = \frac{q}{c}|\vec{v}_\perp \times \delta\vec{B}| \qquad p_{||} = p \; \mu$$

$$\frac{d\mu}{dt} = \frac{q}{pc} v(1-\mu^2)^{1/2} \delta B cos(\Omega t - kx + \psi)$$

Wave form of the magnetic field with a random phase and frequency

$$\Omega = qB_0/mc\gamma$$

Larmor frequency

In the frame in which the wave is at rest we can write $x = v\mu t$

$$\frac{d\mu}{dt} = \frac{q}{pc} v(1-\mu^2)^{1/2} \delta B cos\left[(\Omega - kv\mu)t + \psi\right]$$

It is clear that the mean value of the pitch angle variation over a long enough time vanishes

$$\langle \Delta\mu \rangle_t = 0$$

We want to see now what happens to $\langle \Delta\mu \Delta\mu \rangle$

Let us first average upon the random phase of the waves:

$$\langle \Delta\mu(t')\Delta\mu(t")\rangle_\psi = \frac{q^2v^2(1-\mu^2)\delta B^2}{2c^2p^2} cos\left[(\Omega - kv\mu)(t'-t")\right]$$

And integrating over time:

$$\langle \Delta\mu\Delta\mu\rangle_t = \frac{q^2v^2(1-\mu^2)\delta B^2}{2c^2p^2}\int dt' \int dt" cos\left[(\Omega - kv\mu)(t'-t")\right]$$

$$= \frac{q^2v(1-\mu^2)\delta B^2}{c^2p^2\mu}\delta(k-\Omega/v\mu)\Delta t$$

RESONANCE

Many waves

IN GENERAL ONE DOES NOT HAVE A SINGLE WAVE BUT RATHER A POWER SPECTRUM:

$$P(k) = B_k^2/4\pi$$

THEREFORE INTEGRATING OVER ALL OF THEM:

$$\langle \frac{\Delta\mu\Delta\mu}{\Delta t} \rangle = \frac{q^2(1-\mu^2)\pi}{m^2c^2\gamma^2}\frac{1}{v\mu}4\pi\int dk \frac{\delta B(k)^2}{4\pi}\delta(k-\Omega/v\mu)$$

OR IN A MORE IMMEDIATE FORMALISM:

$$\left\langle \frac{\Delta\mu\Delta\mu}{\Delta t} \right\rangle = \frac{\pi}{2}\Omega\,(1-\mu^2)k_{res}F(k_{res})$$

$$k_{res} = \frac{\Omega}{v\mu}$$

RESONANCE!!!

DIFFUSION COEFFICIENT

THE RANDOM CHANGE OF THE PITCH ANGLE IS DESCRIBED BY A DIFFUSION COEFFICIENT

$$D_{\mu\mu} = \left\langle \frac{\Delta\theta\Delta\theta}{\Delta t} \right\rangle = \frac{\pi}{4}\Omega k_{res} F(k_{res})$$

FRACTIONAL POWER $(\delta B/B_0)^2$ $=G(k_{res})$

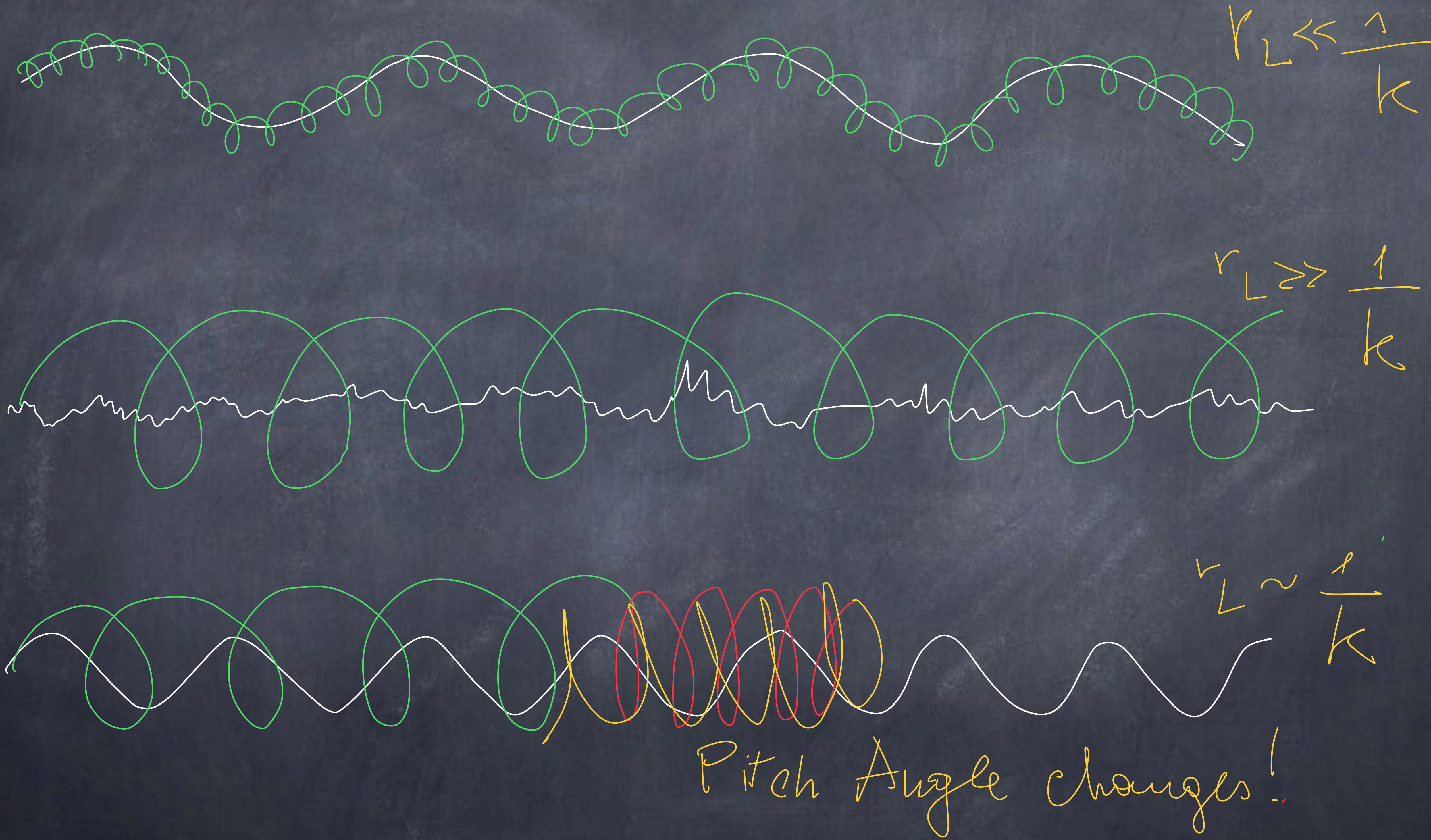
THE DEFLECTION ANGLE CHANGES BY ORDER UNITY IN A TIME:

$$\tau \approx \frac{1}{\Omega\, G(k_{res})}$$

PATHLENGTH FOR DIFFUSION ~ VT

$$\left\langle \frac{\Delta z \Delta z}{\Delta t} \right\rangle \approx v^2\,\tau = \frac{v^2}{\Omega\, G(k_{res})}$$

SPATIAL DIFFUSION COEFF.


$r_L \ll \frac{1}{k}$
$r_L \gg \frac{1}{k}$
$r_L \sim \frac{1}{k}$
Pitch Angle changes!

ACCELERATION OF NONTHERMAL PARTICLES

The presence of non-thermal particles is ubiquitous in the Universe (solar wind, Active galaxies, supernova remnants, gamma ray bursts, Pulsars, micro-quasars)

WHEREVER THERE ARE MAGNETIZED PLASMAS THERE ARE NON-THERMAL PARTICLES

↓

PARTICLE ACCELERATION

BUT THERMAL PARTICLES ARE USUALLY DOMINANT, SO WHAT DETERMINES THE DISCRIMINATION BETWEEN THERMAL AND ACCELERATED PARTICLES?

INJECTION

ALL ACCELERATION MECHANISMS ARE ELECTROMAGNETIC IN NATURE

MAGNETIC FIELD CANNOT MAKE WORK ON CHARGED PARTICLES THEREFORE ELECTRIC FIELDS ARE NEEDED FOR ACCELERATION TO OCCUR

REGULAR ACCELERATION

THE ELECTRIC FIELD IS LARGE SCALE:

$$\langle \vec{E} \rangle \neq 0$$

STOCHASTIC ACCELERATION

THE ELECTRIC FIELD IS SMALL SCALE:

$$\langle \vec{E} \rangle = 0 \quad \langle \vec{E}^2 \rangle \neq 0$$

STOCHASTIC ACCELERATION

$$\langle \vec{E} \rangle = 0 \quad \langle \vec{E}^2 \rangle \neq 0$$

We have seen that the action of random magnetic fluctuations is that of scattering particles when/if a resonance is achieved. In other words, the particle distribution gets isotropized in the reference frame of the waves.

Although in the reference frame of the waves momentum is conserved (there is no E-field! and B-fields do not make work) in the lab frame the particle momentum changes in random direction (+ or -) by

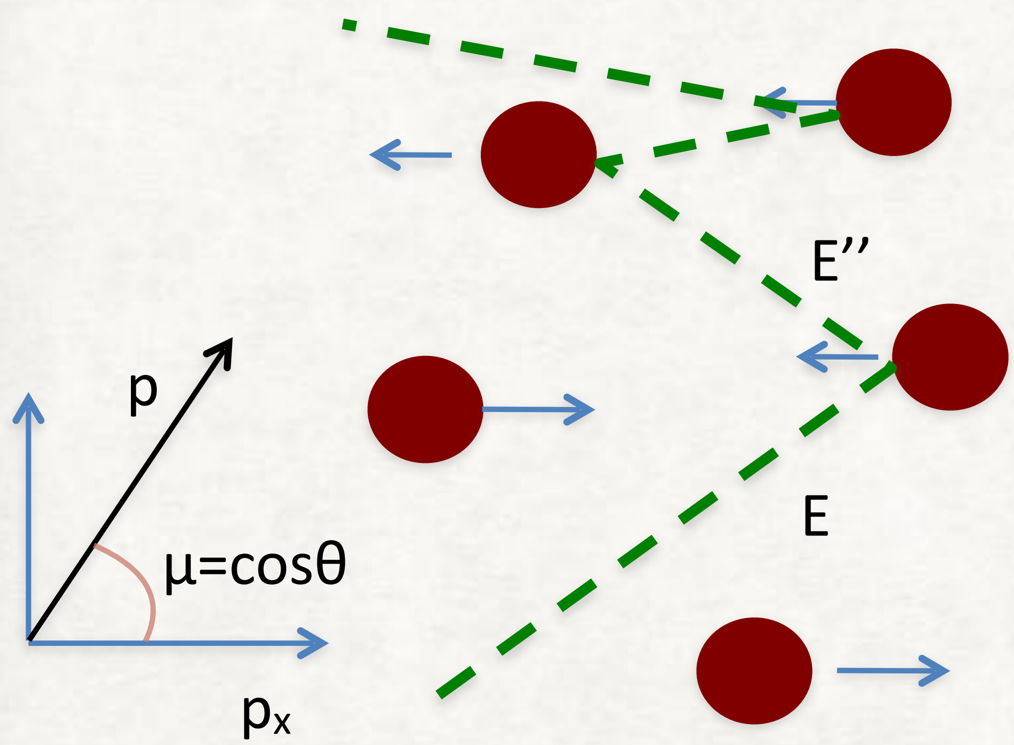
$$\Delta p \sim p \frac{v_A}{c}$$

In a time T which is the diffusion time as found in the last lecture. It follows that

$$D_{pp} = \langle \frac{\Delta P \Delta p}{\Delta t} \rangle \sim p^2 \frac{1}{T} \left(\frac{v_A}{c} \right)^2 \rightarrow \tau_{pp} = \frac{p^2}{D_{pp}} T \left(\frac{c}{v_A} \right)^2 \gg T$$

THE MOMENTUM CHANGE IS A SECOND ORDER PHENOMENON !!!

Second Order Fermi Acceleration



We inject a particle with energy E. In the reference frame of a cloud moving with speed β (and Lorentz factor γ, with γ-1<<1) the particle energy is:

$$E' = \gamma E + \beta\gamma p\mu$$

and the momentum along x is:

$$p'_x = \beta\gamma E + \gamma p\mu$$

Assuming that the cloud is very massive compared with the particle, we can assume that the cloud is unaffected by the scattering, therefore the particle energy in the cloud frame does not change and the momentum along x is simply inverted, so that after 'scattering' $p'_x \rightarrow - p'_x$. The final energy in the Lab frame is therefore:

$$E'' = \gamma E' + \beta\gamma p'_x =$$

$$\gamma^2 E\left(1 + \beta^2 + 2\beta\mu\frac{p}{E}\right)$$

$$\frac{p}{E} = \frac{mv\gamma}{m\gamma} = v$$

Where v is now the dimensionless particle velocity

It follows that: $$E" = \gamma^2 E \left(1 + \beta^2 + 2\beta\mu v\right)$$

and: $$\frac{E" - E}{E} = \gamma^2 \left(1 + 2\beta v\mu + \beta^2\right) - 1$$

and finally, taking the limit of non-relativistic clouds $\gamma \to 1$:

$$\frac{E" - E}{E} \approx 2\beta^2 + 2\beta v\mu$$

We can see that the fractional energy change can be both positive or negative, which means that particles can either gain or lose energy, depending on whether the particle-cloud scattering is head-on or tail-on.

THIS FACT IS SINGLE-HANDEDLY RESPONSIBLE FOR THE SECOND ORDER NATURE OF THE ACCELERATION PROCESS

We need to calculate the probability that a scattering occurs head-on or Tail-on. The scattering probability along direction μ is proportional to the Relative velocity in that direction:

$$P(\mu) = Av_{rel} = A\frac{\beta\mu + v}{1 + v\beta\mu} \rightarrow_{v\rightarrow 1} \approx A(1 + \beta\mu)$$

The condition of normalization to unity:

$$\int_{-1}^{1} P(\mu)d\mu = 1$$

leads to A=1/2. It follows that the mean fractional energy change is:

$$\langle\frac{\Delta E}{E}\rangle = \int_{-1}^{1} d\mu P(\mu)\left(2\beta^2 + 2\beta\mu\right) = \frac{8}{3}\beta^2$$

NOTE THAT IF WE DID NOT ASSUME RIGID REFLECTION AT EACH CLOUD BUT RATHER ISOTROPIZATION OF THE PITCH ANGLE IN EACH CLOUD, THEN WE WOULD HAVE OBTAINED (4/3) β^2 INSTEAD OF (8/3) β^2

THE FRACTIONAL CHANGE IS A SECOND ORDER QUANTITY IN $\beta<<1$. This is the reason for the name SECOND ORDER FERMI ACCELERATION

The acceleration process can in fact be shown to become more important in the relativistic regime where $\beta \rightarrow 1$

THE PHYSICAL ESSENCE CONTAINED IN THIS SECOND ORDER DEPENDENCE IS THAT IN EACH PARTICLE-CLOUD SCATTERING THE ENERGY OF THE PARTICLE CAN EITHER INCREASE OR DECREASE → WE ARE LOOKING AT A PROCESS OF DIFFUSION IN MOMENTUM SPACE

THE REASON WHY ON AVERAGE THE MEAN ENERGY INCREASES IS THAT HEAD-ON COLLISIONS ARE MORE PROBABLE THAN TAIL-ON COLLISIONS

WHAT IS DOING THE WORK?

We just found that particles propagating in a magnetic field can change their momentum (in modulus and direction)...

BUT MAGNETIC FIELDS CANNOT CHANGE THE MOMENTUM MODULUS... ONLY ELECTRIC FIELDS CAN

WHAT IS THE SOURCE OF THE ELECTRIC FIELDS???
Moving Magnetic Fields

The induced electric field (FOR INSTANCE THE ONE CARRIED BY ALFVEN WAVES) is responsible for this first instance of particle acceleration

FOOD FOR THOUGHT: Scattering of particles in pitch angle leads to isotropization... where is the momentum in the x-direction going???

SITUATION AS OF EARLY 1950'S

- Fermi managed to find an acceleration process that energizes charged particles
- But it is second order …
- and it is second order in a quantity $\boldsymbol{\beta} \sim v_A/c << 1$

IT TOOK ABOUT 30 YEARS TO FIGURE OUT A WAY TO TRANSFORM THE PROCESS FROM SECOND TO FIRST ORDER AND TO THINK OF A PARAMETER $\boldsymbol{\beta}$ NOT SO $<<1$

IN PART THE PROGRESS WAS FACILITATED BY THE TREMENDOUS AND WORRISOME EXPERIENCE GROWN DURING THE COLD WAR WITH *EXPLODING THINGS…*

A PLASMA MOVING INTO ANOTHER PLASMA

When a plasma moves towards a hard surface or towards another plasma, you can ask the meaningful question of what happens to it

The most obvious way to answer this question would be to impose the good old conservation laws:

MASS $$\frac{\partial}{\partial x}(\rho u) = 0 \qquad \rho u = \text{constant}$$

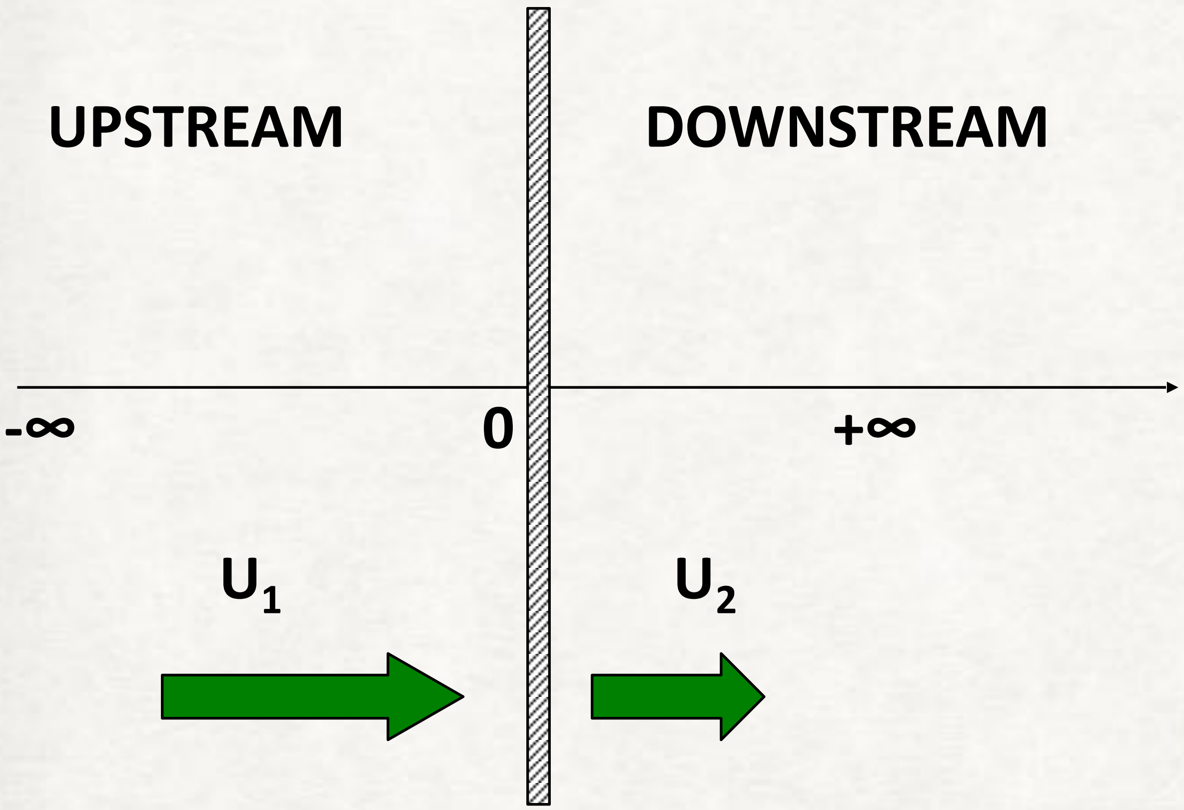
MOMENTUM $$\frac{\partial}{\partial x}(\rho u^2 + P) = 0 \qquad \rho u^2 + P = \text{constant}$$

ENERGY $$\frac{\partial}{\partial x}\left(\frac{1}{2}\rho u^3 + \frac{\gamma}{\gamma - 1}uP\right) = 0 \qquad \frac{1}{2}\rho u^3 + \frac{\gamma}{\gamma - 1}uP = \text{constant}$$

One can see that there is the trivial solution — density, velocity and pressure are the same everywhere … constant…

BUT… there is one more solution that appears only when u/c_s = Mach number>1

THE SHOCK SOLUTION



Let us sit in the reference frame in which the shock is at rest and look for stationary solutions

$$\frac{\partial}{\partial x}(\rho u) = 0$$

$$\frac{\partial}{\partial x}(\rho u^2 + P) = 0$$

$$\frac{\partial}{\partial x}\left(\frac{1}{2}\rho u^3 + \frac{\gamma}{\gamma - 1}uP\right) = 0$$

It is easy to show that aside from the trivial solution in which all quantities remain spatially constant, there is a discontinuous solution:

$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2 + 2}$$

$$\frac{p_2}{p_1} = \frac{2\gamma M_1^2}{\gamma+1} - \frac{\gamma - 1}{\gamma + 1}$$

$$\frac{T_2}{T_1} = \frac{[2\gamma M_1^2 - \gamma(\gamma-1)][(\gamma-1)M_1^2 + 2]}{(\gamma+1)^2 M_1^2}$$

M_1 is the upstream Fluid Mach number and MUST BE >1

STRONG SHOCKS M_1>>1

In the limit of strong shock fronts these expressions get substantially simpler and one has:

$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{\gamma+1}{\gamma-1} \rightarrow 4$$

$$\frac{p_2}{p_1} = \frac{2\gamma M_1^2}{\gamma+1} \rightarrow \text{becomes divergently large}$$

$$\frac{T_2}{T_1} = \frac{2\gamma(\gamma-1)}{(\gamma+1)^2} M_1^2, \quad T_2 = 2\frac{\gamma-1}{(\gamma+1)^2} m u_1^2$$

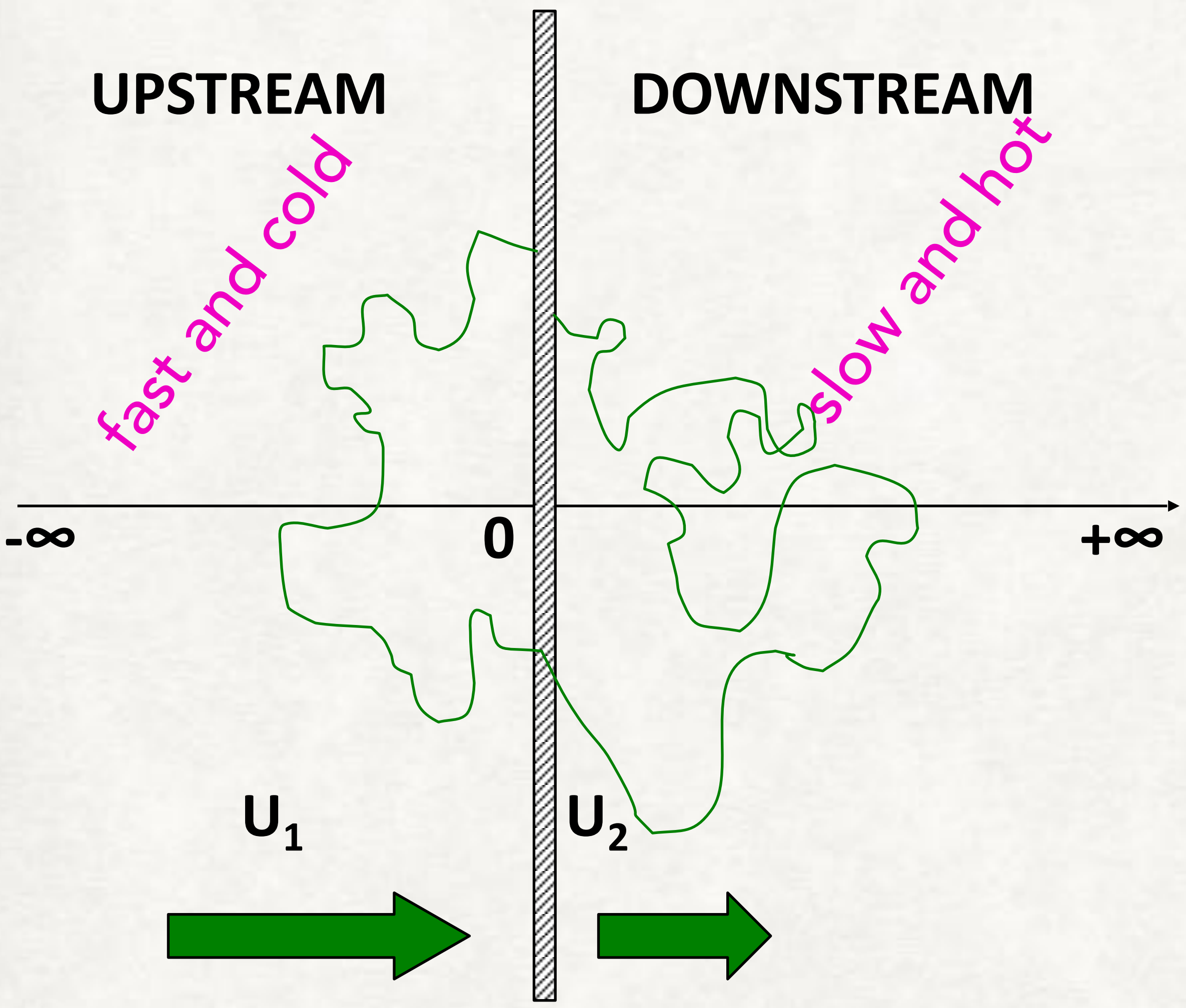
ONE CAN SEE THAT SHOCKS BEHAVE AS VERY EFFICIENT HEATING MACHINES IN THAT A LARGE FRACTION OF THE INCOMING RAM PRESSURE IS CONVERTED TO INTERNAL ENERGY OF THE GAS BEHIND THE SHOCK FRONT...

THE PLASMA IS SLOWED DOWN AND HEATED UP

DIFFUSIVE SHOCK ACCELERATION

WHY?

BOUNCING BETWEEN APPROACHING MAGNETIC MIRRORS



Let us take a relativistic particle with energy E~p upstream of the shock. In the downstream frame:

$$E_d = \gamma E(1 + \beta\mu) \quad 0 \leq \mu \leq 1$$

where β = u_1-u_2>0. In the downstream frame the direction of motion of the particle is isotropized and reapproaches the shock with the same energy but pitch angle μ'

$$E_u = \gamma E_d - \beta E_d \gamma \mu' = \gamma^2 E(1 + \beta\mu)(1 - \beta\mu')$$

$$-1 \leq \mu' \leq 0$$

In the non-relativistic case the particle distribution is, at zeroth order, isotropic
Therefore:

TOTAL FLUX

$$J = \int_0^1 d\Omega \frac{N}{4\pi} v\mu = \frac{Nv}{4} \quad \Rightarrow \quad P(\mu)d\mu = \frac{ANv\mu}{\frac{Nv}{4}} d\mu = 2\mu d\mu$$

The mean value of the energy change is therefore:

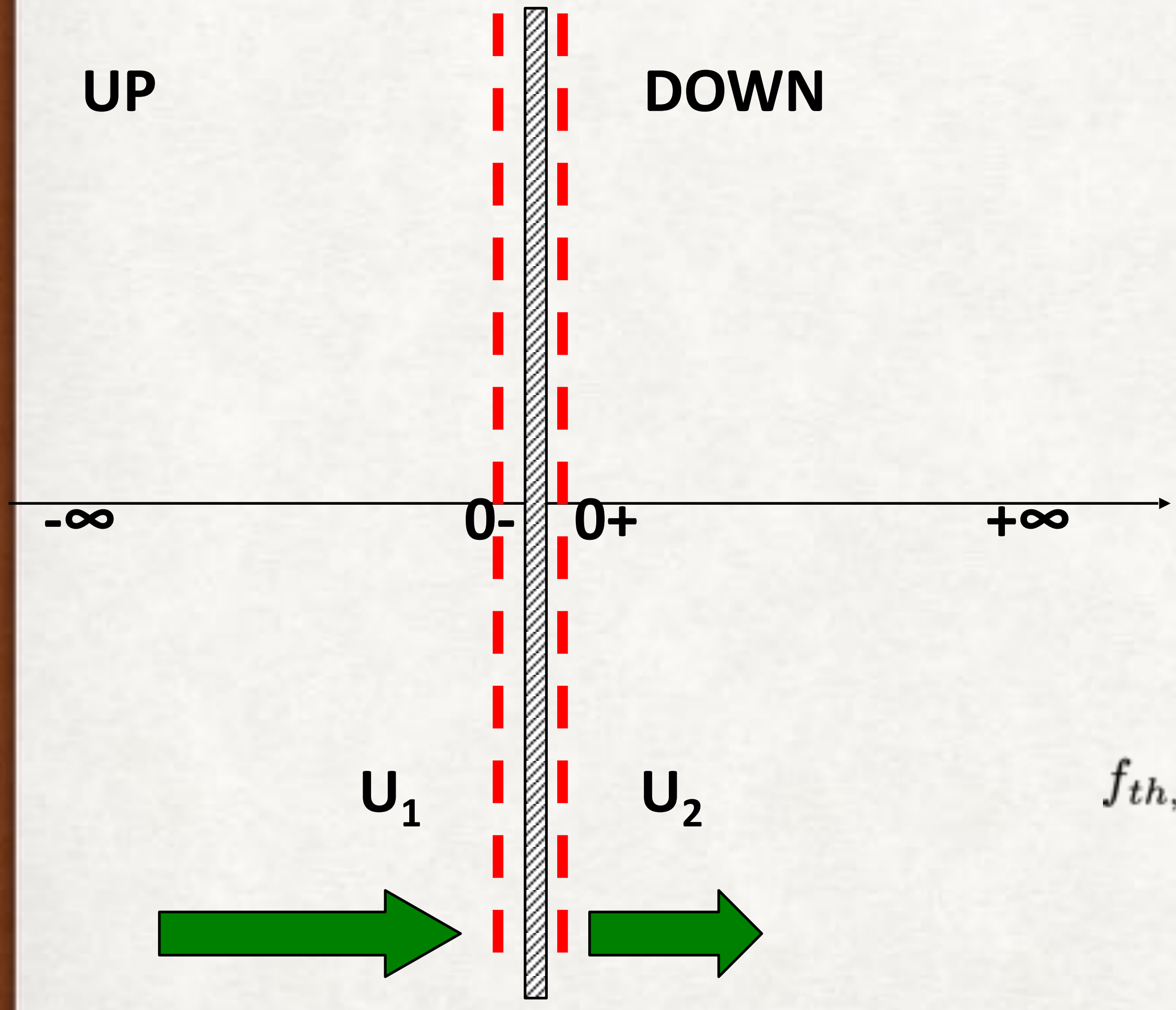
$$\langle \frac{E_u - E}{E} \rangle = -\int_0^1 d\mu 2\mu \int_{-1}^0 d\mu' 2\mu' \left[\gamma^2(1+\beta\mu)(1-\beta\mu') - 1\right] \approx \frac{4}{3}\beta = \frac{4}{3}(u_1 - u_2)$$

- THERE ARE NO CONFIGURATIONS THAT LEAD TO LOSSES
- THE MEAN ENERGY GAIN IS NOW FIRST ORDER IN β
- $\beta=(u_1-u_2)/c<1$ BUT NOW >>> v_A/c
- THE ENERGY GAIN IS BASICALLY INDEPENDENT OF ANY DETAIL ON HOW PARTICLES SCATTER BACK AND FORTH!!!

THE TRANSPORT EQUATION APPROACH

$$\frac{\partial f}{\partial t}=\frac{\partial}{\partial x}\left[D\frac{\partial f}{\partial x}\right]-u\frac{\partial f}{\partial x}+\frac{1}{3}\frac{du}{dx}p\frac{\partial f}{\partial p}+Q(x,p,t)$$

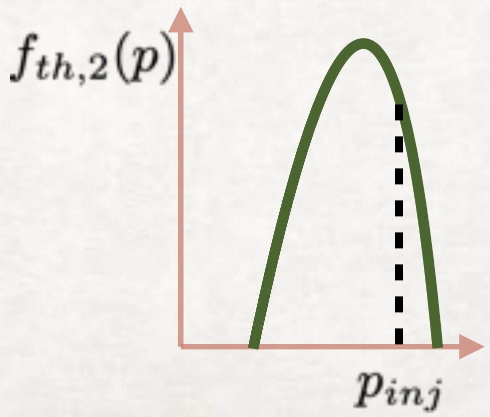
DIFFUSION ADVECTION COMPRESSION INJECTION



THE PHYSICS OF INJECTION IS CLEARLY VERY COMPLEX AND WE WILL DISCUSS SOME OF IT LATER. BUT WE CAN THINK OF INJECTION HERE IN A PHYSICAL WAY

THE SHOCK IS COLLISIONLESS—>ITS THICKNESS MUST BE OF ORDER THE LARMOR RADIUS OF THERMAL PARTICLES BEHIND THE SHOCK

FOR A PARTICLE TO BE INJECTED IT HAS TO CROSS THE THICKNESS OF THE SHOCK —> ONLY PARTICLES ON THE TAIL OF THE THERMAL DISTRIBUTION CAN BE INJECTED



$$Q(x,p)=\frac{\eta n_1 u_1}{4\pi p_{inj}^2}\delta(p-p_{inj})\delta(x)$$

UPSTREAM SOLUTION

LET US ASSUME STATIONARITY (LATER WE SHALL DISCUSS IMPLICATIONS)

IN THE UPSTREAM THE EQUATION READS

$$\frac{\partial}{\partial x}\left[D\frac{\partial f}{\partial x}\right] - u\frac{\partial f}{\partial x} = 0 \quad \rightarrow \quad \frac{\partial}{\partial x}\left[D\frac{\partial f}{\partial x} - uf\right] = 0$$

FLUX IS CONSERVED!

THE SOLUTION THAT HAS VANISHING f AND VANISHING DERIVATIVE AT UPSTREAM INFINITY IS

$$f(x,p) = f_0 \exp\left[\frac{u_1 x}{D}\right] \quad \rightarrow \quad D\frac{\partial f}{\partial x}|_{x\rightarrow 0^-} = u_1 f_0(p)$$

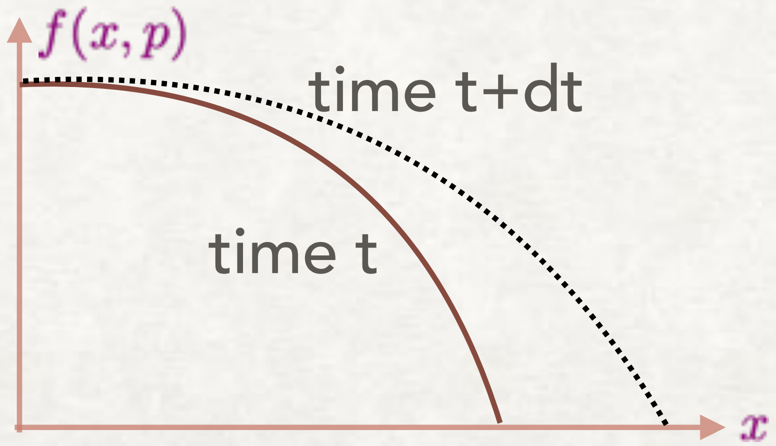
DOWNSTREAM SOLUTION

IN THE DOWNSTREAM THE EQUATION READS

$$\frac{\partial}{\partial x}\left[D\frac{\partial f}{\partial x}\right] - u\frac{\partial f}{\partial x} = 0 \quad \rightarrow \quad \frac{\partial}{\partial x}\left[D\frac{\partial f}{\partial x} - uf\right] = 0$$

FLUX IS CONSERVED!

NOTICE THAT WE HAVE REQUIRED STATIONARITY AND OBVIOUSLY THE ONLY SOLUTION THAT IS CONSISTENT WITH THAT ASSUMPTION IS



$$f(x,p) = \text{constant} = \mathrm{f_0(p)}$$

$$D\frac{\partial f}{\partial x}|_{x\rightarrow 0^+} = 0$$

AROUND THE SHOCK

INTEGRATING THE TRANSPORT EQUATION IN A NARROW NEIGHBORHOOD OF THE SHOCK WE GET

$$D\frac{\partial f}{\partial x}|_{x\to 0^+} - D\frac{\partial f}{\partial x}|_{x\to 0^-} + \frac{1}{3}(u_2 - u_1)p\frac{df_0}{dp} + \frac{\eta n_1 u_1}{4\pi p_{inj}^2}\delta(p - p_{inj})$$

WHERE WE USED $du/dx=(u_2-u_1)\boldsymbol{\delta}(x)$

REPLACING THE EXPRESSIONS FOR THE DERIVATIVES DERIVED BEFORE:

$$-u_1 f_0(p) + \frac{1}{3}(u_2 - u_1)p\frac{df_0}{dp} = 0 \qquad p > p_{inj}$$

WHICH HAS THE SOLUTION:

$$f_0(p) = Kp^{-\alpha} \qquad \alpha = \frac{3u_1}{u_1 - u_2} = \frac{3r}{r-1}$$

THE SPECTRUM IS A POWER LAW IN MOMENTUM

INTEGRATION OF THIS SIMPLE EQUATION GIVES:

$$f_0(p) = \frac{3u_1}{u_1 - u_2} \frac{N_{inj}}{4\pi p_{inj}^2} \left(\frac{p}{p_{inj}} \right)^{\frac{-3u_1}{u_1 - u_2}}$$

DEFINE THE COMPRESSION FACTOR $r = u_1/u_2 \rightarrow 4$ (strong shock)

THE SLOPE OF THE SPECTRUM IS

$$\frac{3u_1}{u_1 - u_2} = \frac{3}{1 - 1/r} \rightarrow 4 \quad \text{if r} \rightarrow 4$$

THE SPECTRUM OF THE PARTICLES ACCELERATED AT A STRONG SHOCK IS UNIVERSAL AND IS ALWAYS PROPORTIONAL TO p^{-4}

IT IS **NOT** A POWER LAW IN ENERGY!!! UNLESS YOU ARE EITHER…

ULTRA-RELATIVISTIC $N(E)dE = 4\pi p^2 f(p) dp \rightarrow N(E) \propto E^{-2}$

NON-RELATIVISTIC $N(E)dE = 4\pi p^2 f(p) dp \rightarrow N(E) \propto E^{-3/2}$

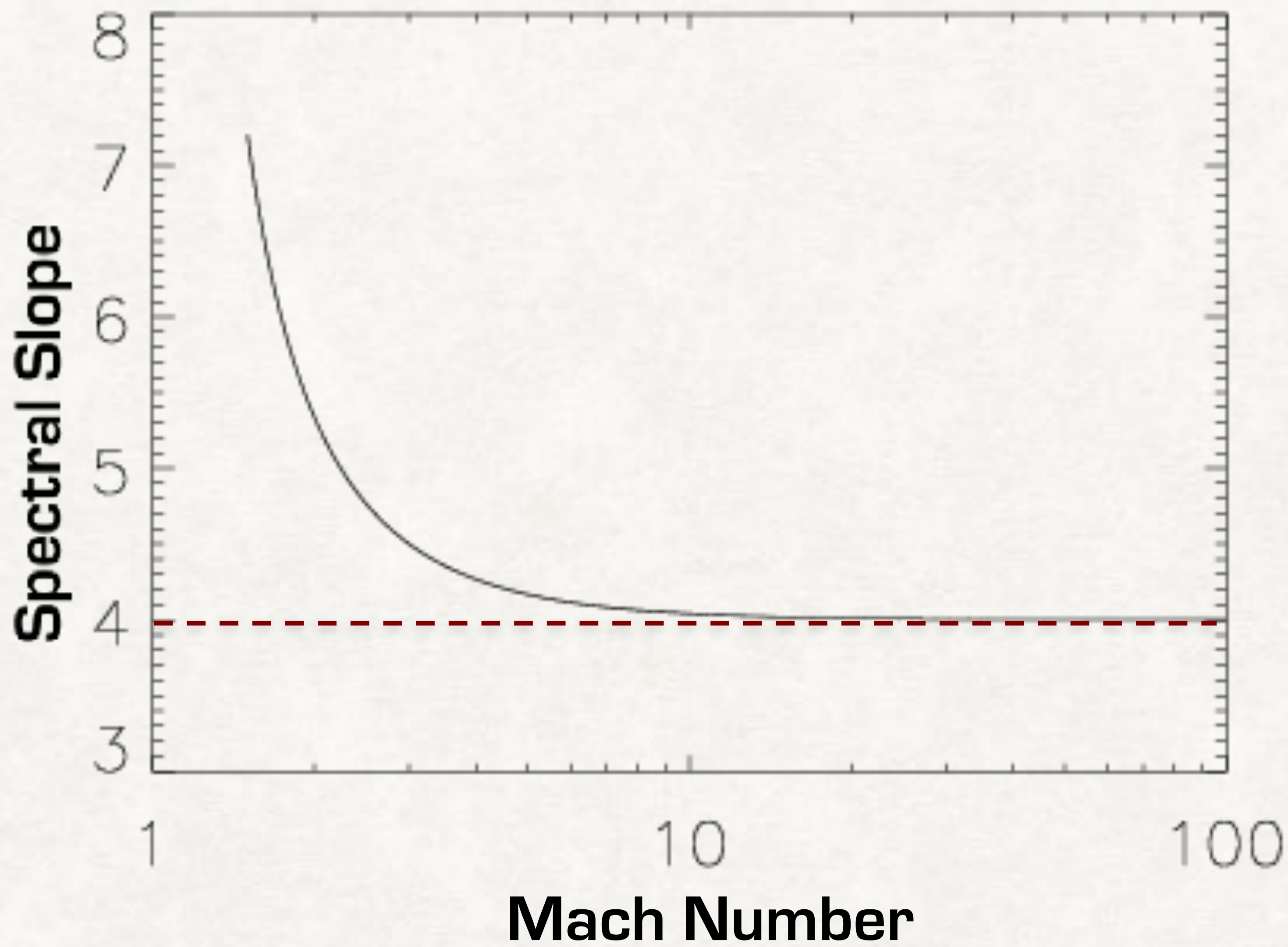
THE SPECTRUM IS A POWER LAW IN MOMENTUM

- THE SPECTRUM OF ACCELERATED PARTICLES IS A POWER LAW IN MOMENTUM
- THE POWER LAW EXTENDS TO INFINITE MOMENTA!!!
- THE SLOPE DEPENDS UNIQUELY ON THE COMPRESSION FACTOR AND IS INDEPENDENT OF THE DIFFUSION PROPERTIES
- NO DEPENDENCE UPON DIFFUSION (MICRO-PHYSICS)

AND HERE IS WHEN YOU START GETTING CONCERNED…

- ☑ *ASSUMPTION OF STATIONARITY —> THERE IS NO MAXIMUM ENERGY! (if there were one, at a time t+dt it would be higher, violating stationarity)*
- ☑ *…BUT THE TOTAL ENERGY CARRIED BY PARTICLES IS* $E_{tot} \gtrsim \int_m^{E_{max}} dE E^{-2} E \to \ln(\infty)$ *(contradicting the assumption of test particles)*
- ☑ *EVEN IF THE TOTAL ENERGY WERE NOT INFINITE, THERE IS NO CHECK THAT IT IS NOT LARGER THAT* ρu^2*, THE TOTAL WE CAN TAP FROM*

TEST PARTICLE SPECTRUM



WE WILL SEE LATER THAT THIS APPARENT UNIVERSALITY IS VIOLATED BY SEVERAL MICROPHYSICAL EFFECTS, WHICH ARE INDISPENSABLE FOR THE THEORY TO CONFRONT OBSERVATIONS

MAXIMUM ENERGY

In a real system, the maximum energy is set by either the age of the accelerator compared with the acceleration time or the size of the system compared with the diffusion length D(E)/u. The hardest condition is the one that dominates.

Using the diffusion coefficient in the ISM derived from the B/C ratio:

$$D(E) \approx 3 \times 10^{28} E_{GeV}^{1/3} cm^2/s$$

and the velocity of a SNR shock as u=5000 km/s one sees that:

$$t_{acc} \sim D(E)/u^2 \sim 4 \times 10^3 E_{GeV}^{1/3} \; years$$

Too long for any useful acceleration → **NEED FOR SMALLER D(E)!!!**

$$t_{acc}(p) = \langle t \rangle = \frac{3}{u_1 - u_2} \int_{p_0}^{p} \frac{dp'}{p'} \left[\frac{D_1(p')}{u_1} + \frac{D_2(p')}{u_2} \right]$$

Drury 1983

ENERGY LOSSES AND ELECTRONS

For electrons, energy losses make acceleration even harder.

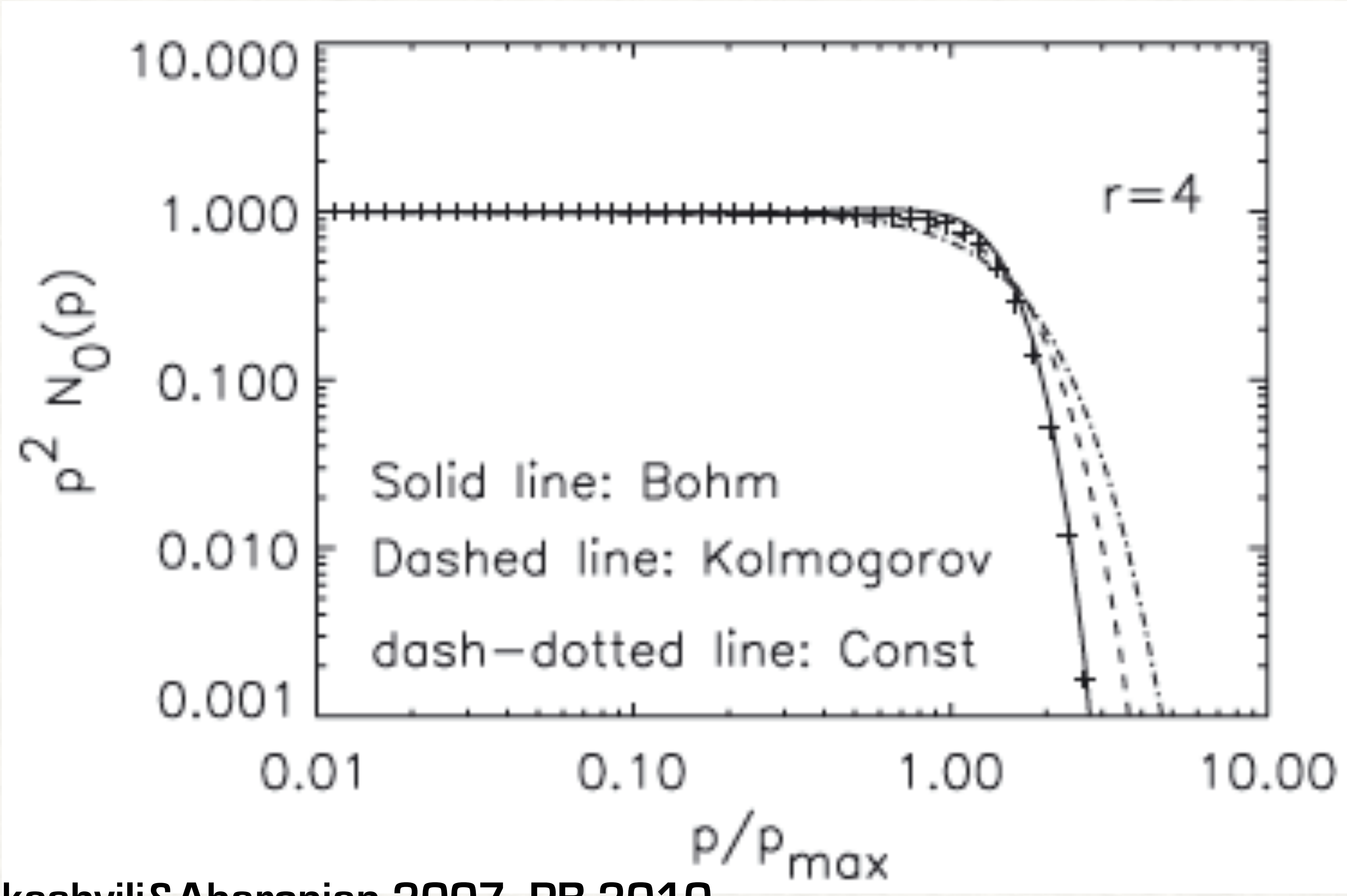
The maximum energy of electrons is determined by the condition:

$$t_{acc} \leq Min\left[Age, \tau_{loss}\right]$$

Where the losses are mainly due to synchrotron and inverse Compton Scattering.

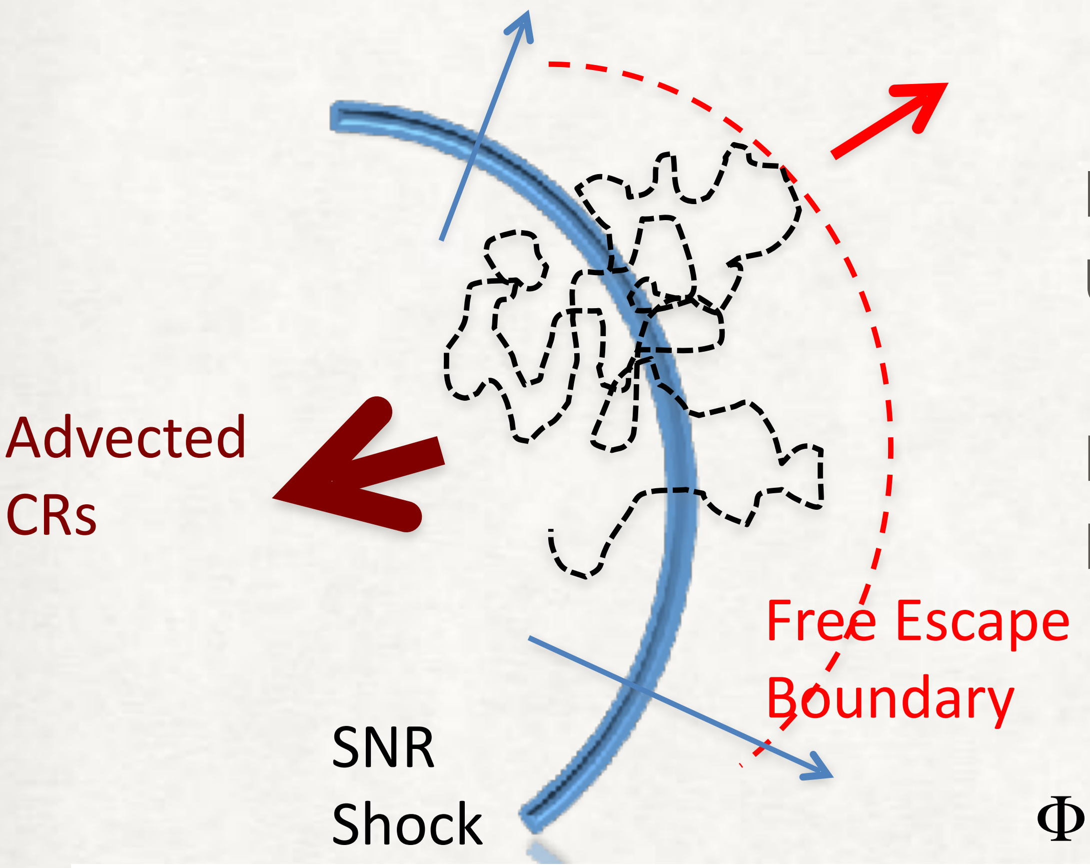
ELECTRONS IN ONE SLIDE

THE SHAPE OF THE LOSS-RELATED CUTOFF DEPENDS ON D(E)



Zirakashvili&Aharonian 2007, PB 2010

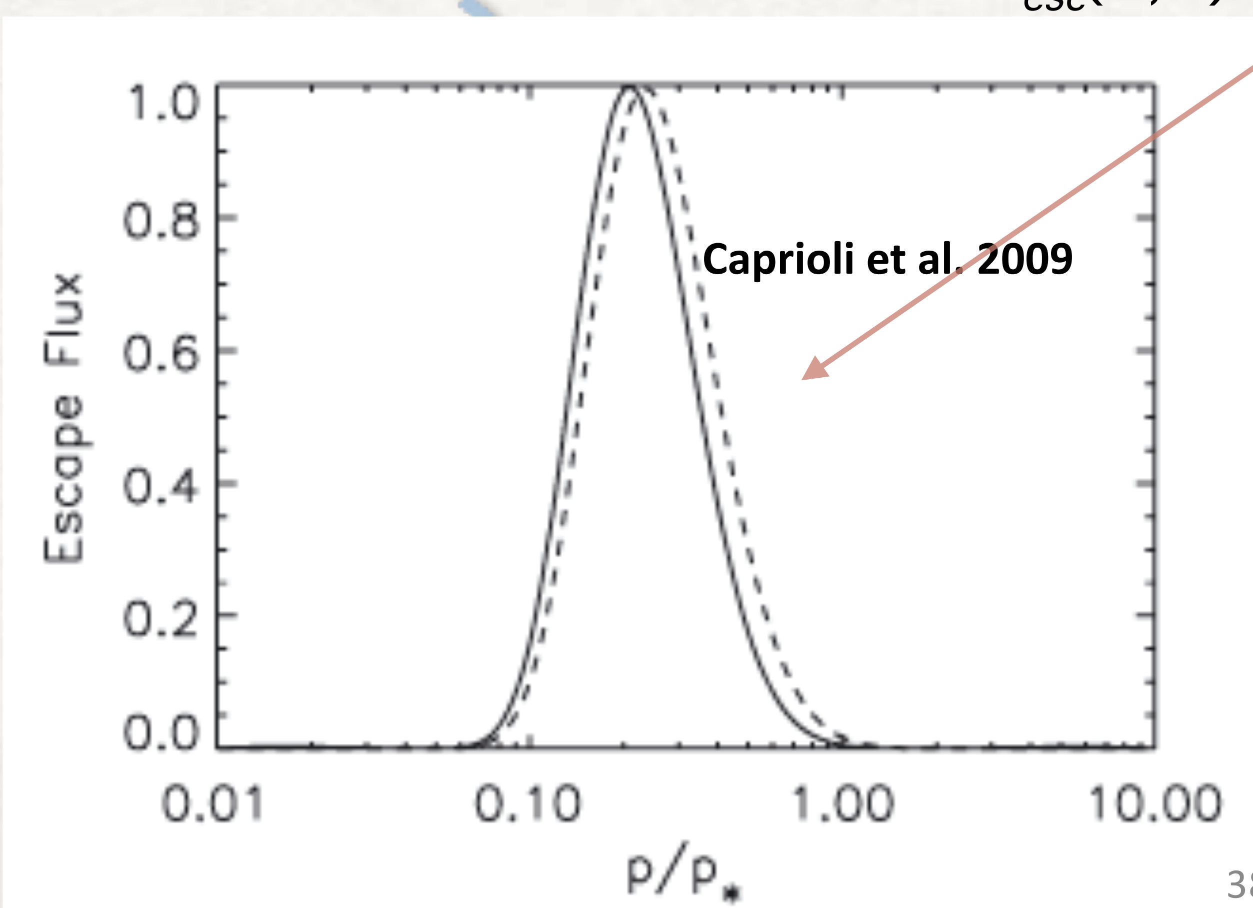
THE PROBLEM OF ESCAPE FROM THE ACCELERATOR

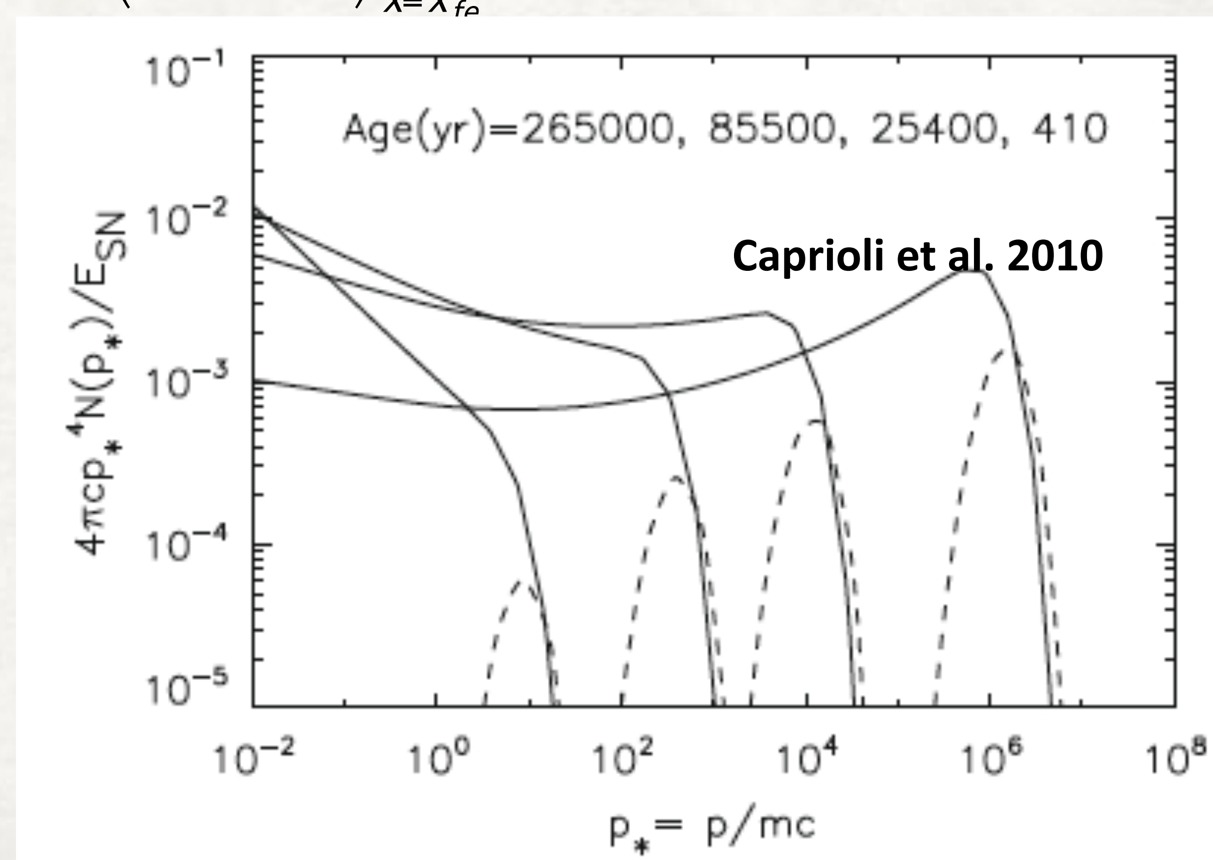


IN STANDARD DSA THERE IS NO ESCAPE FROM UPSTREAM

ESCAPE CAN BE FORCED BY IMPOSING A FREE ESCAPE BOUNDARY CONDITION

$$\Phi_{esc}(E,x) = D(E)\left(\frac{\partial f(E,x)}{\partial x}\right)_{x=x_{fe}}$$





ESCAPE: THE MISSING CONNECTION BETWEEN ACCELERATED PARTICLES AND COSMIC RAYS

- The spectrum of CR from SNR is a convolution of particles escaping and particles trapped
- As we will see the problem of MAX energy is intrinsically related to that of escape
- Defining the escape of CR from a source is crucial also for the description of the first stages of CR transport, close to the source but yet in the ISM (e.g. teV halos)
- Despite its importance, escape is the weak link in this story

NON LINEAR THEORY OF DSA

WHY DO WE NEED A NON LINEAR THEORY?

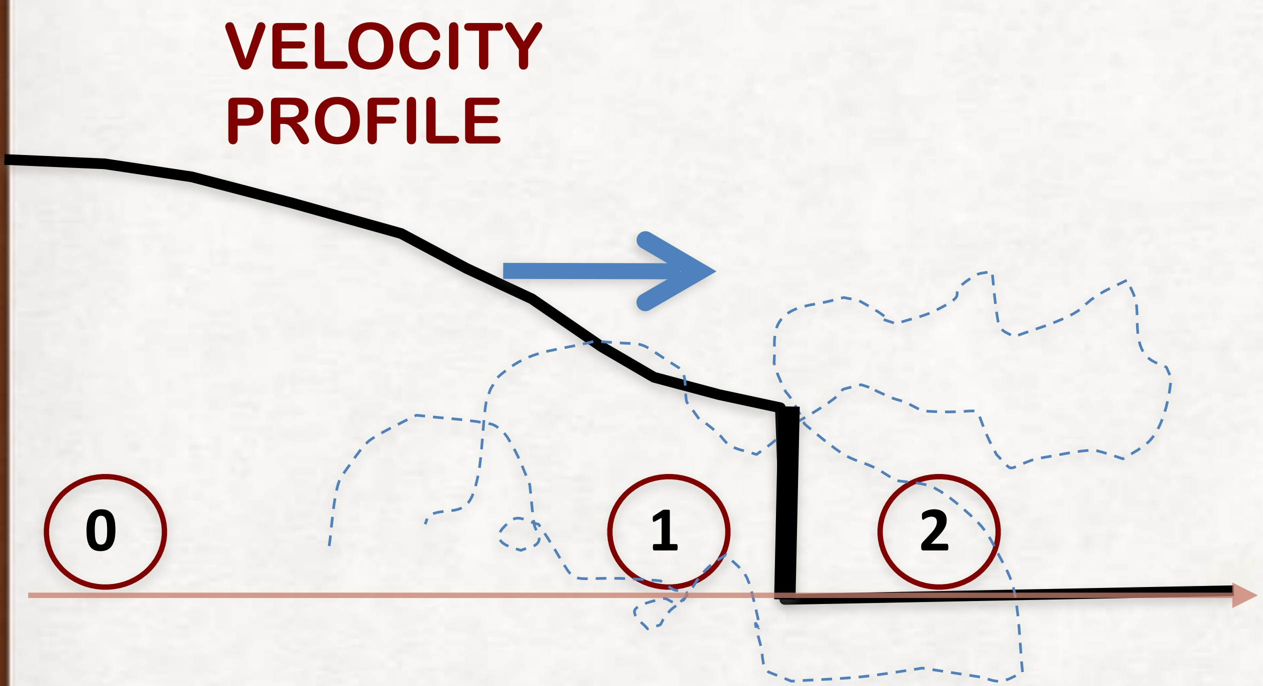
TEST PARTICLE THEORY PREDICTS ENERGY DIVERGENT SPECTRA

THE TYPICAL EFFICIENCY EXPECTED OF A SNR (~10%) IS SUCH THAT TEST PARTICLE THEORY IS A BAD APPROXIMATION

THE MAX MOMENTUM CAN ONLY BE INTRODUCED BY HAND IN TEST PARTICLE THEORY

SIMPLE ESTIMATES SHOW THAT EMAX IS VERY LOW UNLESS CR TAKE PART IN THE ACCELERATION PROCESS, BY AFFECTING THEIR OWN SCATTERING

DYNAMICAL REACTION OF ACCELERATED PARTICLES



Particle transport is described by using the usual transport equation including diffusion and advection

But now dynamics is important too:

$$\rho_0 u_0 = \rho_1 u_1$$

Conservation of Mass

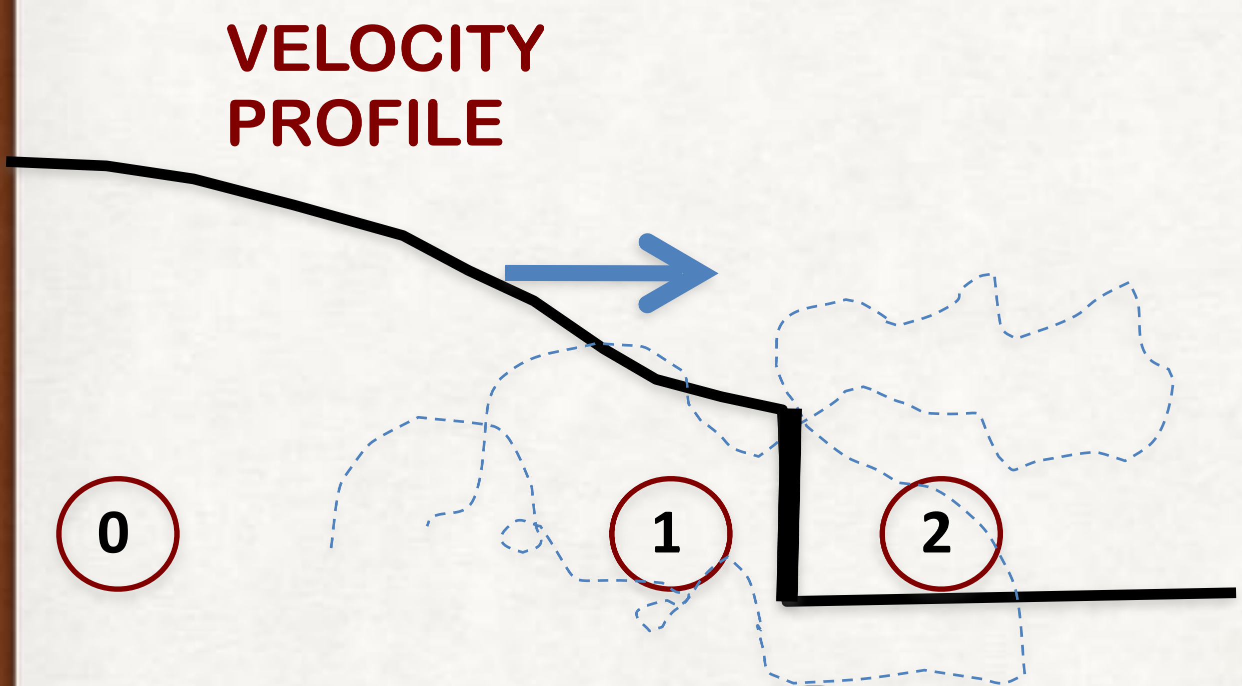
$$\rho_0 u_0^2 + P_{g,0} = \rho_1 u_1^2 + P_{g,1} + P_{c,1}$$

Conservation of Momentum

Conservation of Energy

$$\frac{1}{2}\rho_0 u_0^3 + \frac{P_{g,0} u_0 \gamma_g}{\gamma_g - 1} - F_{esc} = \frac{1}{2}\rho_1 u_1^3 + \frac{P_{g,1} u_1 \gamma_g}{\gamma_g - 1} + \frac{P_{c,1} u_1 \gamma_c}{\gamma_c - 1}$$

FORMATION OF A PRECURSOR - SIMPLIFIED



$$\frac{\partial}{\partial x}[\rho u] = 0 \rightarrow \rho(x)u(x) = \rho_0 u_0$$

$$\frac{\partial}{\partial x}\left[P_g + \rho u^2 + P_{CR}\right] = 0$$

$$P_g(x) + \rho u^2 + P_{CR} = P_{g,0} + \rho_0 u_0^2$$

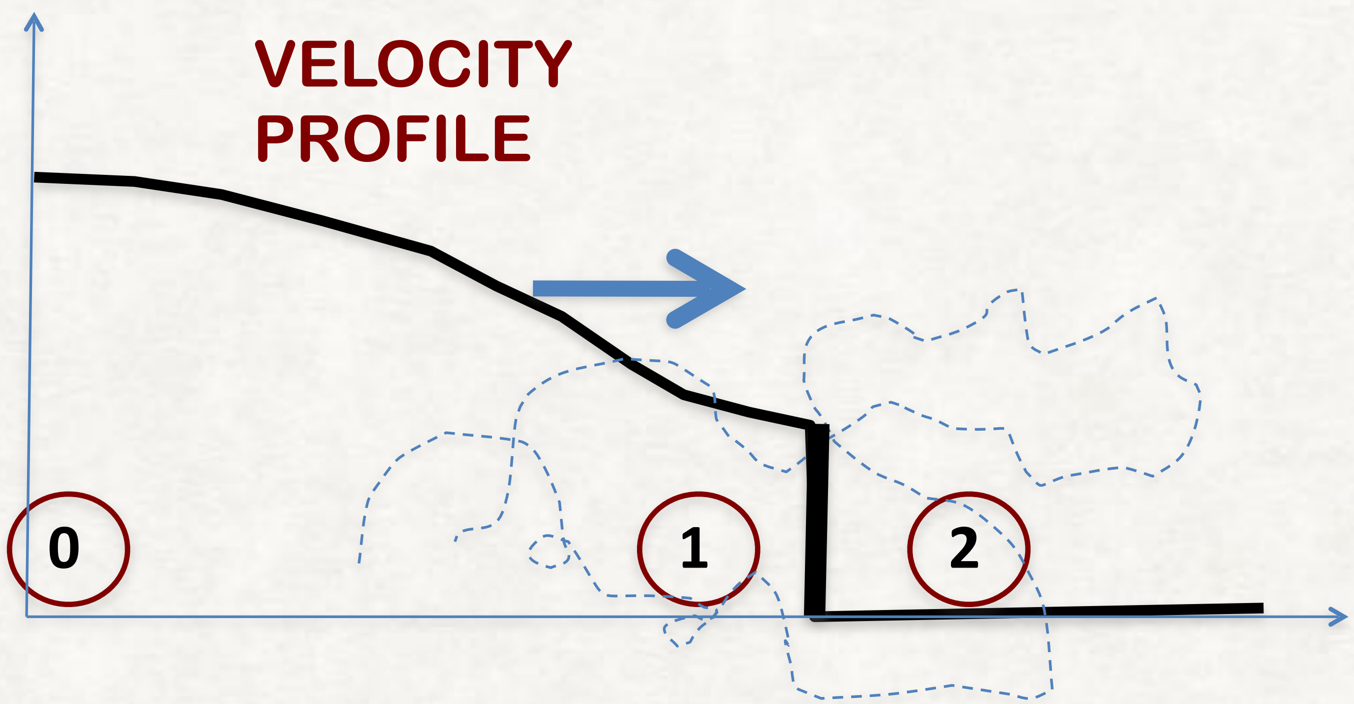
AND DIVIDING BY THE RAM PRESSURE AT UPSTREAM INFINITY:

$$\frac{P_g}{\rho_0 u_0^2} + \frac{u}{u_0} + \frac{P_{CR}}{\rho_0 u_0^2} = \frac{P_{g,0}}{\rho_0 u_0^2} + 1 \Rightarrow \frac{u}{u_0} \approx 1 - \xi_{CR}(x)$$

WHERE WE NEGLECTED TERMS OF ORDER $1/M^2$

$$\xi_{CR}(x) = \frac{P_{CR}(x)}{\rho_0 u_0^2}$$

BASIC PREDICTIONS OF NON LINEAR THEORY



COMPRESSION FACTOR BECOMES FUNCTION OF ENERGY

SPECTRA ARE NOT PERFECT POWER LAWS (CONCAVE)

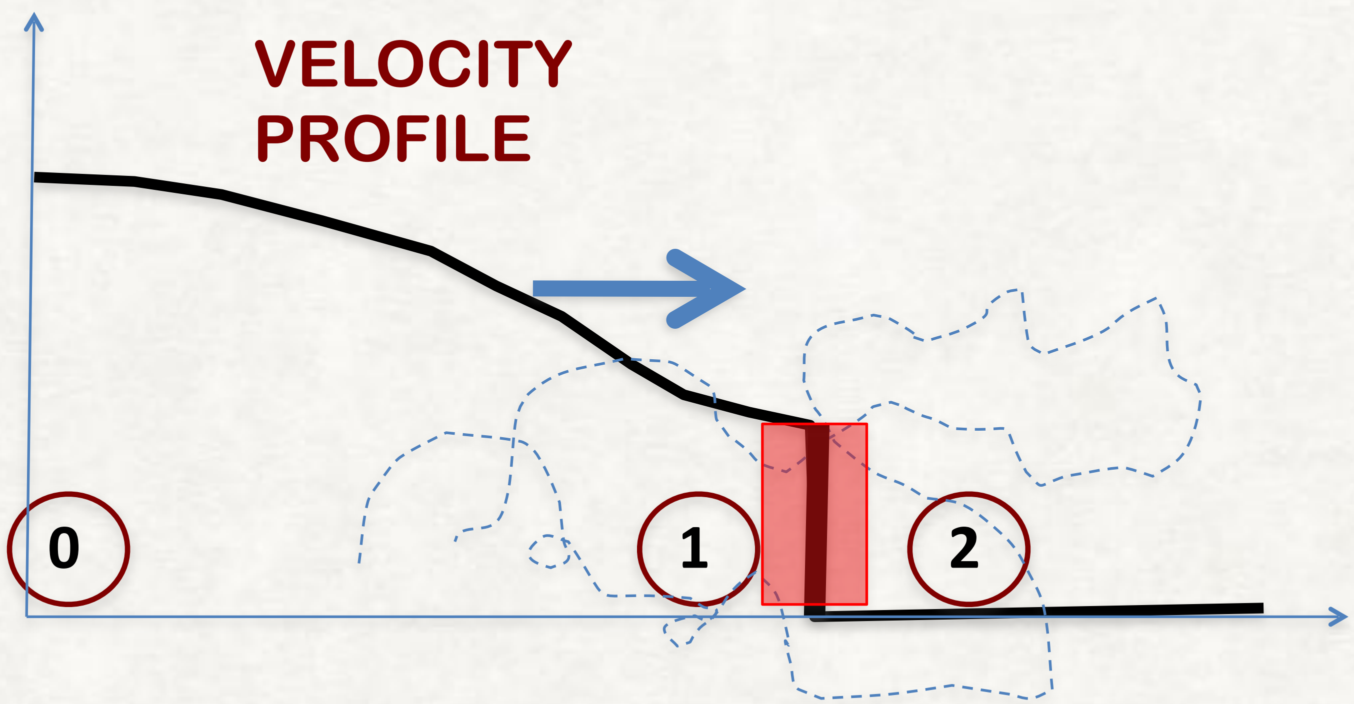
GAS BEHIND THE SHOCK IS COOLER FOR EFFICIENT SHOCK ACCELERATION

SYSTEM SELF REGULATED

EFFICIENT GROWTH OF B-FIELD IF ACCELERATION EFFICIENT

PB+2010

BASIC PREDICTIONS OF NON LINEAR THEORY



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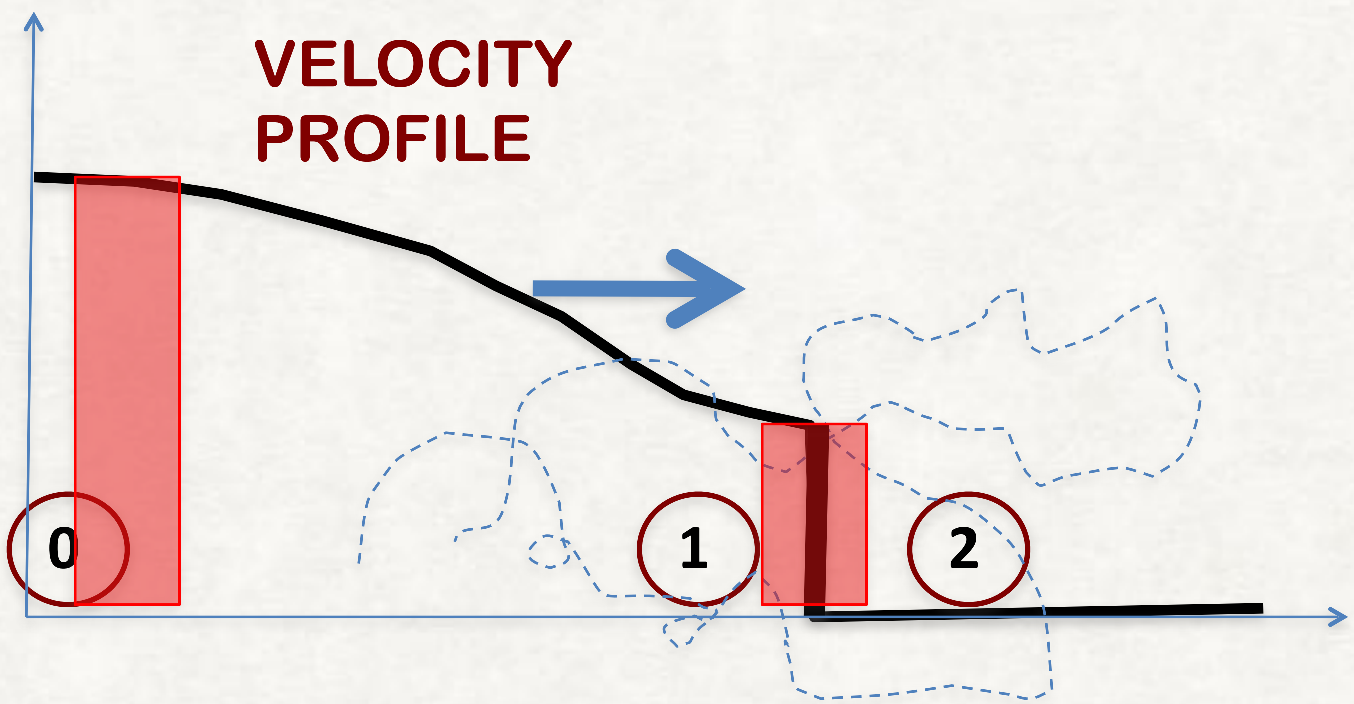

PB+2010

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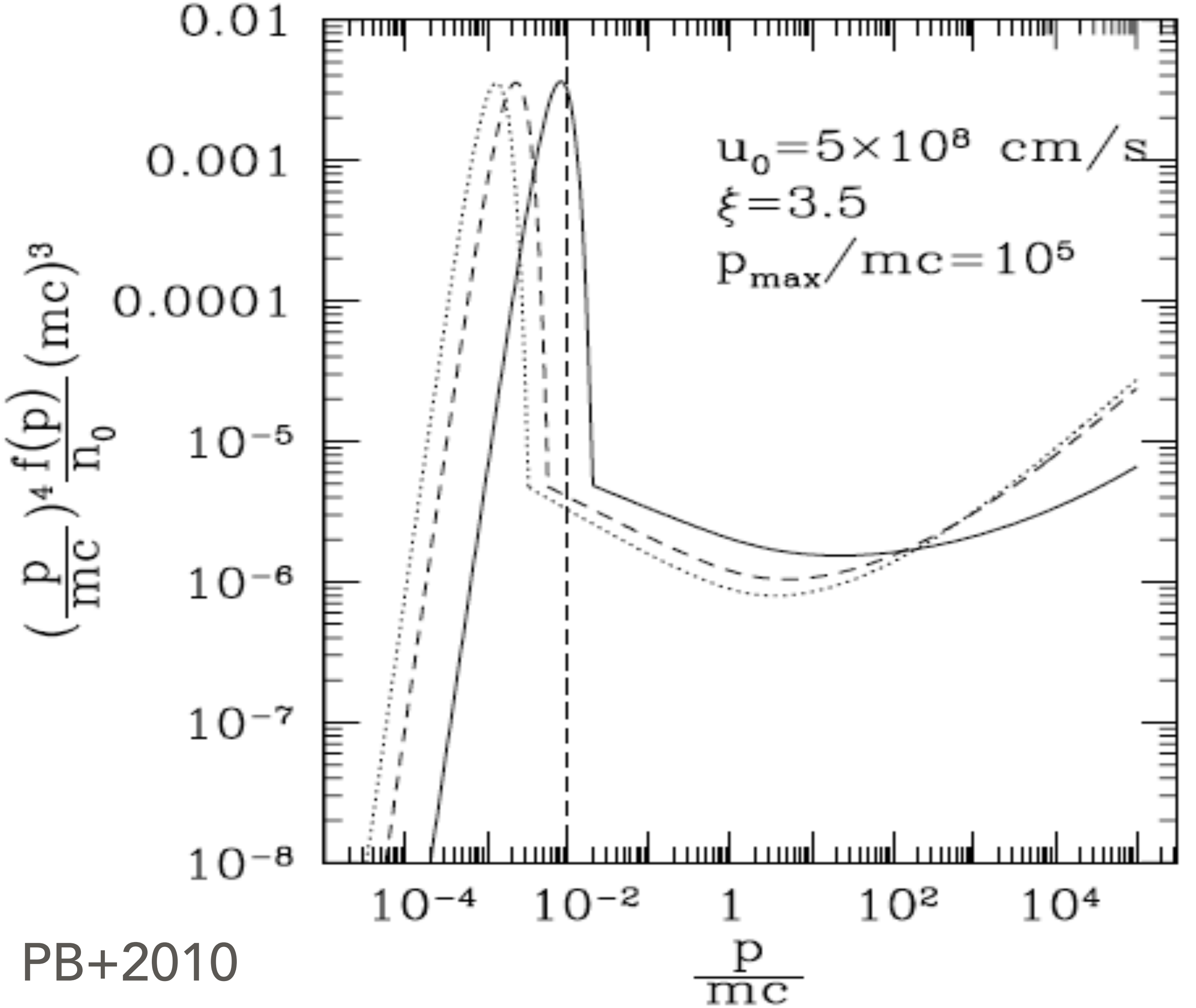

PB+2010

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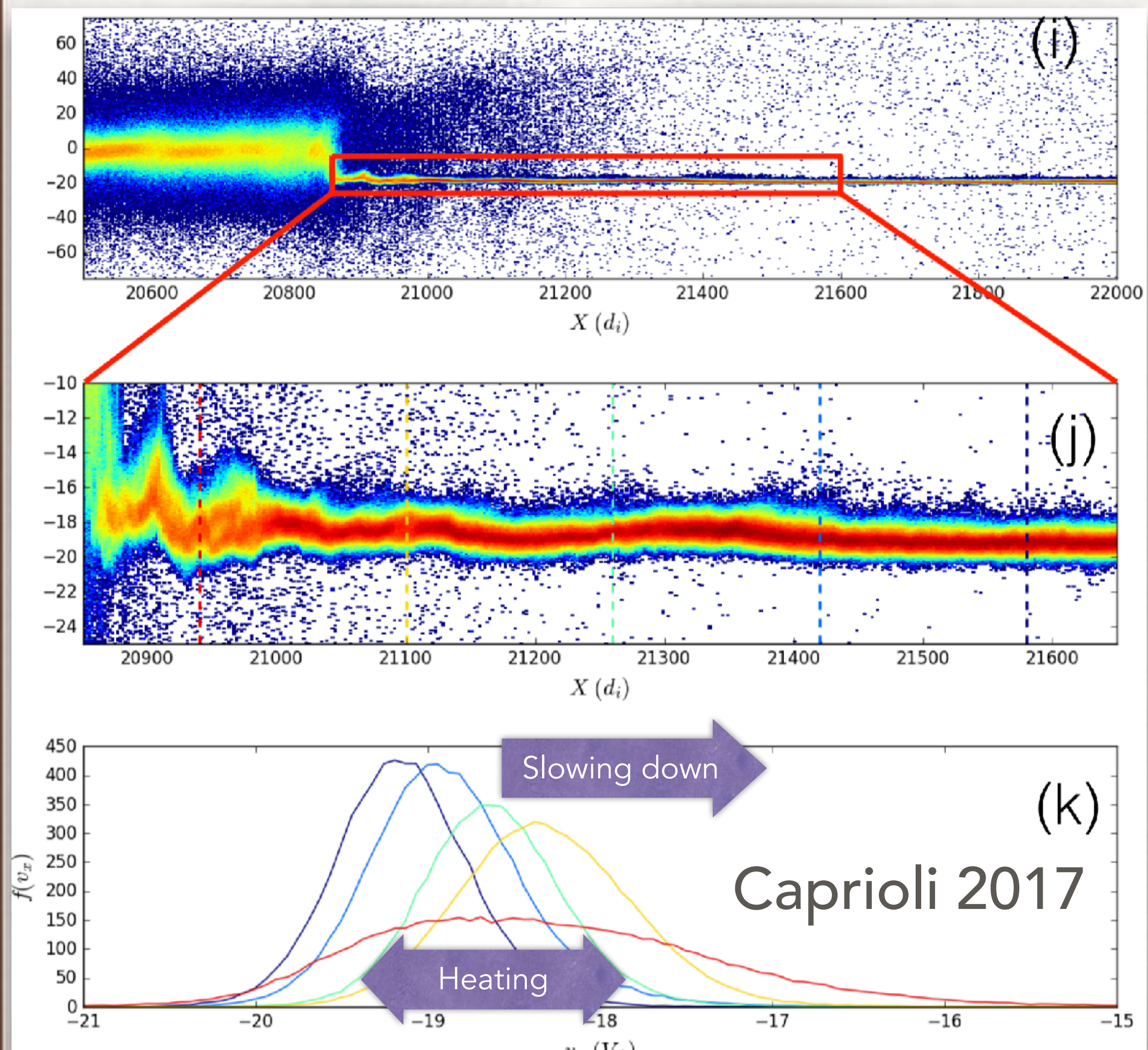
EFFICIENT GROWTH OF B-FIELD IF ACCELERATION EFFICIENT

EFFECT OF TURBULENT DAMPING

AT LEAST A FRACTION OF THE ENERGY OF CR UPSTREAM IS TRANSFERRED TO THE THERMAL ENERGY OF THE BACKGROUND PLASMA

THIS PROCESS (TURBULENT HEATING) LEADS TO A REDUCTION OF THE MACH NUMBER IN THE PRECURSOR —> SMOOTHER PRECURSOR —> **SPECTRA AGAIN CLOSE TO E^{-2}**

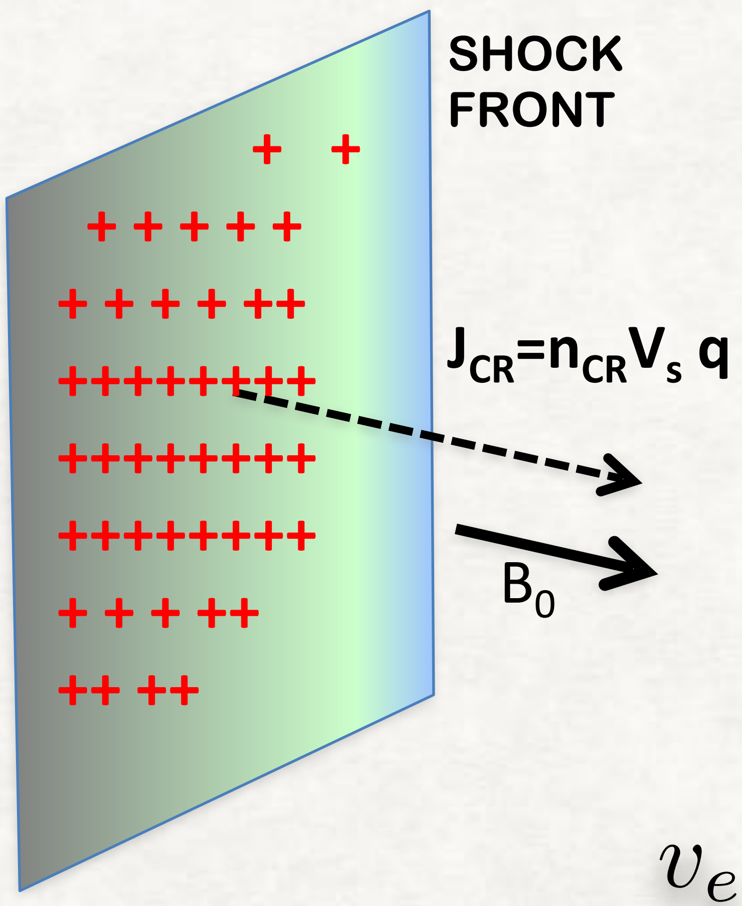


HYBRID SIMS SHOW THIS EFFECT IN THE FORM OF A SLOWING DOWN OF THE PLASMA AND HEATING

YET NO APPRECIABLE DEVIATION FROM E^{-2}

HOWEVER THESE SIMULATIONS ARE NON RELATIVISTIC

BASICS OF CR STREAMING INSTABILITY



THE UPSTREAM PLASMA REACTS TO THE UPCOMING CR CURRENT BY CREATING A RETURN CURRENT TO COMPENSATE THE POSITIVE CR CHARGE

THE SMALL INDUCED PERTURBATIONS MAY BE UNSTABLE (ACHTERBERG 1983, ZWEIBEL 1978, BELL 1978, BELL 2004, AMATO & PB 2009)

$$n_p + n_{CR} = n_e$$

$$n_{CR} v_{shock} = n_e v_e$$

$$v_e = \frac{n_{CR}}{n_{CR}+n_p} v_{shock} \approx v_{shock} \frac{n_{CR}}{n_p}$$

CR MOVE WITH THE SHOCK SPEED (>> V_A). THIS UNSTABLE SITUATION LEADS THE PLASMA TO REACT IN ORDER TO SLOW DOWN CR TO <V_A BY SCATTERING PARTICLES IN THE PERP DIRECTION (B-FIELD GROWTH)

STREAMING INSTABILITY - THE SIMPLE VIEW

CR streaming with the shock leads to growth of waves. The general idea is simple to explain:

$$n_{CR} m v_D \to n_{CR} m V_A \Rightarrow \frac{dP_{CR}}{dt} = \frac{n_{CR} m (v_D - V_A)}{\tau} \qquad \frac{dP_w}{dt} = \gamma_W \frac{\delta B^2}{8\pi} \frac{1}{V_A}$$

and assuming equilibrium:

$$\gamma_W = \sqrt{2}\, \frac{n_{CR}}{n_{gas}} \frac{v_D - V_A}{V_A} \Omega_{cyc}$$

And for parameters typical of SNR shocks:

$$\gamma_W \simeq \sqrt{2}\, \xi_{CR} \left(\frac{V_s}{c}\right)^2 \frac{V_s}{V_A} \Omega_{cyc} \sim \mathcal{O}(10^{-4}\ seconds^{-1})$$

BRANCHES OF THE CR INDUCED STREAMING INSTABILITY

A CAREFUL ANALYSIS OF THE INSTABILITY REVEALS THAT THERE ARE TWO BRANCHES

RESONANT	NON RESONANT
MAX GROWTH AT K=1/LARMOR	MAX GROWTH AT K>>1/LARMOR

THE MAX GROWTH CAN ALWAYS BE WRITTEN IN THE FORM

$$\gamma_{max} = k_{max} v_A$$

WHERE THE WAVENUMBER IS DETERMINED BY THE TENSION CONDITION:

$$k_{max} B_0 \approx \frac{4\pi}{c} J_{CR} \rightarrow k_{max} \approx \frac{4\pi}{c B_0} J_{CR}$$

THE SEPARATION BETWEEN THE TWO REGIMES IS AT $k_{MAX}\ r_L=1$

IF WE WRITE THE CR CURRENT AS $$J_{CR} = n_{CR}(> E)ev_D$$

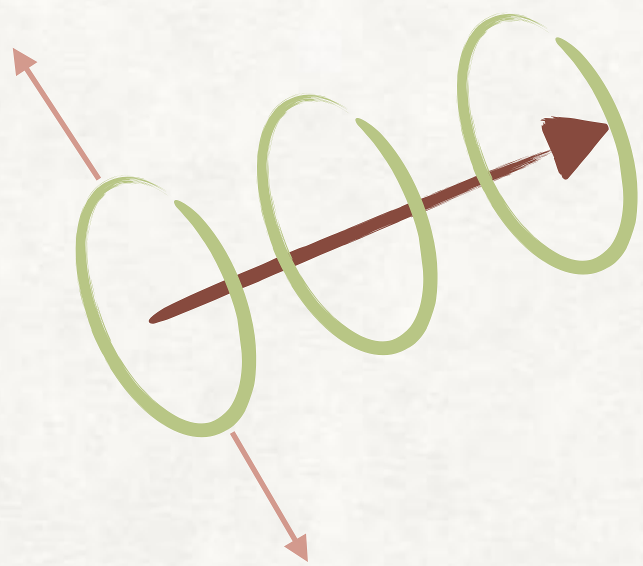
WHERE E IS THE ENERGY OF THE PARTICLES DOMINATING THE CR CURRENT, WE CAN WRITE THE CONDITION ABOVE AS

$$\frac{U_{CR}}{U_B} = \frac{c}{v_D} \qquad U_{CR} = n_{CR}(> E)E \qquad U_B = \frac{B^2}{4\pi}$$

IN CASE OF SHOCKS **V_D=SHOCK VELOCITY** AND THE CONDITION SAYS THAT THE NON-RESONANT MODES DOMINATED WHEN THE SHOCK IS VERY FAST AND ACCELERATION IS EFFICIENT —- *FOR TYPICAL CASES THIS IS ALWAYS THE CASE*

BUT RECALL! THE WAVES THAT GROW HAVE K MUCH LARGER THAN THE LARMOR RADIUS OF THE PARTICLES IN THE CURRENT —> NO SCATTERING BECAUSE EFFICIENT SCATTERING REQUIRES RESONANCE!!!

THE EASY WAY TO SATURATION OF GROWTH



The current exerts a force of the background plasma

$$\rho \frac{dv}{dt} \sim \frac{1}{c} J_{CR} \delta B$$

which translates into a plasma displacement:

$$\Delta x \sim \frac{J_{CR}}{c\rho} \frac{\delta B(0)}{\gamma_{max}^2} exp(\gamma_{max} t)$$

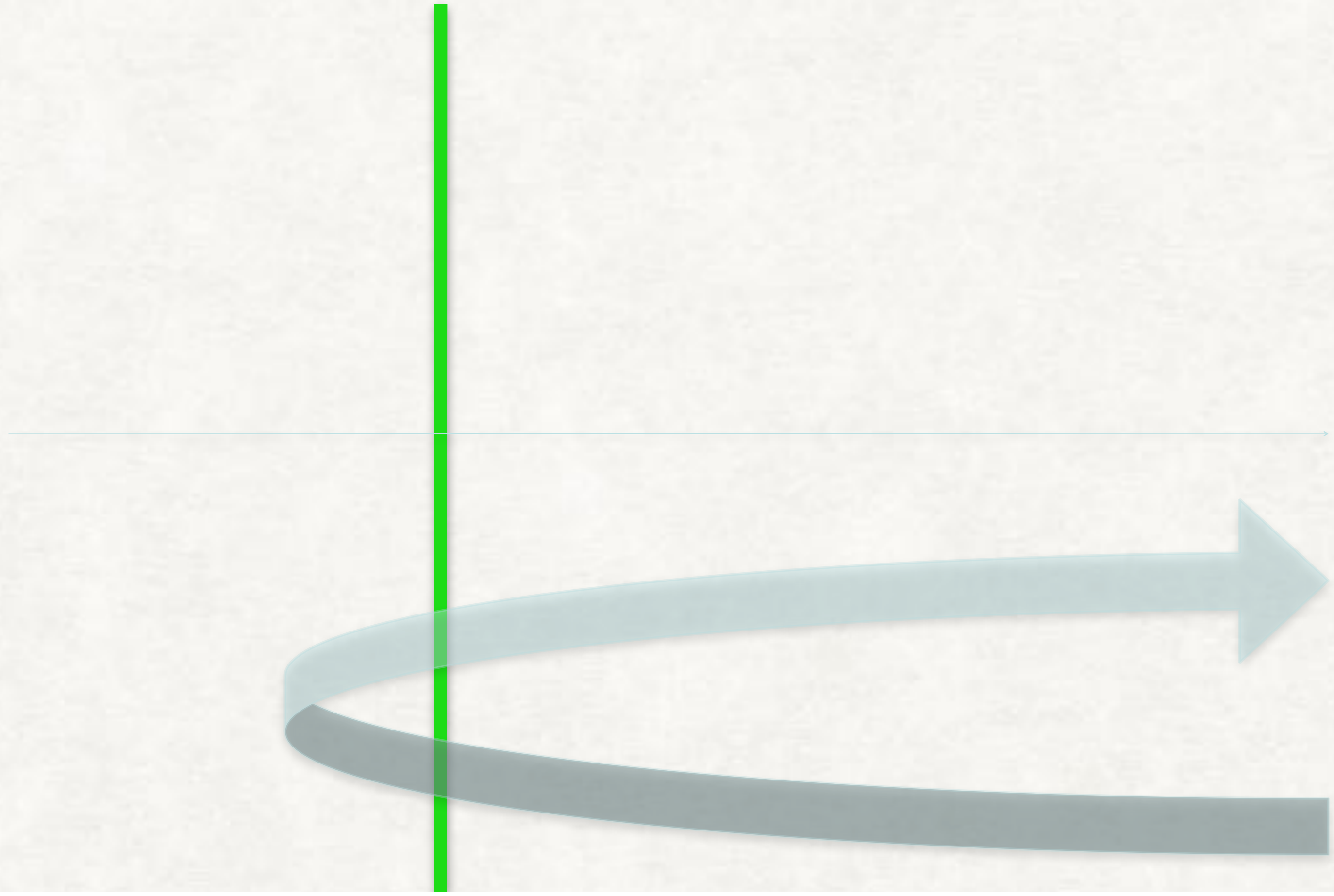
which stretches the magnetic field line by the same amount…

The saturation takes place when the displacement equals the Larmor radius of the particles in the field **δ**B … imposing this condition leads to:

$$\frac{\delta B^2}{4\pi} = \frac{\xi_{CR}}{\Lambda} \rho v_s^2 \frac{v_s}{c} \qquad \Lambda = \ln(E_{max}/E_{min})$$

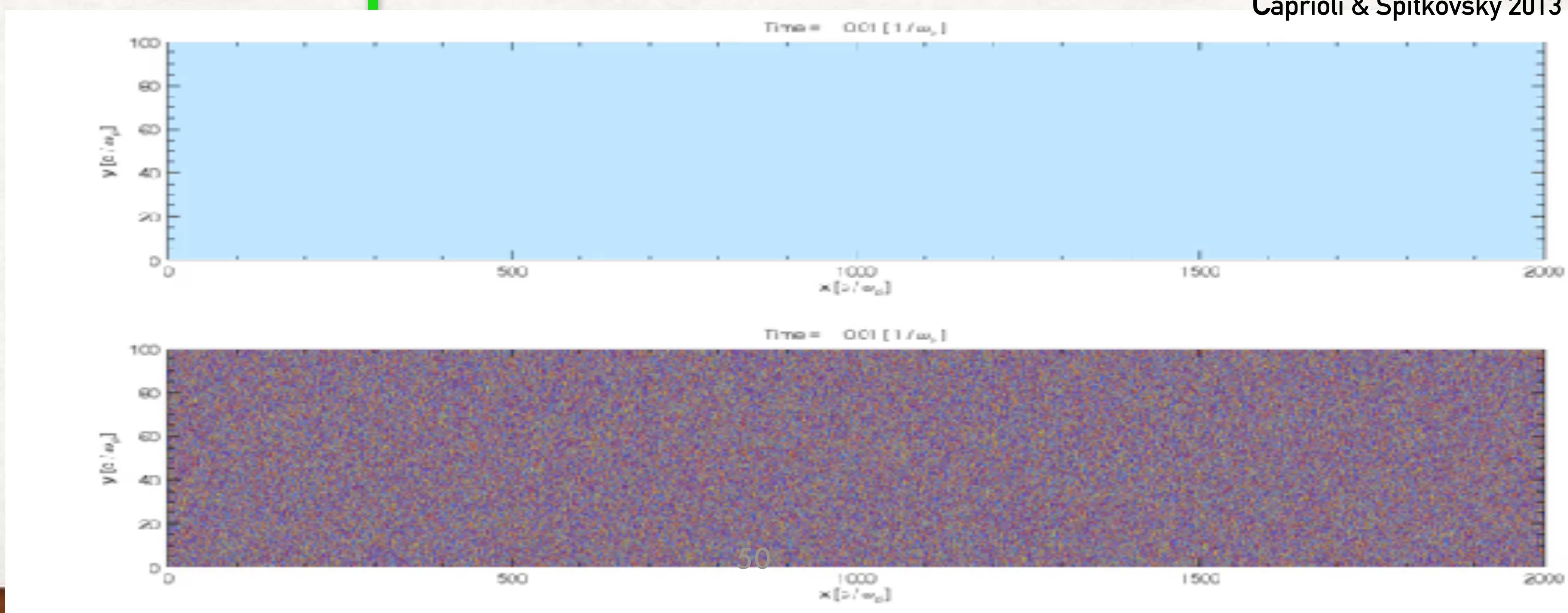
specialized to a strong shock and a spectrum E^{-2}

A QUALITATIVE PICTURE OF ACCELERATION



Bell & Schure 2013
Cardillo, Amato & PB 2015

Caprioli & Spitkovsky 2013



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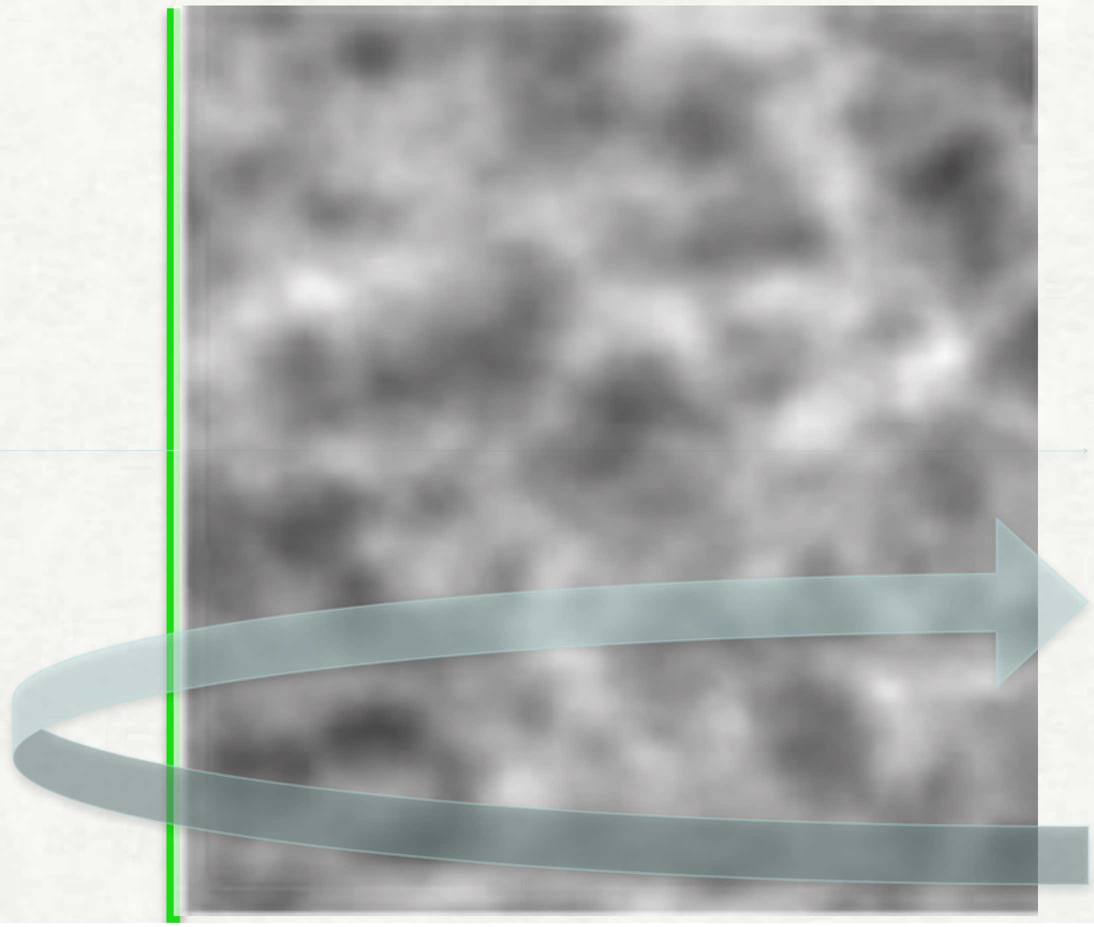

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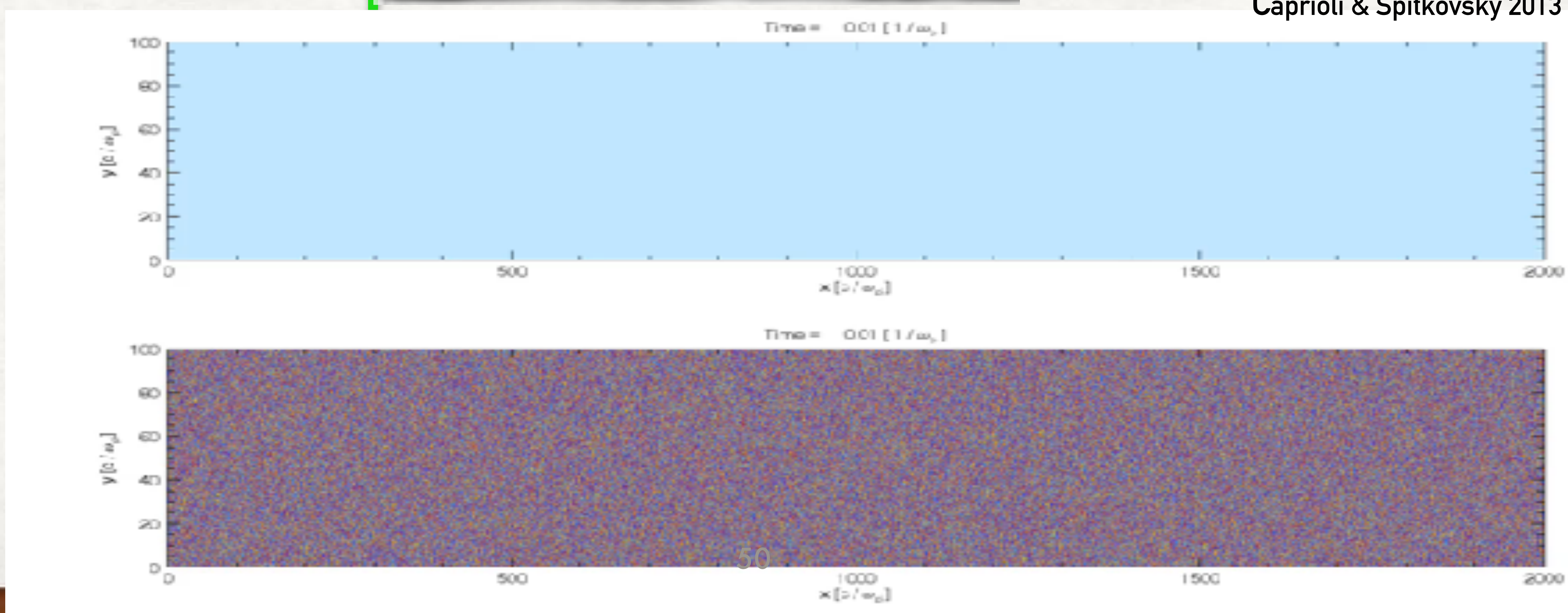

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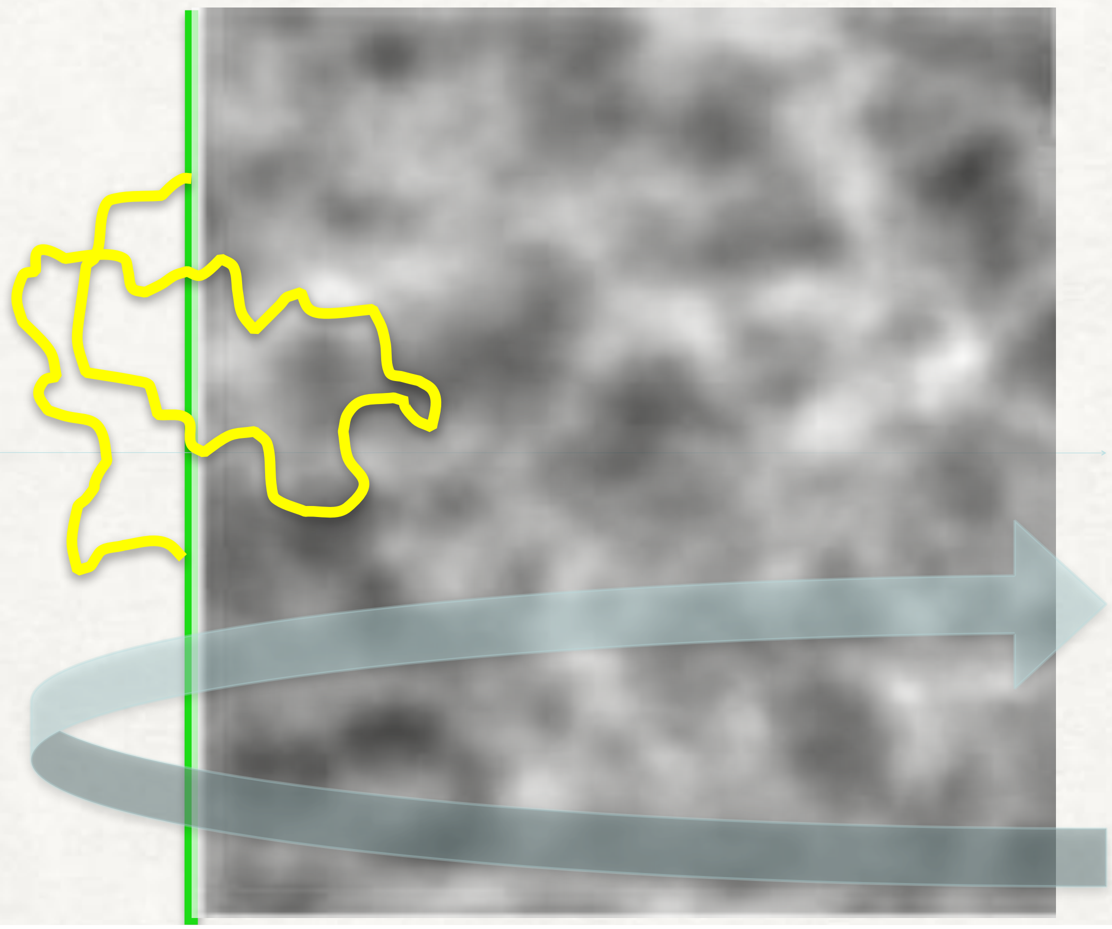

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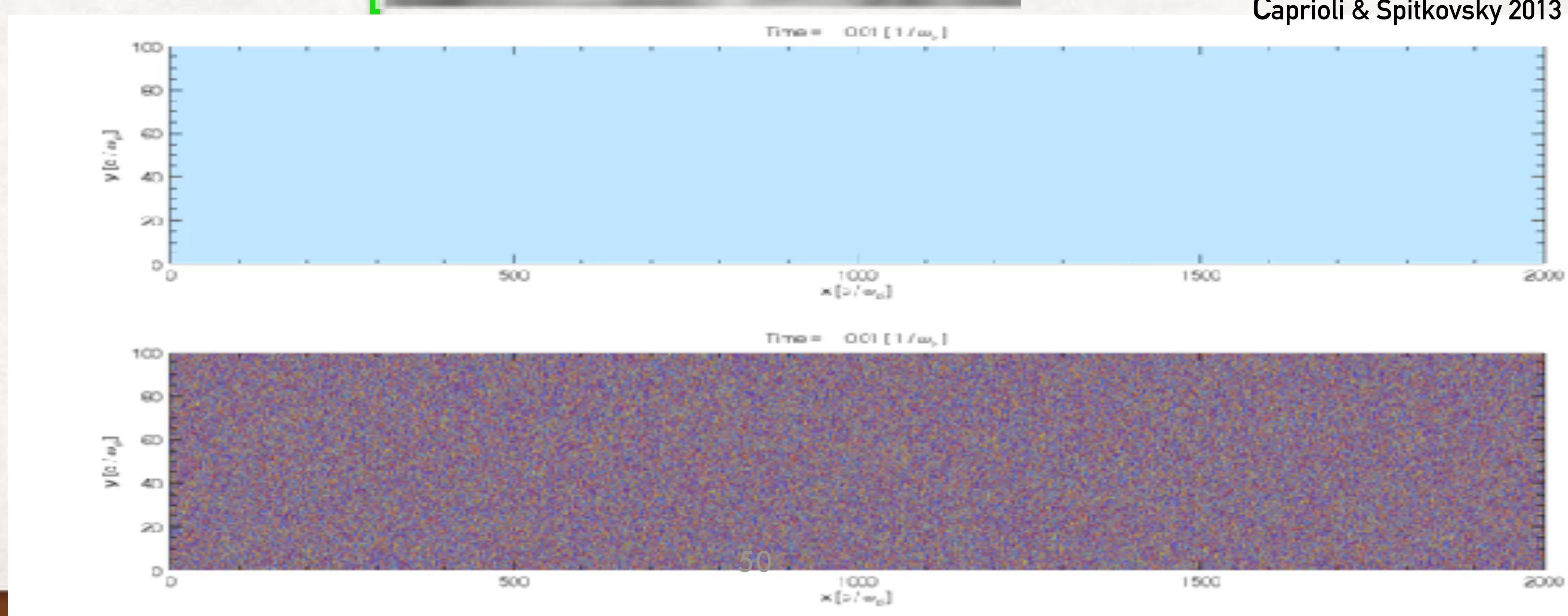

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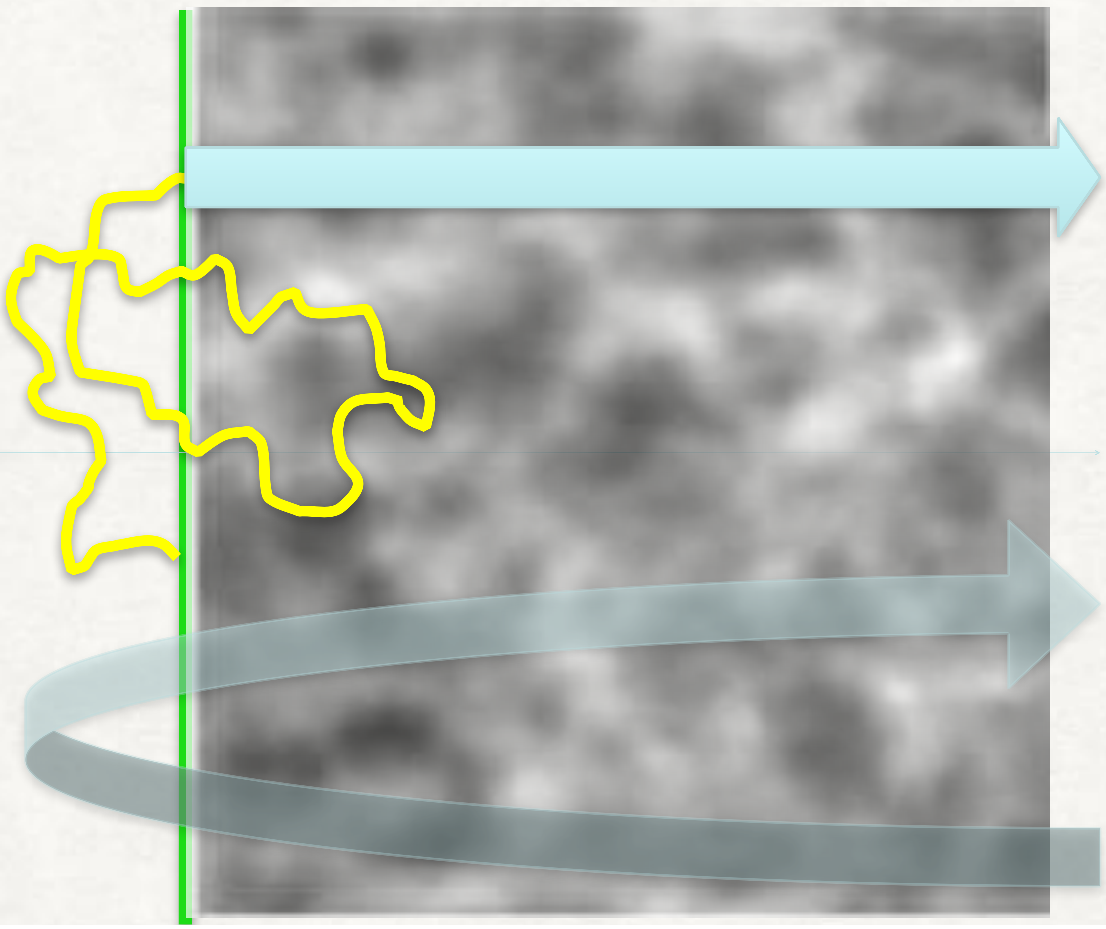

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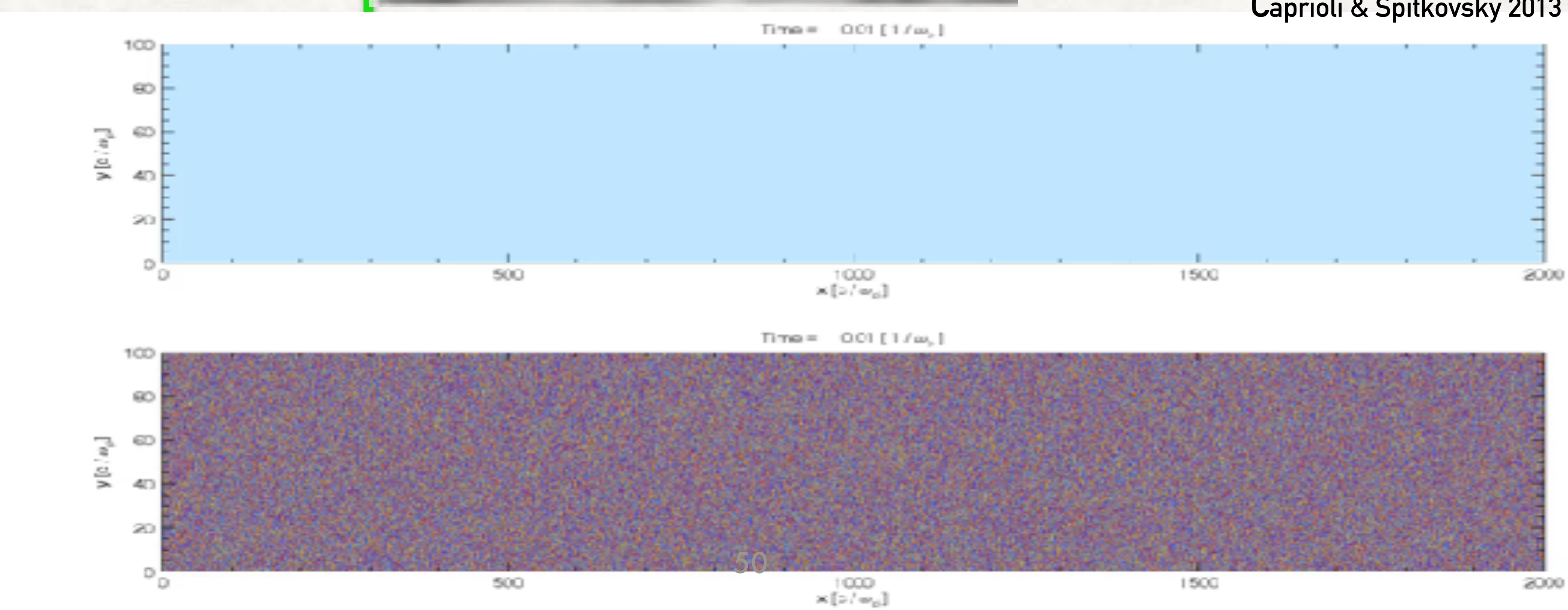

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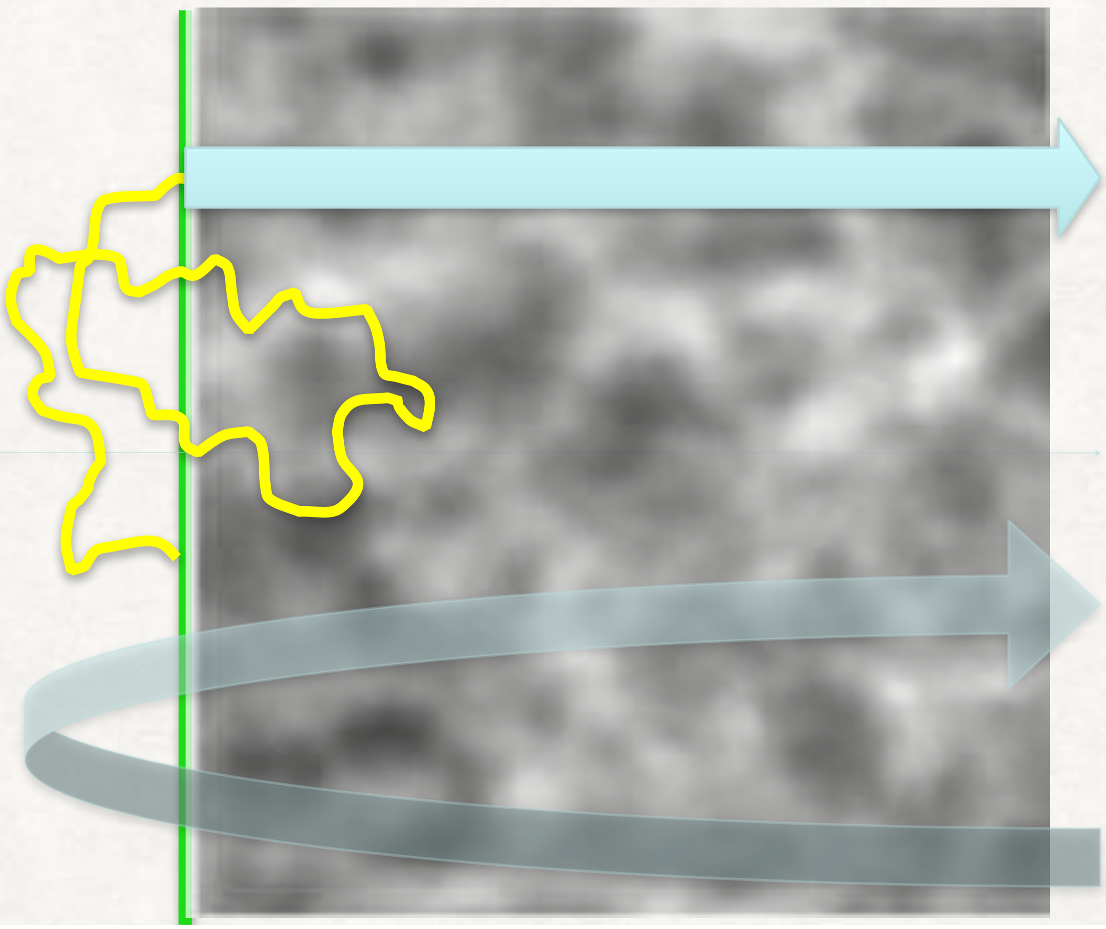

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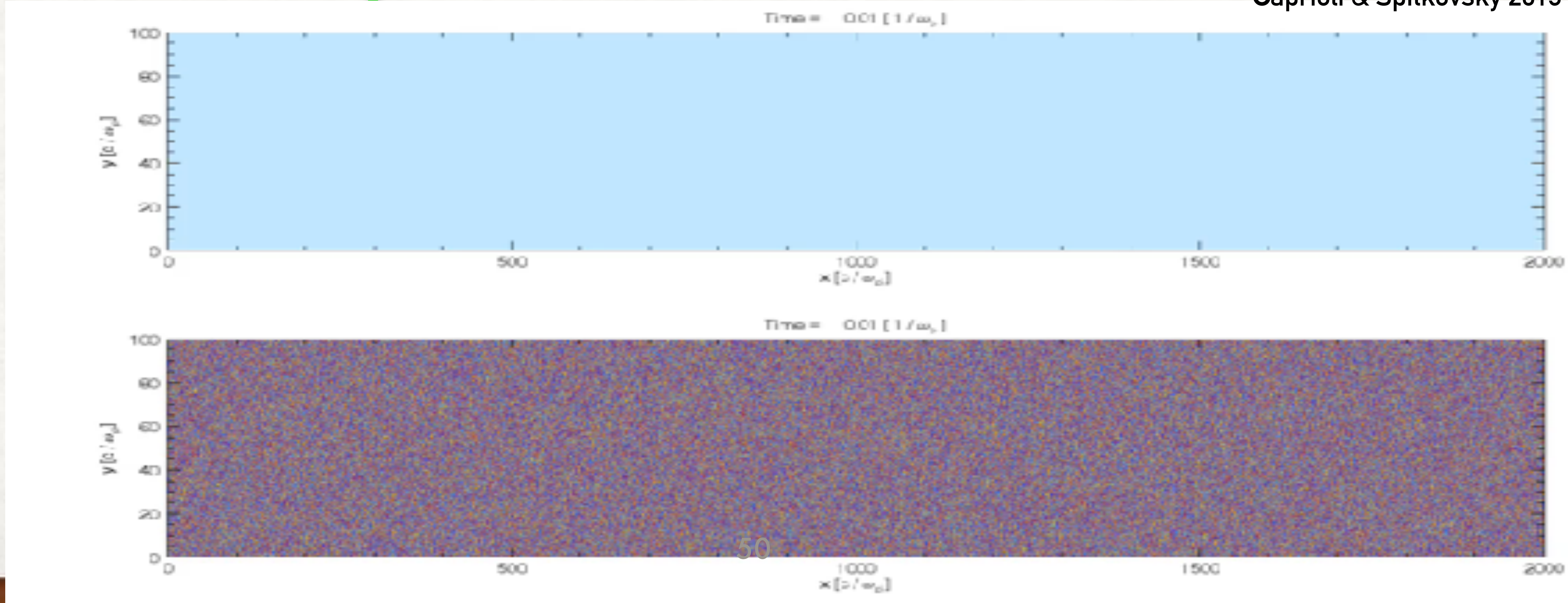


X-ray rims and B-field amplification

TYPICAL THICKNESS OF FILAMENTS: ~ 10^{-2} pc

The synchrotron limited thickness is:

$$\Delta x \approx \sqrt{D(E_{max})\tau_{loss}(E_{max})} \approx 0.04\ B_{100}^{-3/2}\ \text{pc}$$

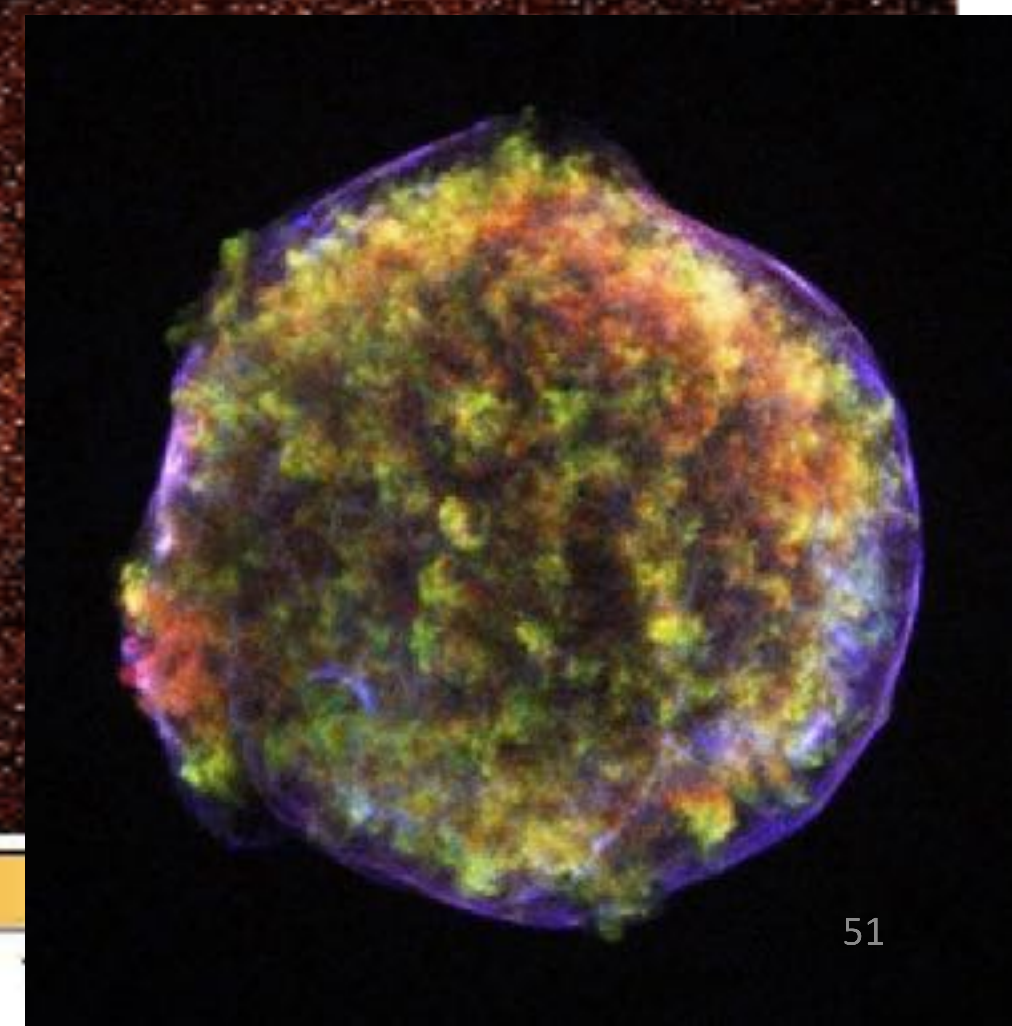
$$B \approx 100\ \mu\text{Gauss}$$

$$E_{max} \approx 10\ B_{100}^{-1/2}\ u_8\ \text{TeV}$$

$$\nu_{max} \approx 0.2\ u_8^2\ \text{keV}$$

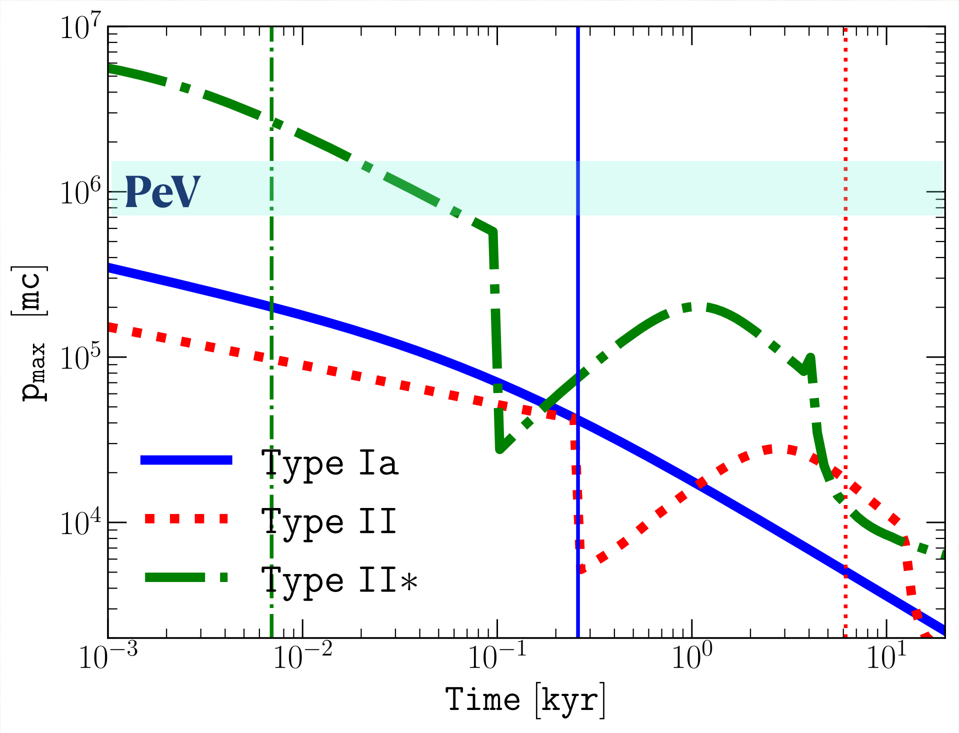
In some cases the strong fields are confirmed by time variability of X-rays

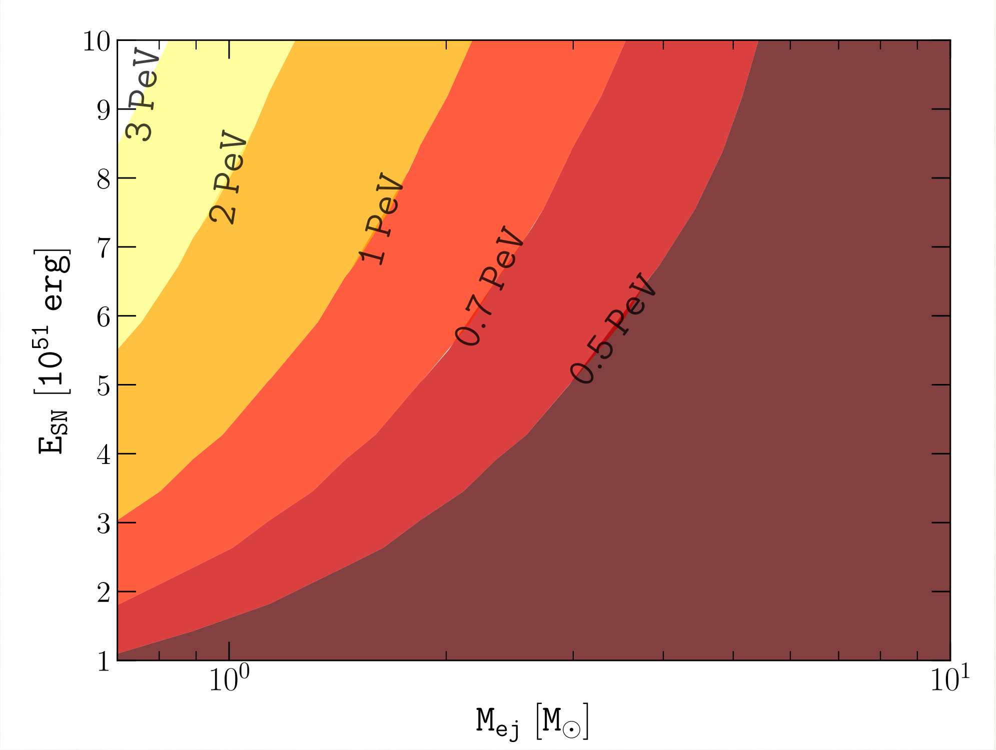
Uchiyama & Aharonian, 2007



SNRS AS PEVATRONS?

Cristofari, PB & Amato 2020

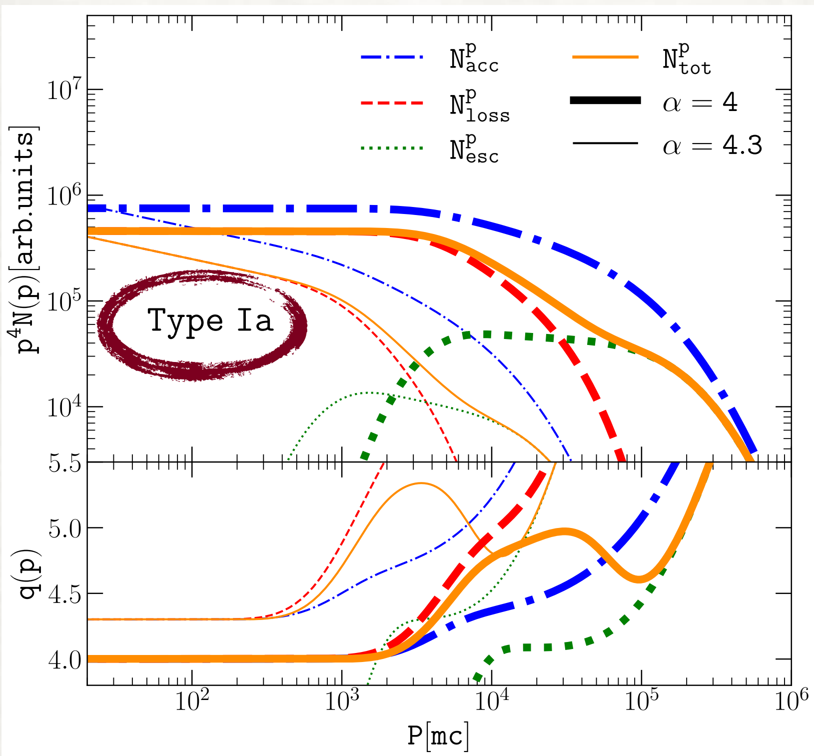


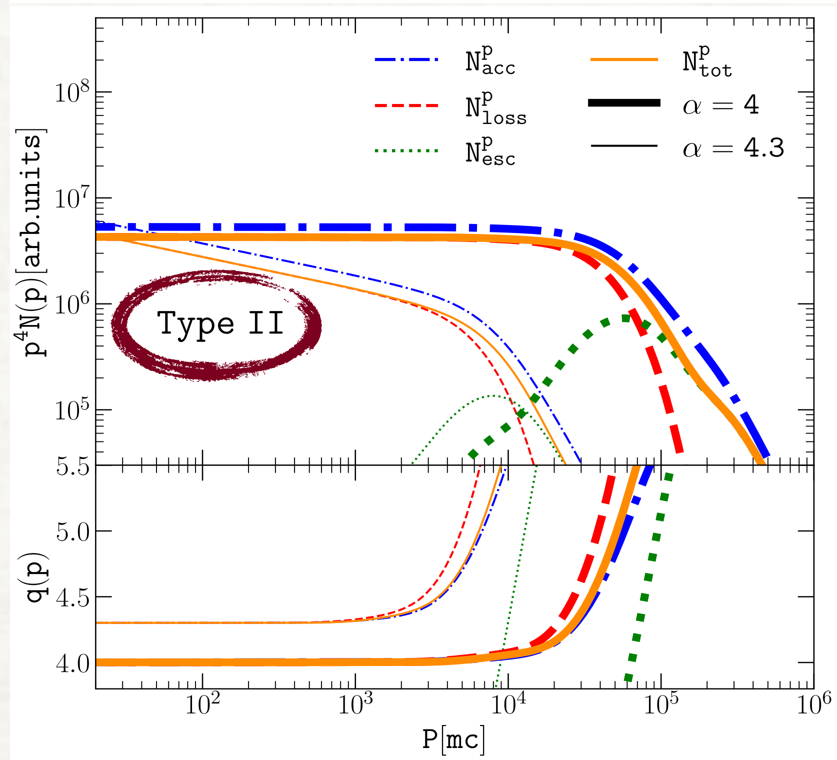


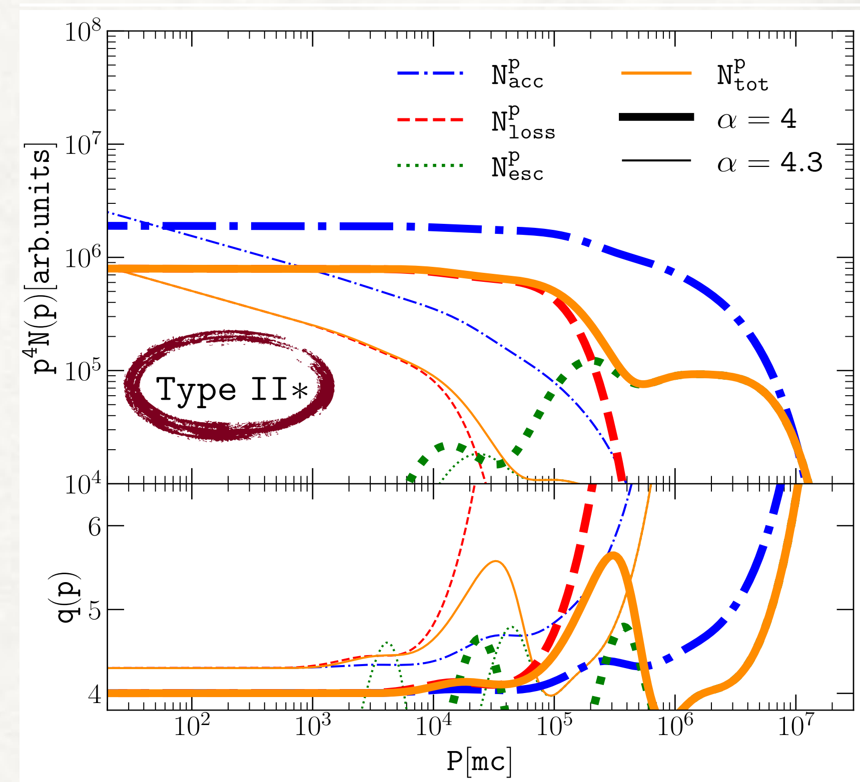
- THE HIGHEST ENERGIES ARE REACHED AT VERY EARLY EVOLUTIONARY STAGES! (Implications for gamma ray observations!)
- ...BUT THE FLUX CONTRIBUTED IN THOSE STAGES IS LOW, AND IN FACT THIS CORRESPONDS TO THE VERY STEEP PART OF THE SPECTRA RELEASED INTO THE ISM
- FOR CORE COLLAPSE SNR THE TEMPORAL EVOLUTION OF THE MAXIMUM ENERGY IS IN GENERAL RATHER COMPLEX
- THE EFFECTIVE E_{MAX} IS THE ONE CORRESPONDING TO THE BEGINNING OF THE SEDOV-TAYLOR PHASE (vertical lines)

SNRS AS PEVATRONS?

Cristofari, PB & Caprioli 2021, Cristofari, PB & Amato 2020

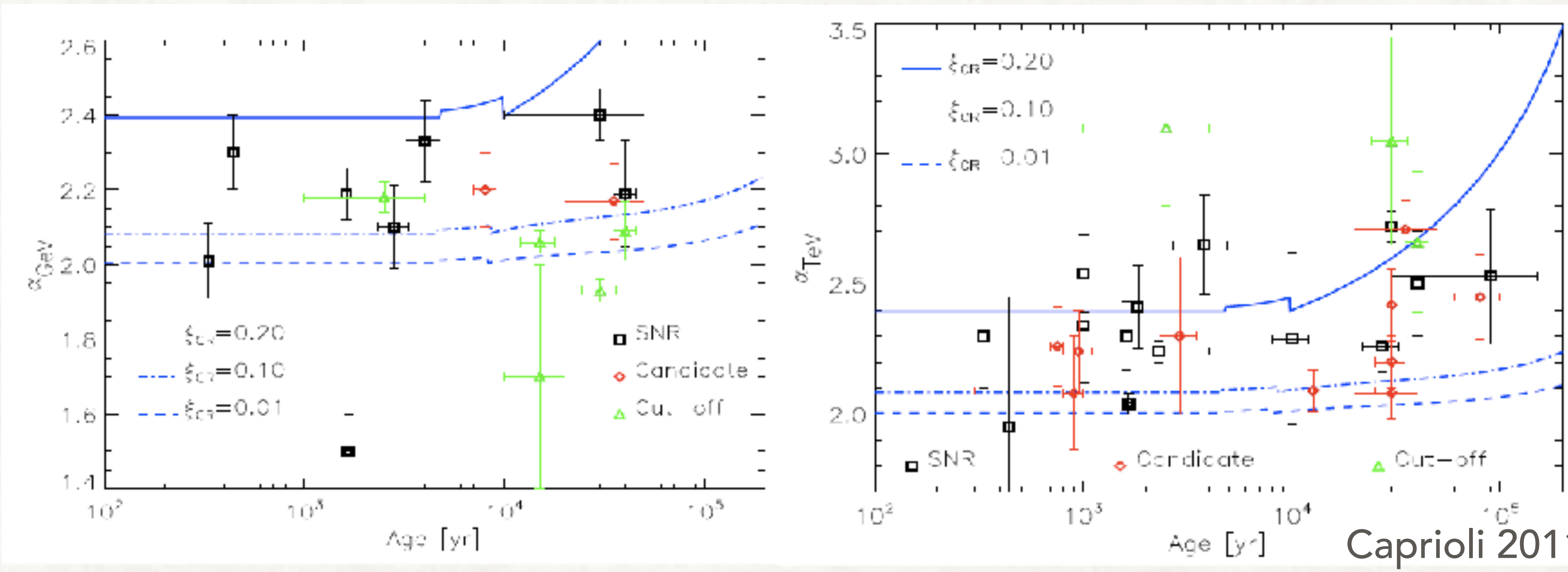






- ☑ THE SPECTRUM RELEASED INTO THE ISM IS THE SUM OF CR ESCAPING FROM UPSTREAM AND THE ONES TRAPPED DOWNSTREAM (COMPLEX SPECTRAL SHAPES)
- ☑ THE EFFECTIVE MAX ENERGY FOR IA AND II IS <100 TEV
- ☑ PEVATRONS ONLY FROM EXTREMELY POWERFUL AND RARE SUPERNOVA REMNANTS
- ☑ EITHER WAY, THE SUPPRESSION IS NOT EXPONENTIAL!!!

ISSUES WITH SPECTRA INSIDE SNR



Caprioli 2011

BOTH GAMMA RAY OBSERVATIONS AND CR TRANSPORT SUGGEST THAT THE SPECTRUM CONTRIBUTED BY SNR IS STEEPER THAN E^{-2} BUT THIS SEEMS INCOMPATIBLE WITH THEORETICAL EXPECTATIONS!

THESE SUBTLE FEATURES ARE SENSITIVE TO THE MICROPHYSICS...

SUBTLE ASPECT OF DSA

$$\frac{\partial}{\partial x}\left[D\frac{\partial f}{\partial x}\right] - (u+v_A)\frac{\partial f}{\partial x} + \frac{1}{3}\frac{d(u+v_A)}{dx}p\frac{\partial f}{\partial p} = 0$$

Velocity of scattering centers

If the velocity of the scatterers is not zero on either side of the shock the implications on the spectrum are quite remarkable

This effect is especially important in situations in which the perturbations are large as expected for Bell modes

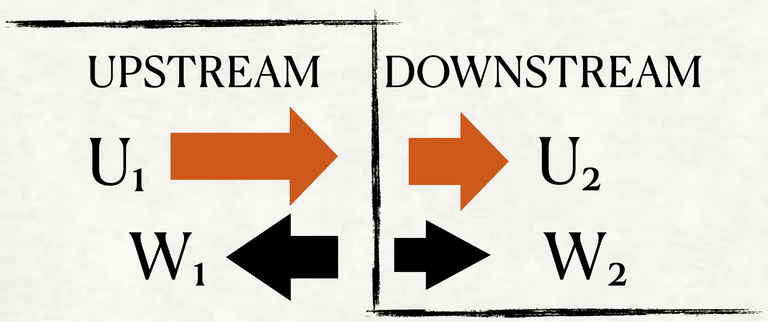
The effective compression factor and the spectrum become:

$$\tilde{r} = \frac{u_1 + v_{A,1}}{u_2 + v_{A,2}} \qquad f(p) \propto p^{-\frac{3\tilde{r}}{\tilde{r}-1}}$$

POSTCURSORS

- THE ACTION OF COSMIC RAYS IS IN GENERAL OF INCREASING THE COMPRESSION FACTOR AT THE SHOCK DUE TO THE CHANGE OF ADIABATIC INDEX (AND OTHER EFFECTS, **PRECURSOR**) —> SPECTRUM SHOULD BECOME HARDER THAN STANDARD DSA

- HOWEVER, THE AMPLIFICATION OF THE MAGNETIC FIELD MAKES ANOTHER EFFECT APPEAR:



THE VELOCITY OF THE WAVES UPSTREAM IS $U_1 - W_1 \approx U_1$

THE WAVES DOWNSTREAM ARE SEEN IN SIMULATIONS TO MOVE IN THE SAME DIRECTION AS THE PLASMA, WITH APPROXIMATELY THE ALFVEN SPEED IN THE AMPLIFIED FIELD (**POSTCURSOR**)

$$W_2 \approx \frac{\delta B}{\sqrt{4\pi\rho}} = \alpha U_2 \quad \Rightarrow \quad q \approx \frac{3R}{R-1-\alpha}$$

THE SPECTRUM BECOMES STEEPER

Caprioli, Haggerty & PB 2020