

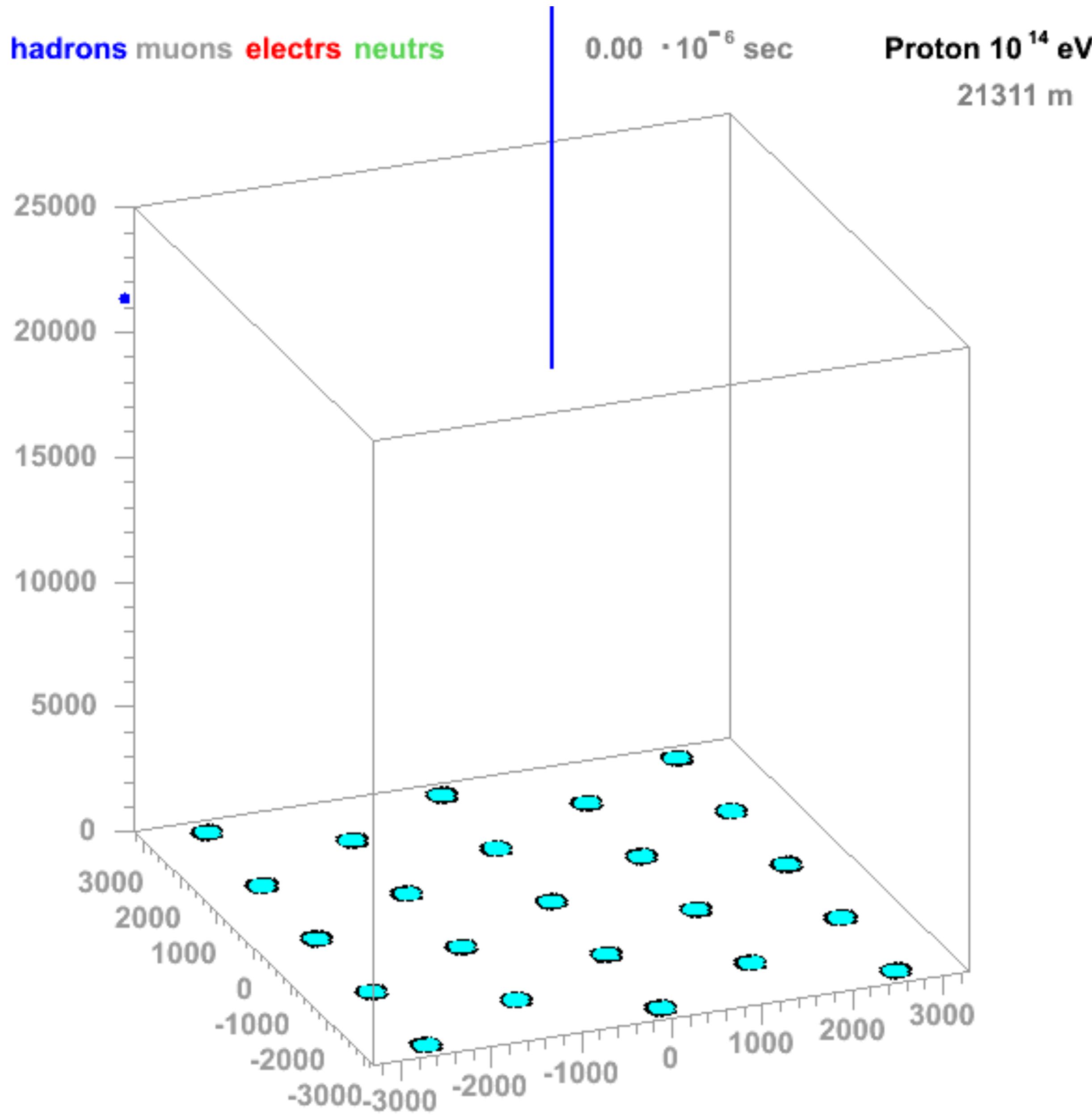
Physics of High-Energy Showers

Lecture 1

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(F. Schmidt & J. Knapp)

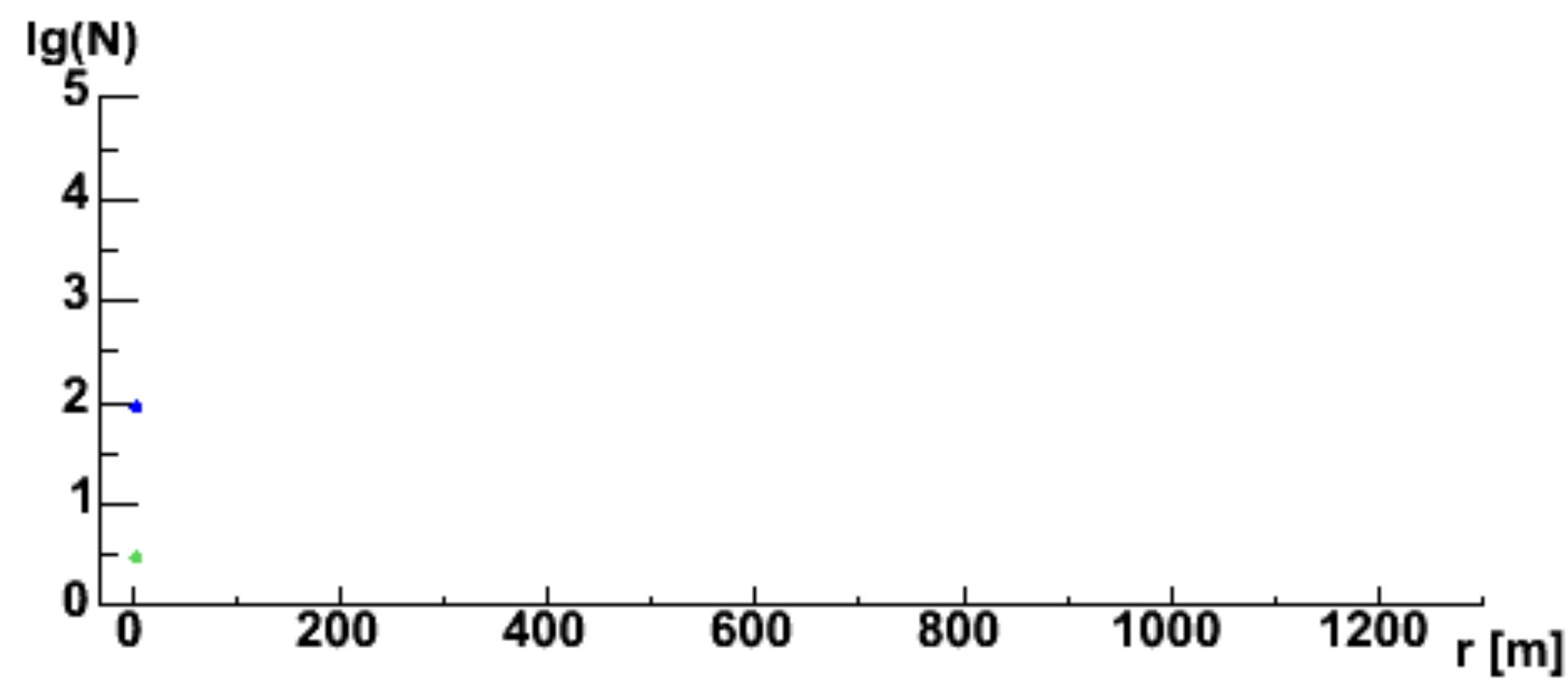
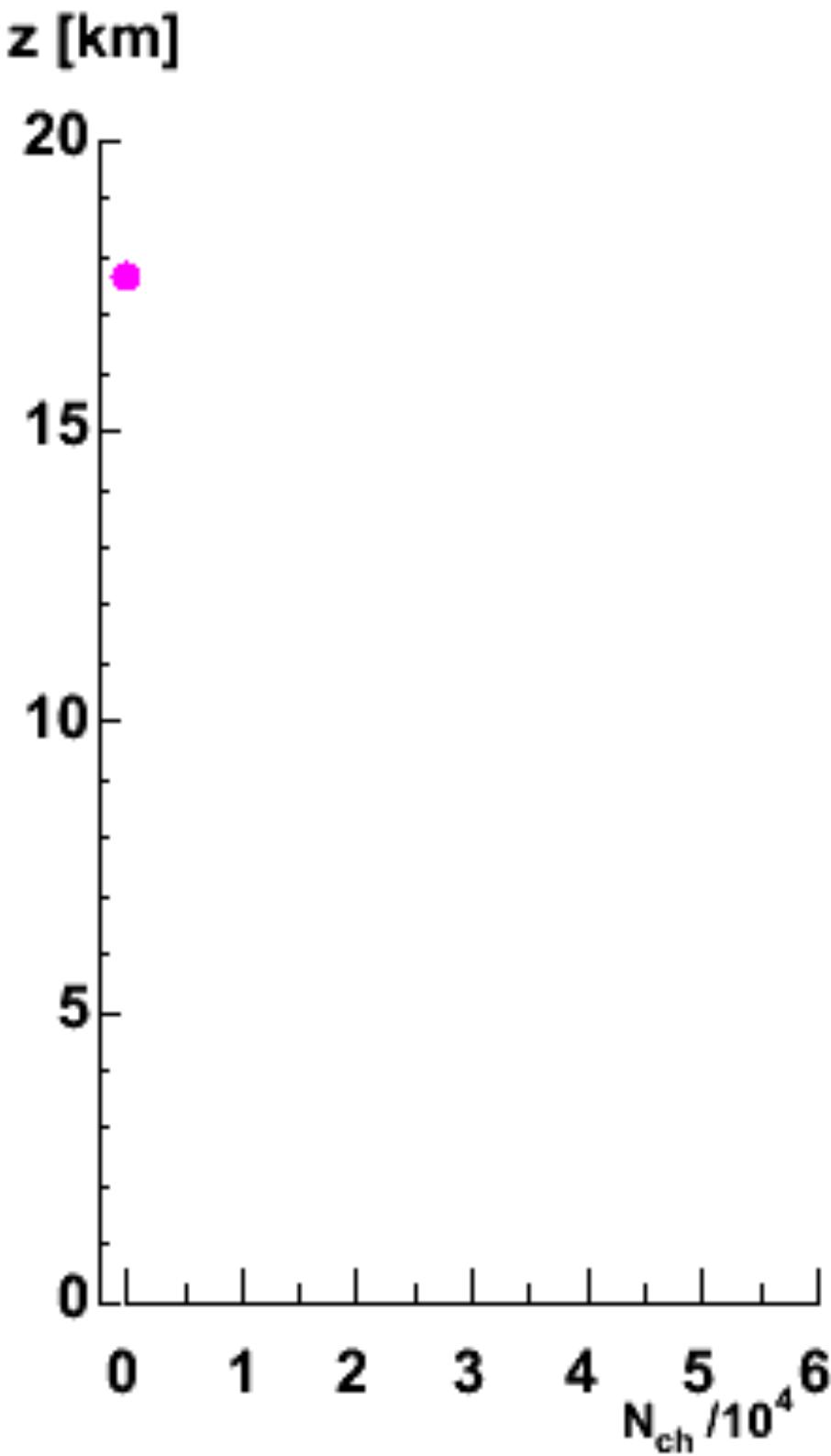
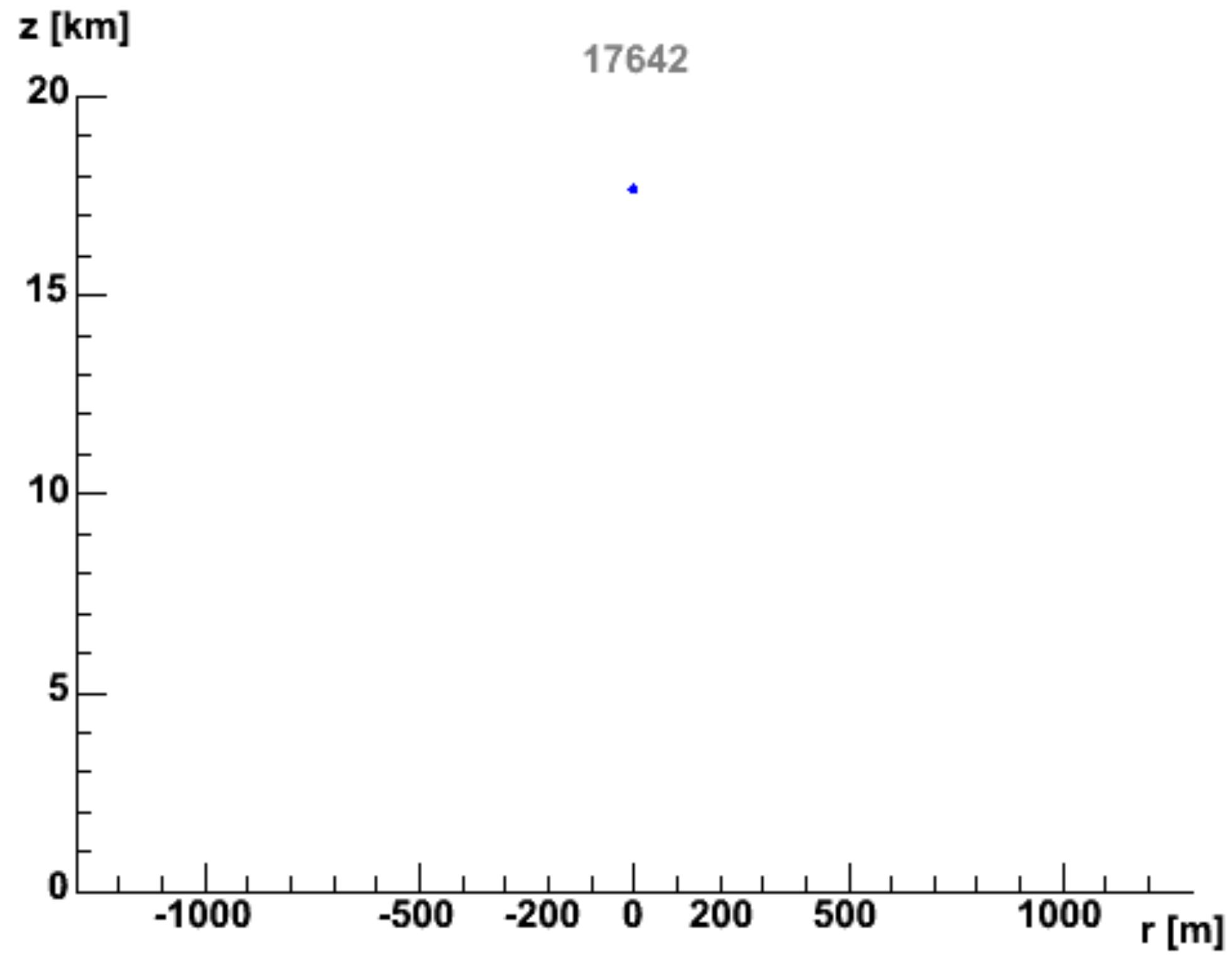


Simulation of shower development (i)

Realistic simulation with CORSIKA

Proton shower of low energy (knee region)

Simulation of shower development (ii)



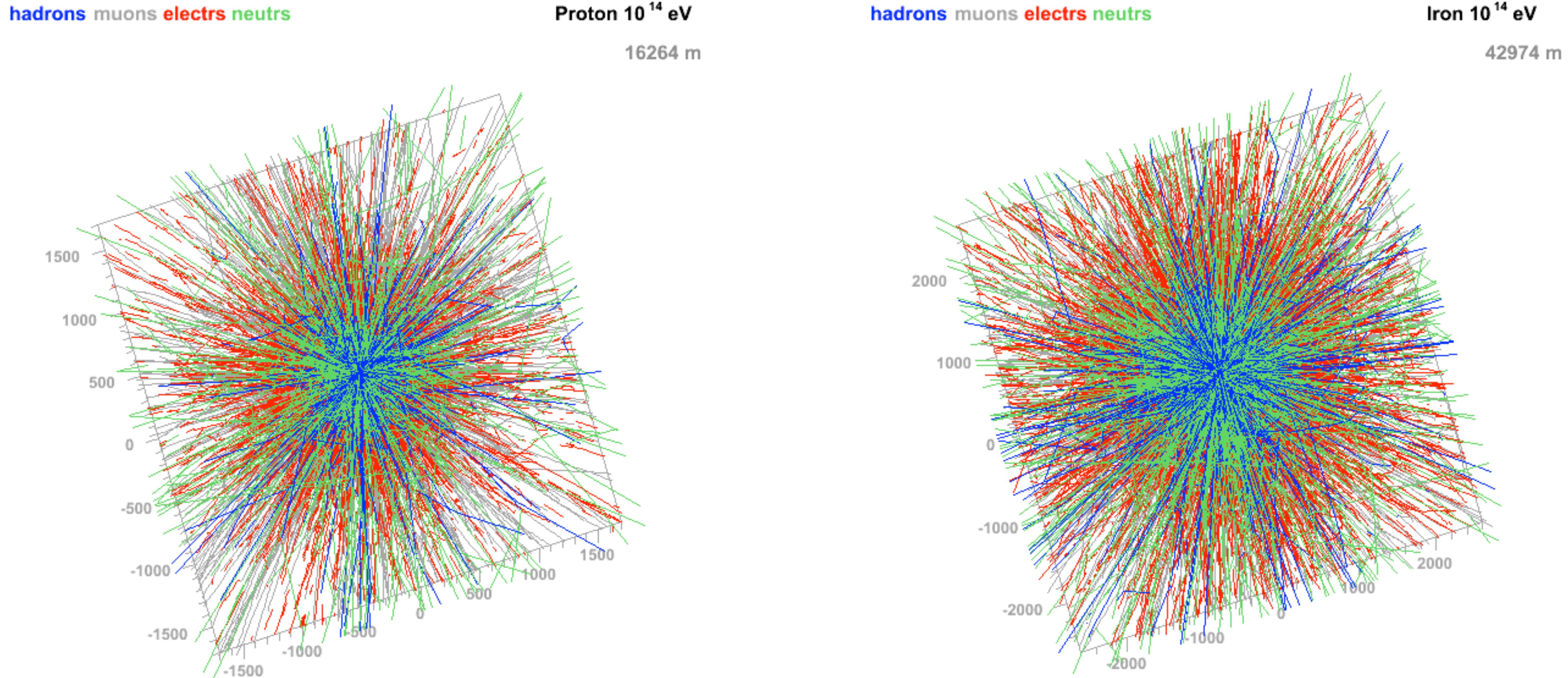
Proton 10^{14} eV

$h^{1st} = 17642$ m

hadrons muons

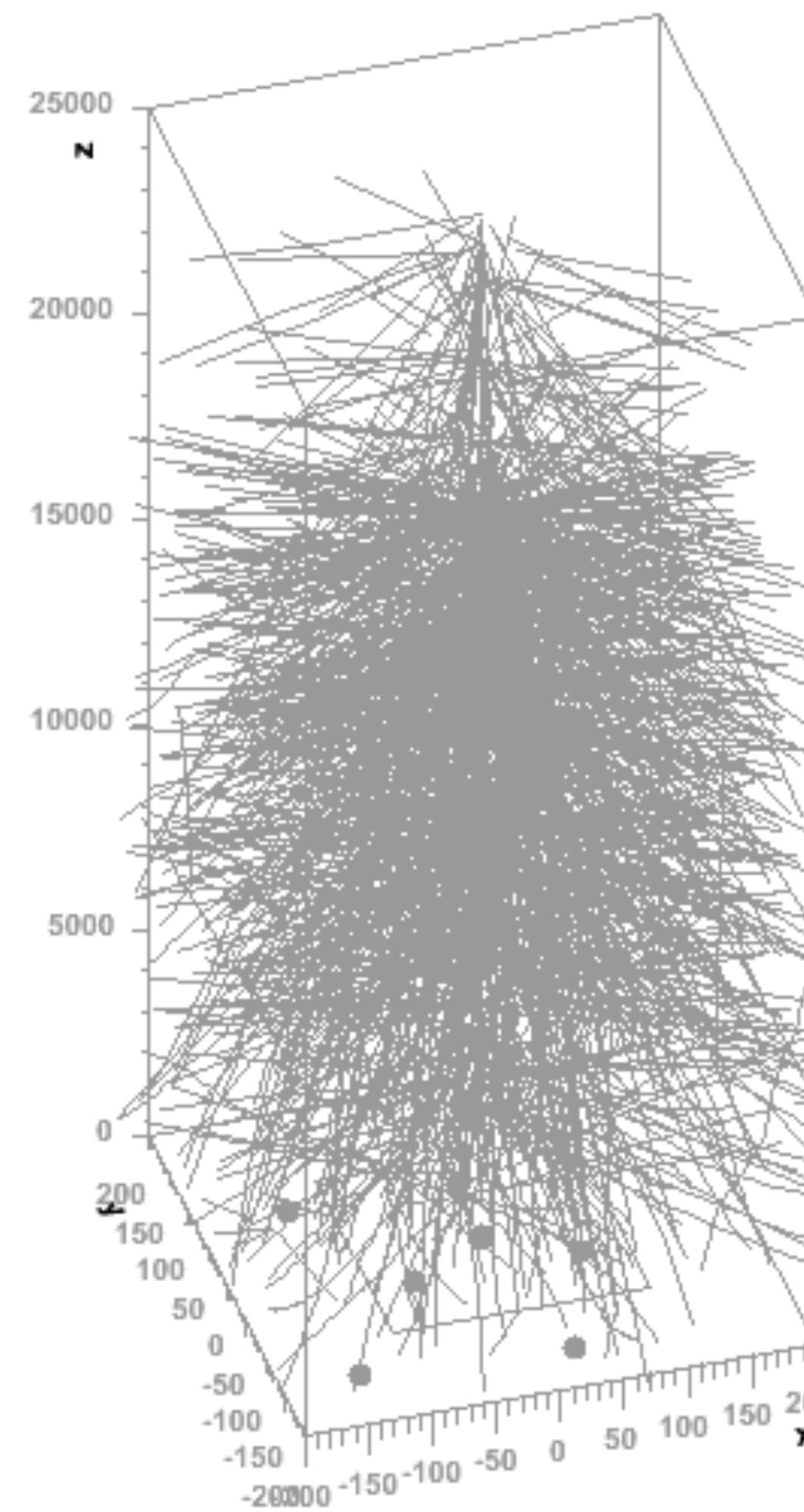
neutrons electrons

Simulation of air shower tracks (i)

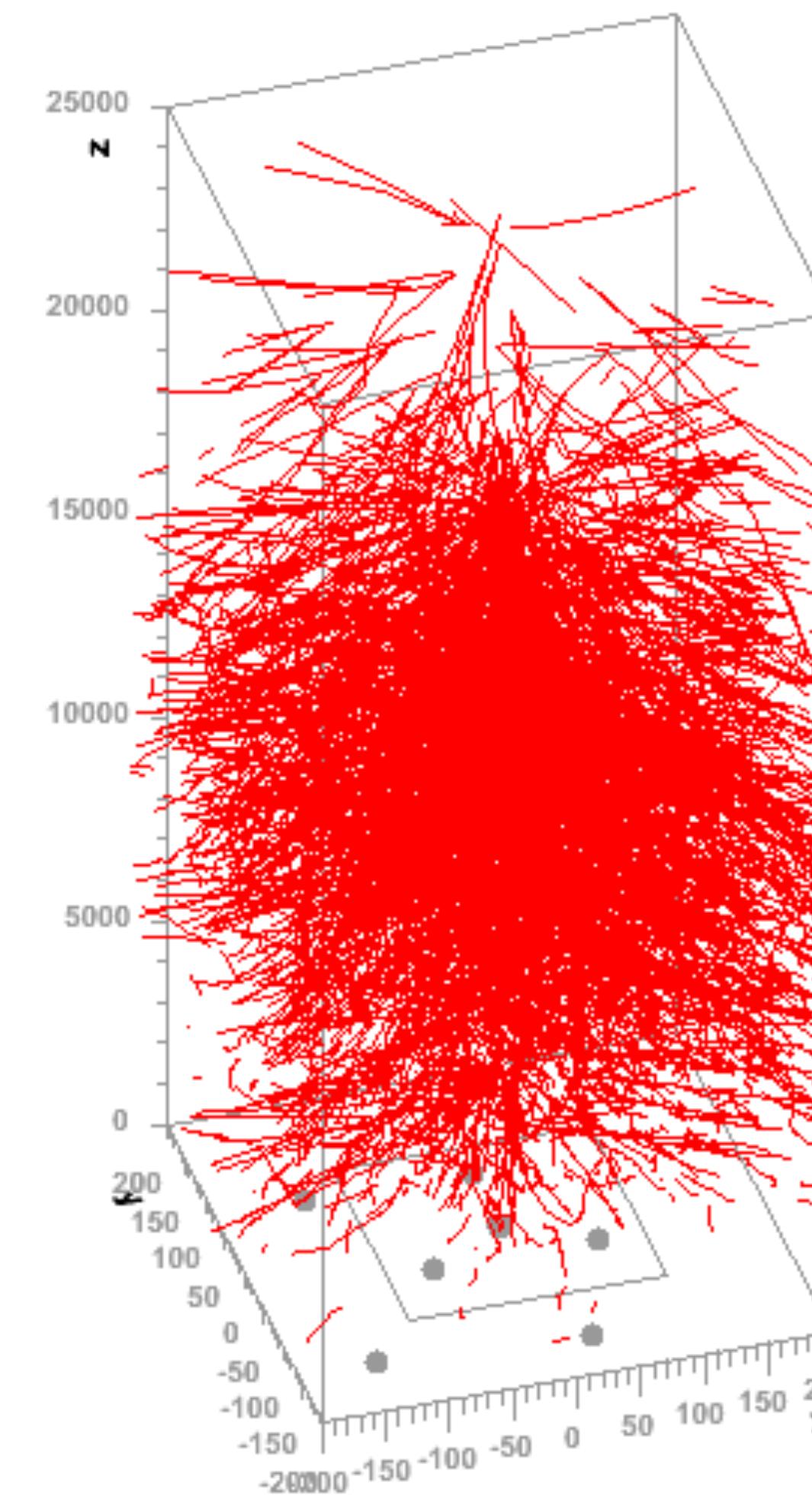


Particles of an iron shower

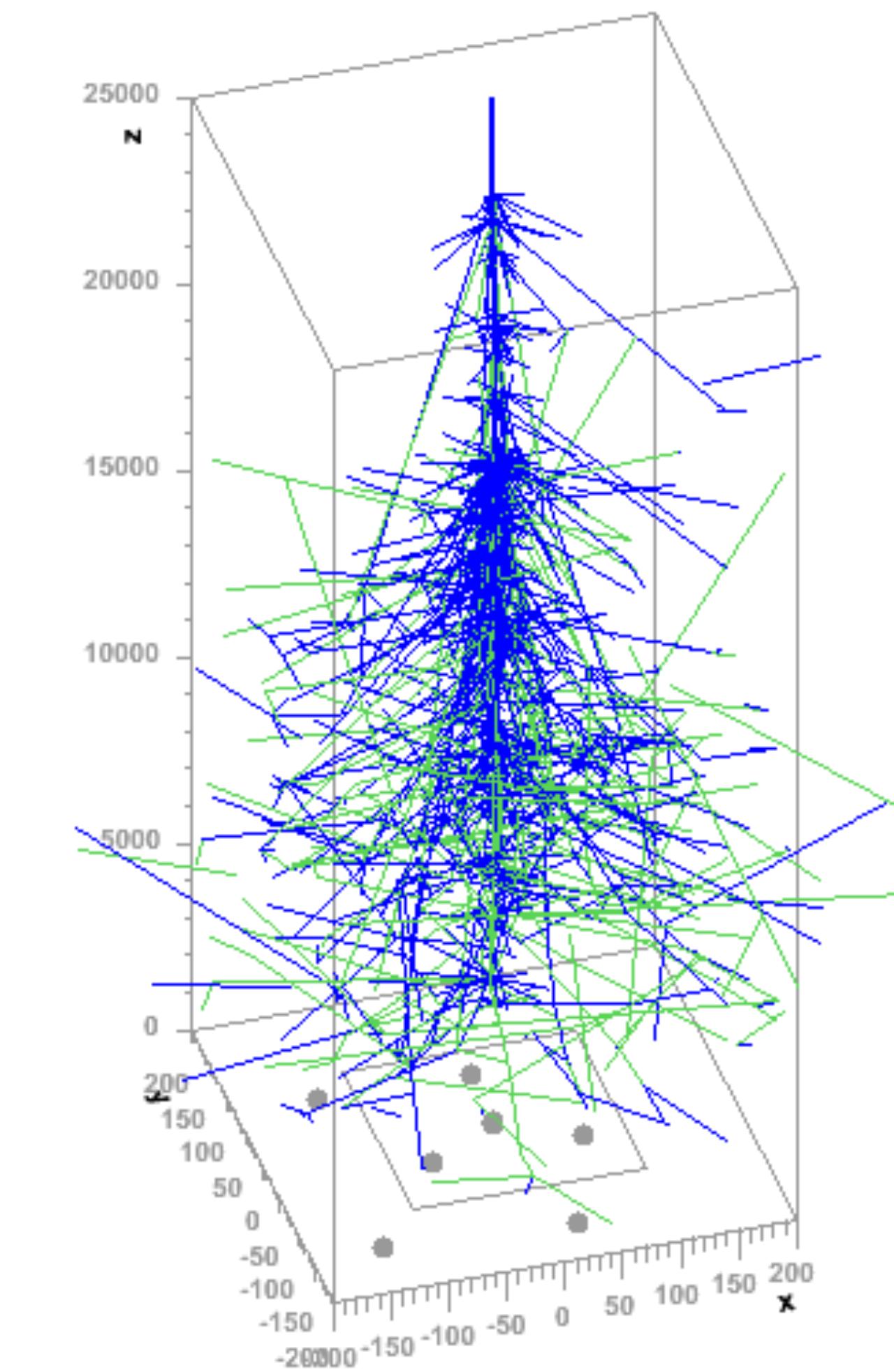
muons



electrs

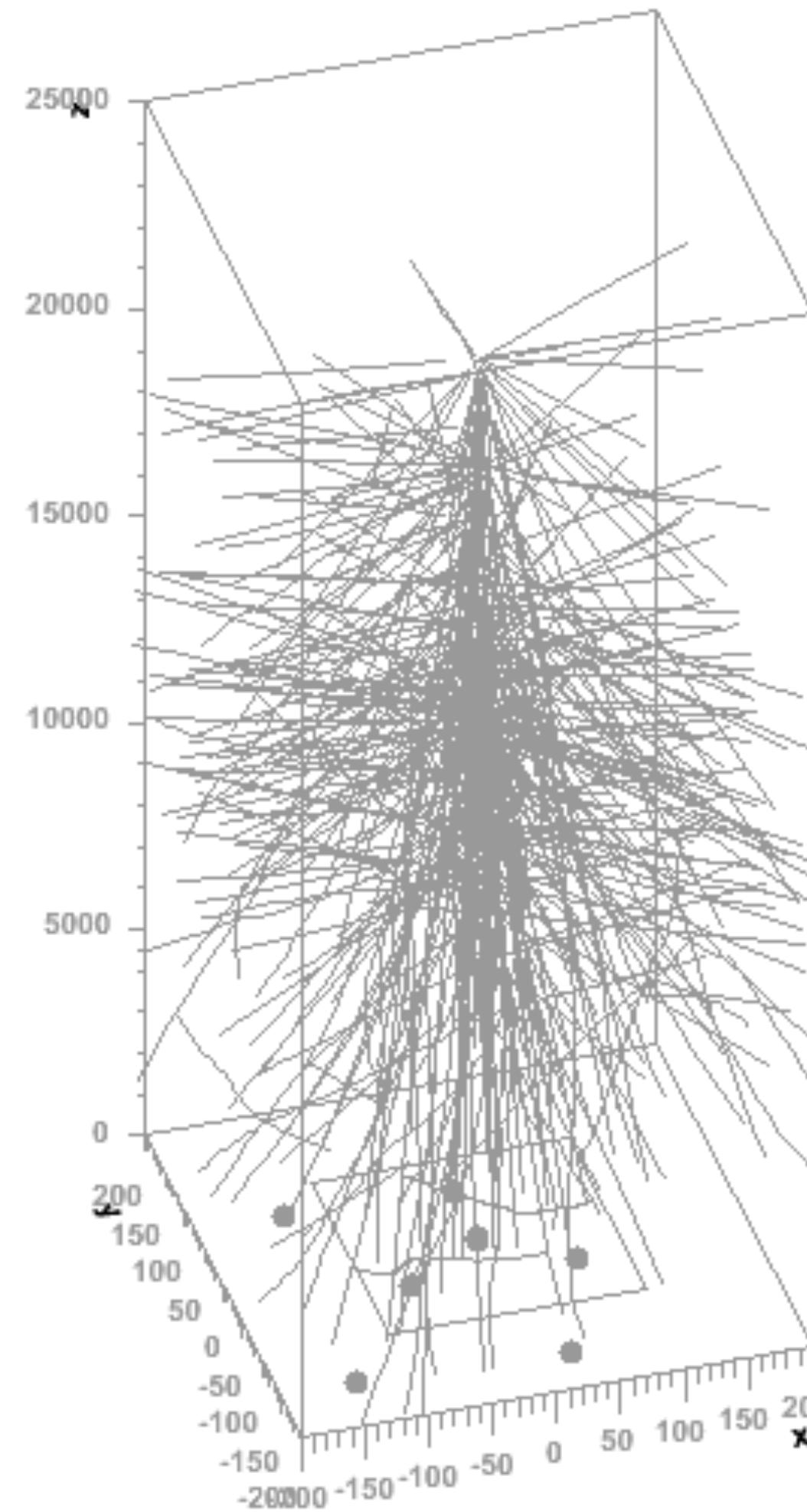


hadrons neutrals

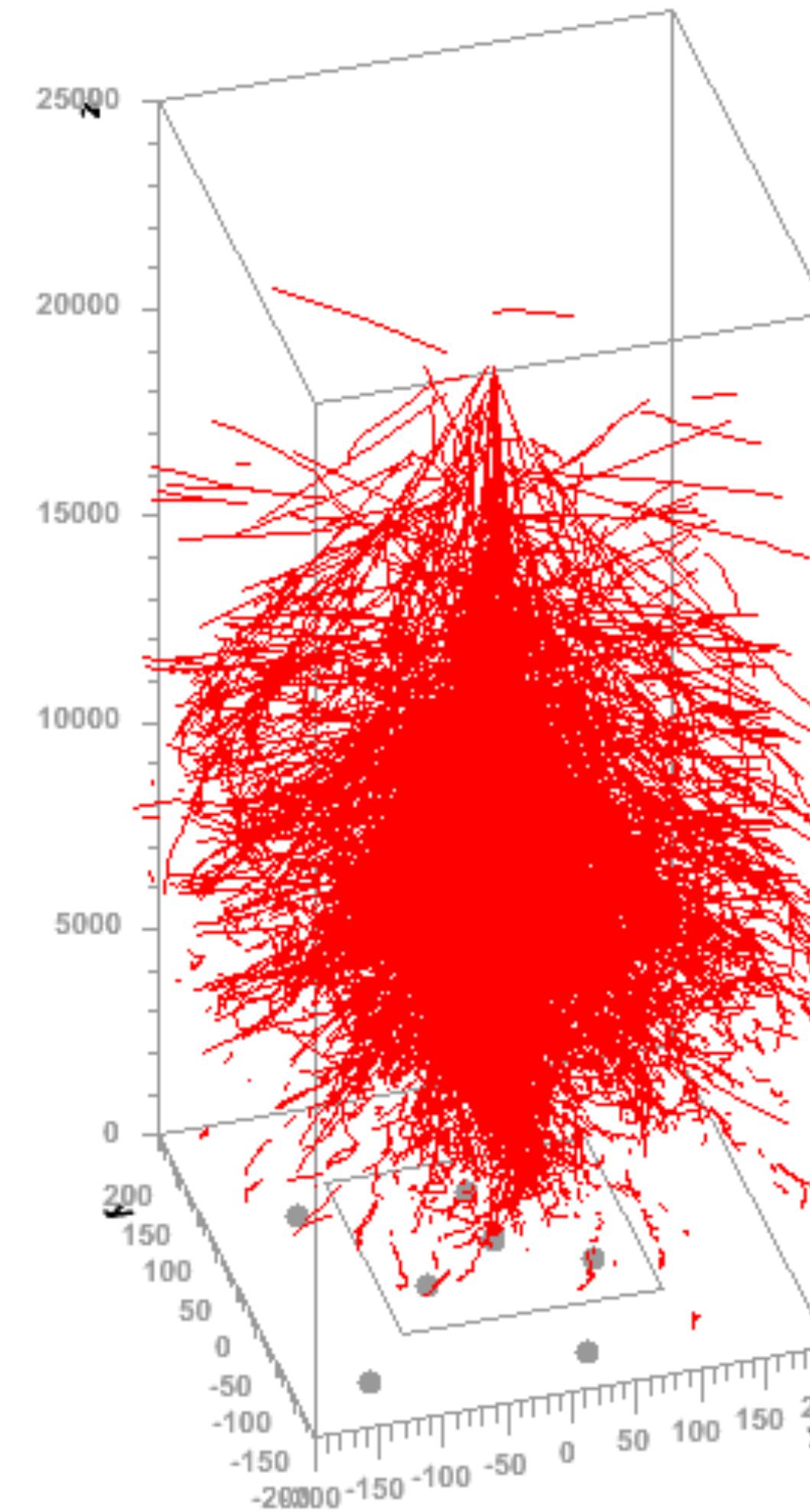


Particles of an proton shower

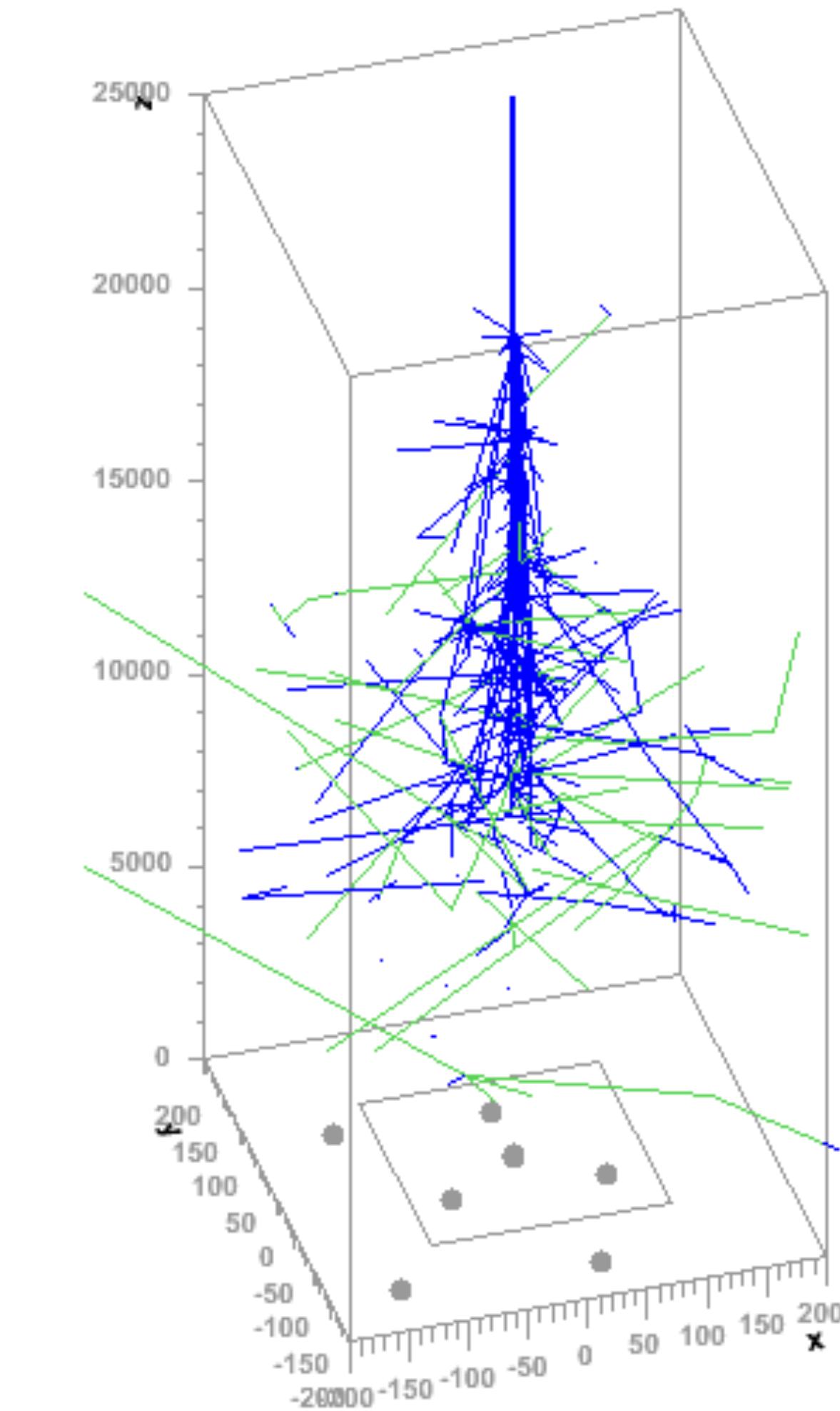
muons



electrs

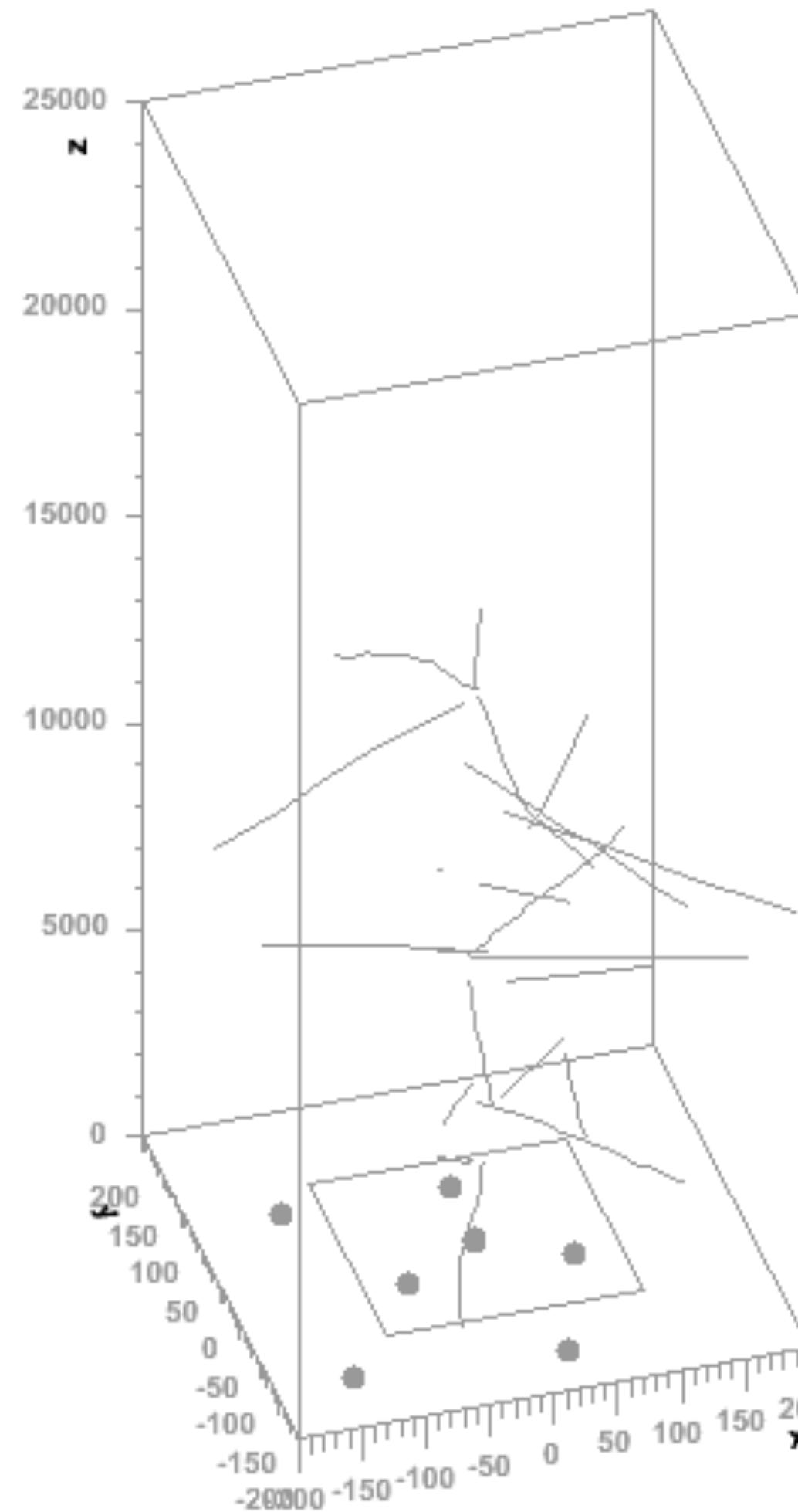


hadrons neutrals

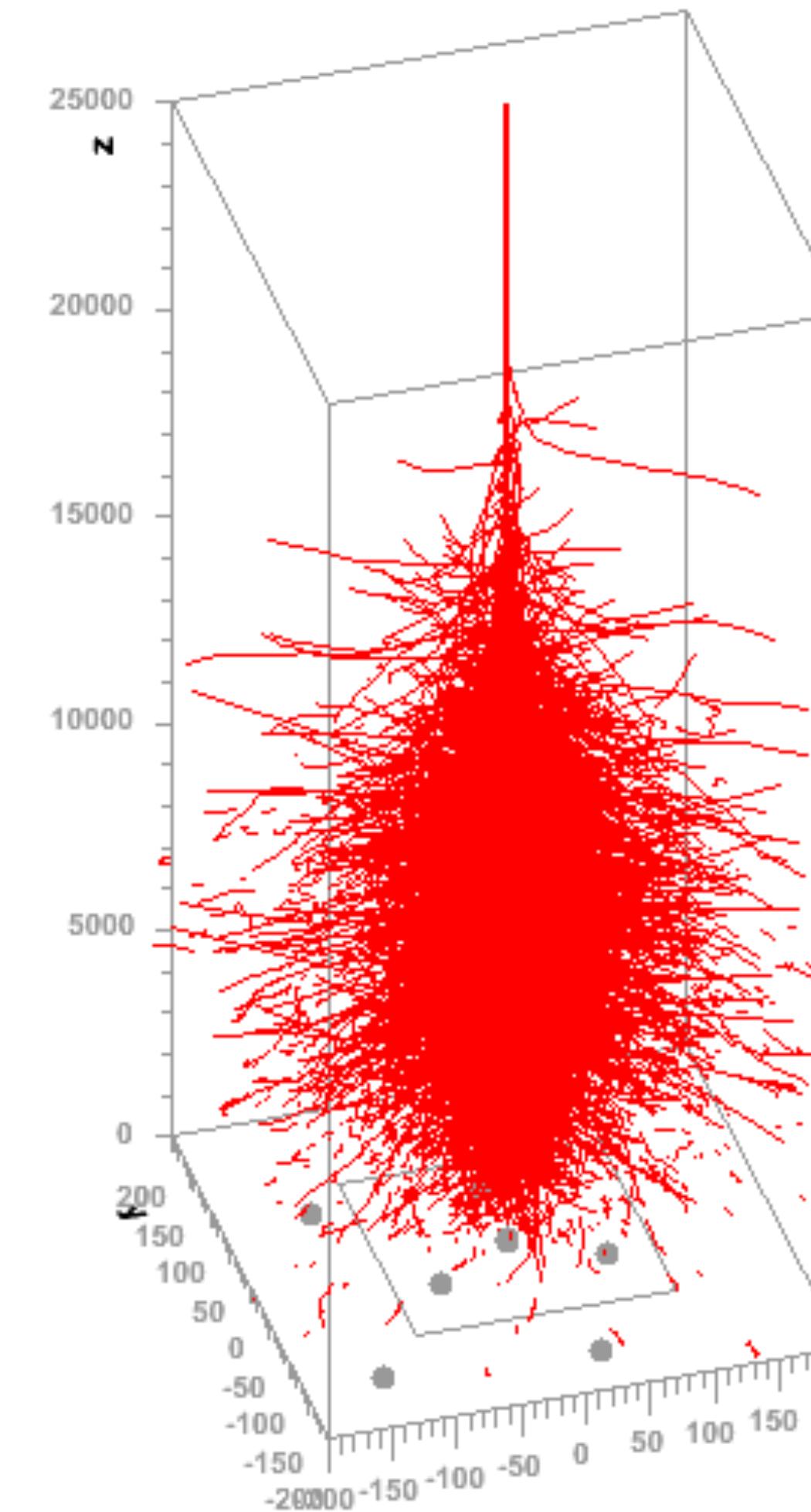


Particles of a gamma-ray shower

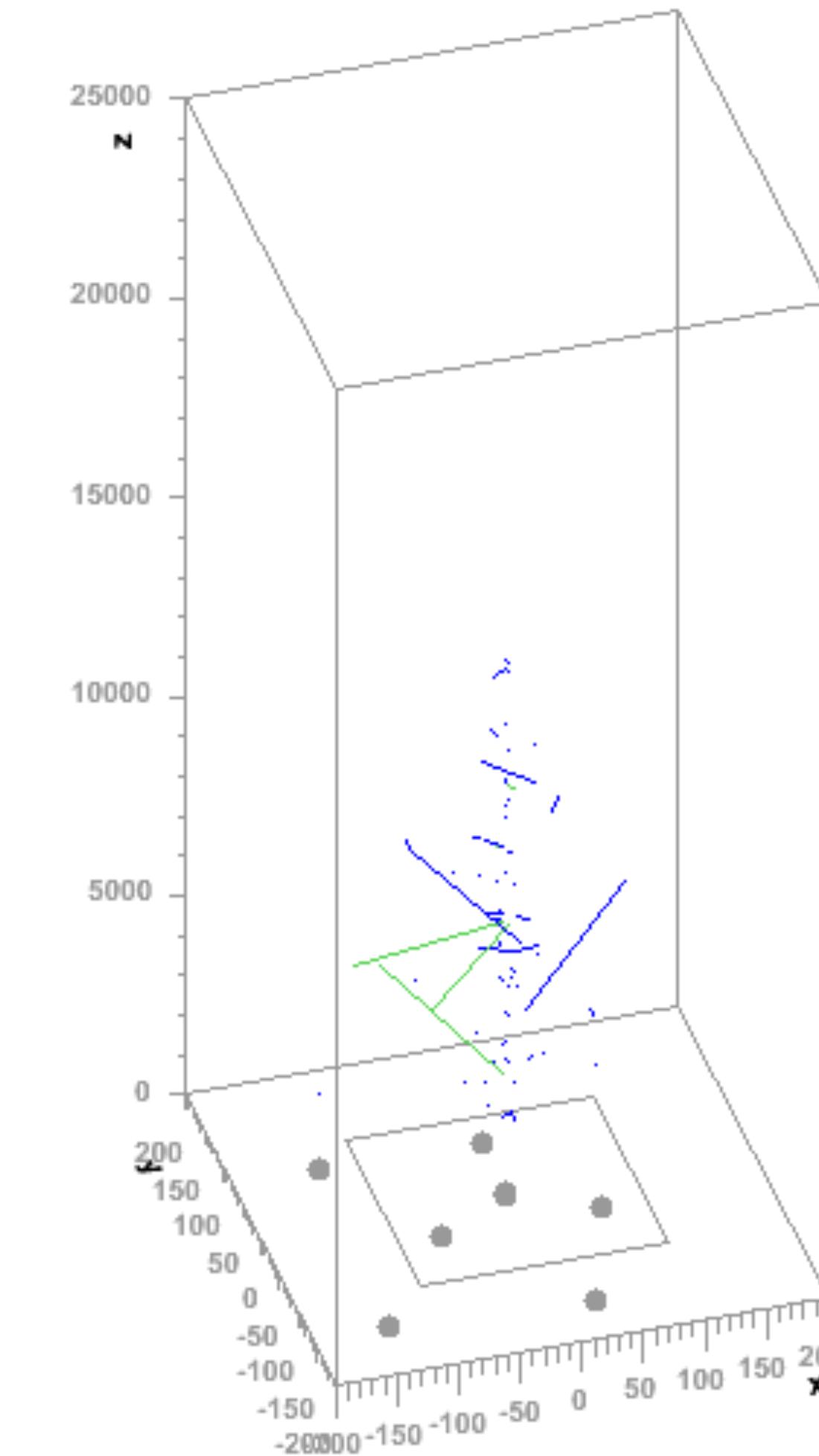
muons



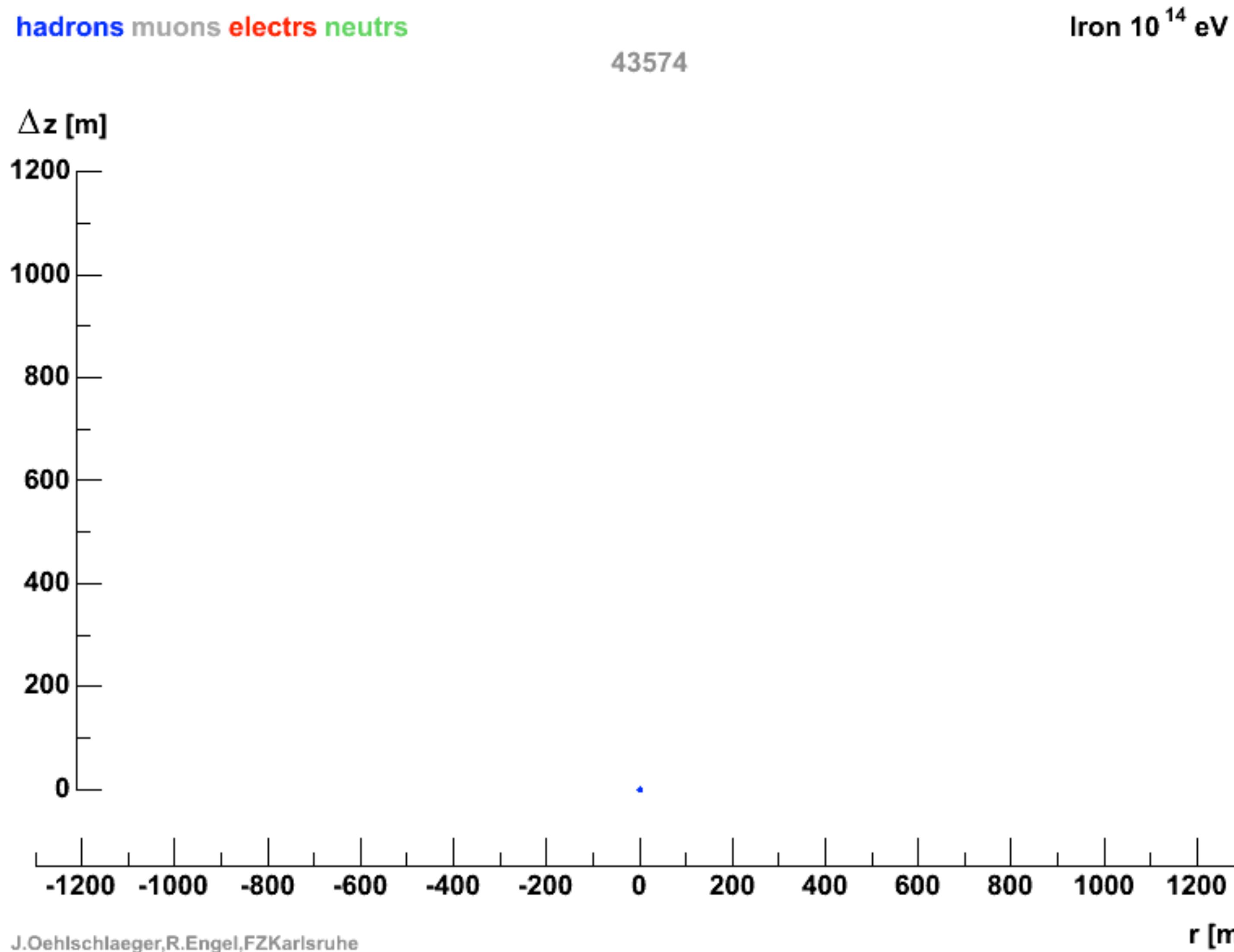
electrs



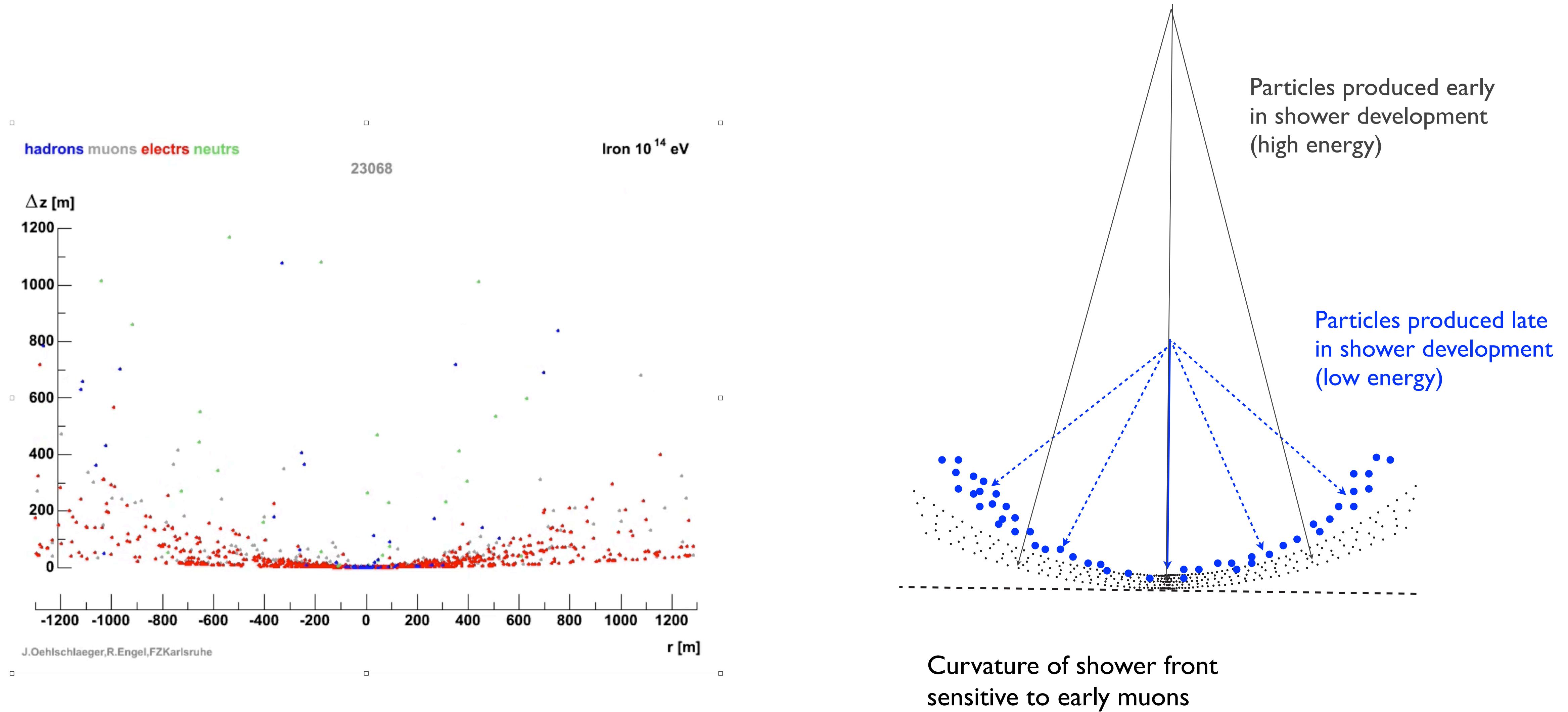
hadrons neutr



Time structure of shower disk



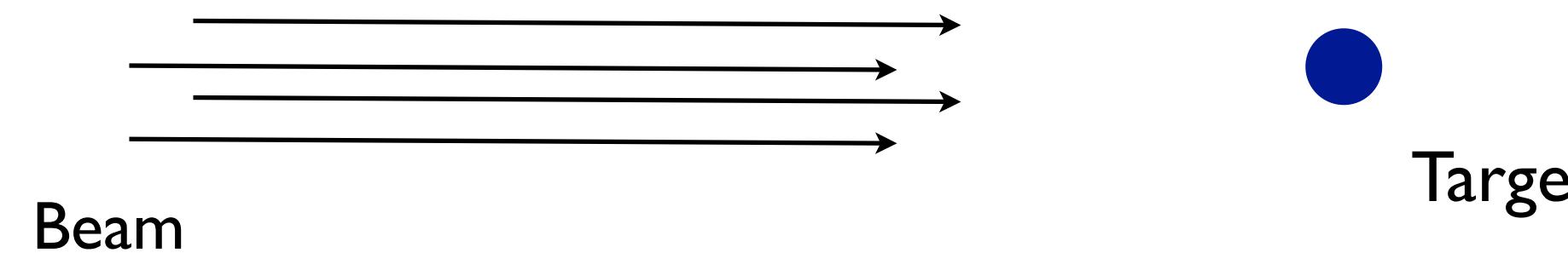
Time structure of shower disk



1. Basics

Cross section, interaction rate, interaction length

$$\Phi = \frac{dN_{\text{beam}}}{dA \ dt}$$



Definition

Flux of particles
on single target

$$\sigma = \frac{1}{\Phi} \frac{dN_{\text{int}}}{dt}$$

Interaction rate

(Units: 1 barn = 10^{-28} m^2
1 mb = 10^{-27} cm^2)

Interaction length

$$\lambda_{\text{int}} = \frac{\langle m_{\text{target}} \rangle}{\sigma}$$

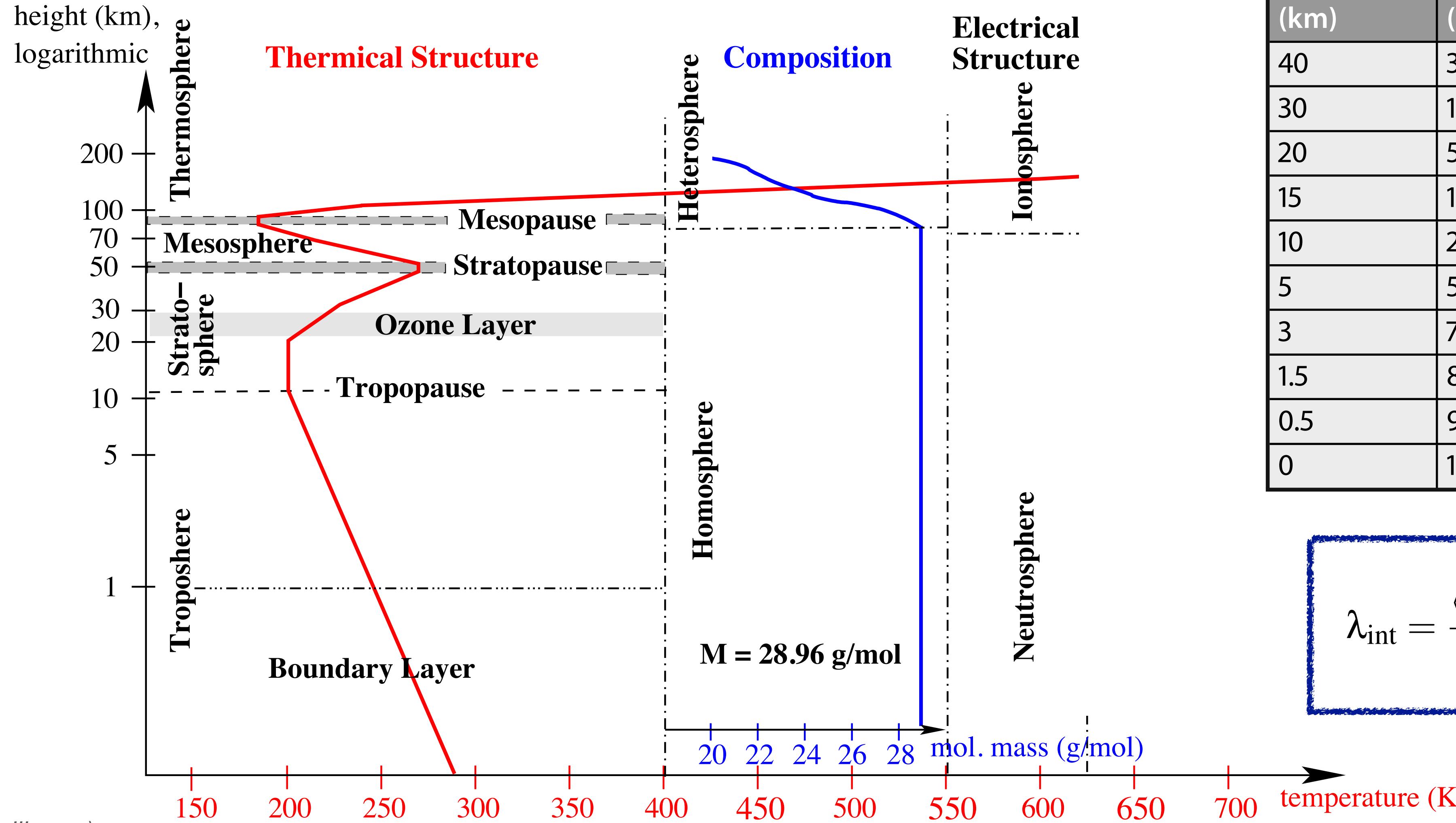
$$\frac{dN_{\text{int}}}{dt dV} = \frac{\rho_{\text{target}}}{\langle m_{\text{target}} \rangle} \sigma \Phi$$

$$dX = \rho_{\text{target}} dl$$

$$\frac{dN_{\text{int}}}{dt dV} = \frac{dN_{\text{int}}}{dl dt dA} = -\rho_{\text{target}} \frac{d\Phi}{dX}$$

$$\frac{d\Phi}{dX} = -\frac{\sigma}{\langle m_{\text{target}} \rangle} \Phi = -\frac{1}{\lambda_{\text{int}}} \Phi$$

Molecular atmosphere of Earth

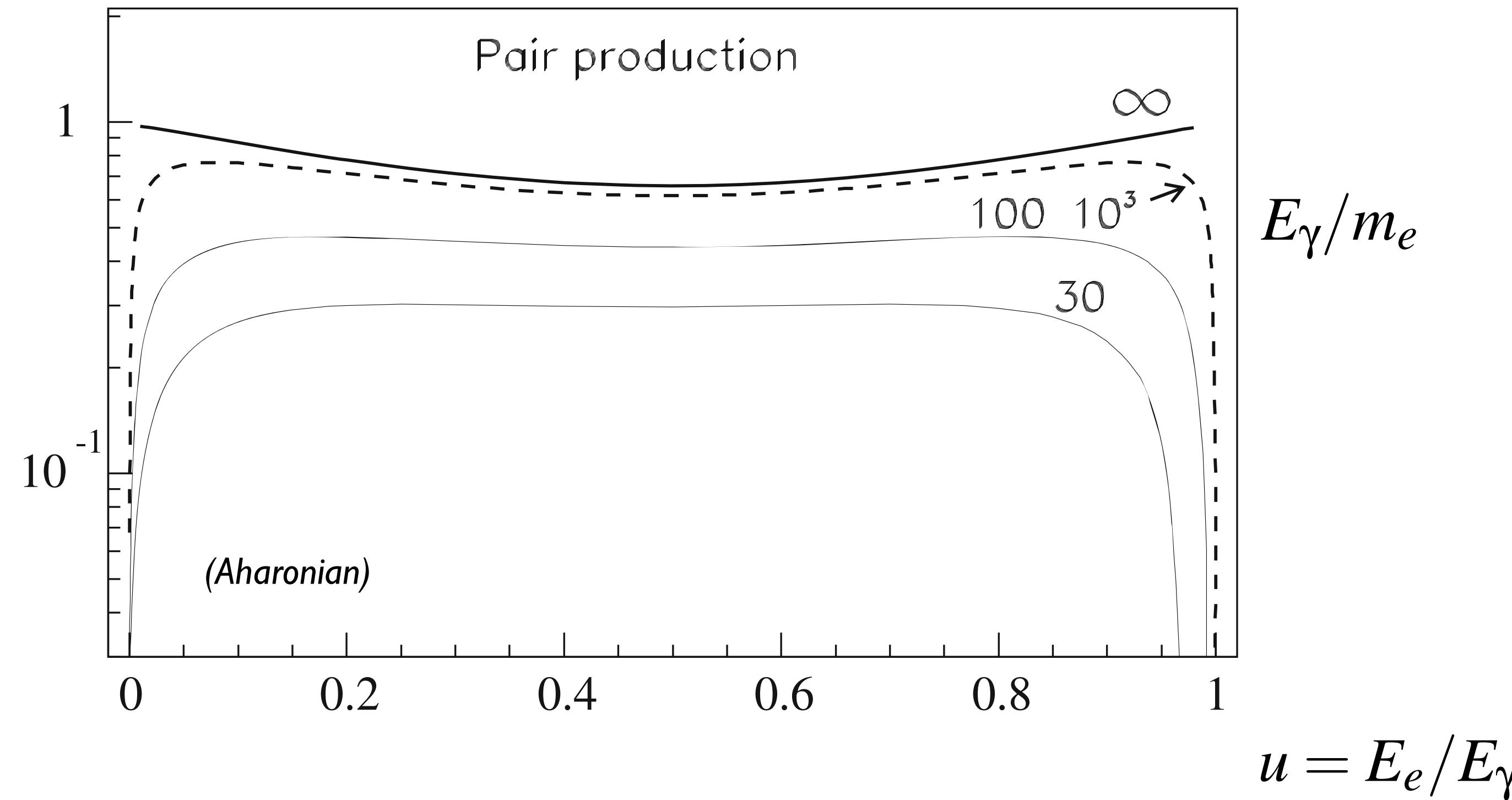


Altitude (km)	Vertical depth (g/cm^2)	Local density (10^{-3} g/cm^3)
40	3	3.8×10^{-3}
30	11.8	1.8×10^{-2}
20	55.8	8.8×10^{-2}
15	123	0.19
10	269	0.42
5	550	0.74
3	715	0.91
1.5	862	1.06
0.5	974	1.17
0	1,032	1.23

$$\lambda_{\text{int}} = \frac{\langle m_{\text{air}} \rangle}{\sigma_{\text{int}}} = \frac{24160 \text{ mb g/cm}^2}{\sigma_{\text{int}}}$$

2. Electromagnetic Showers

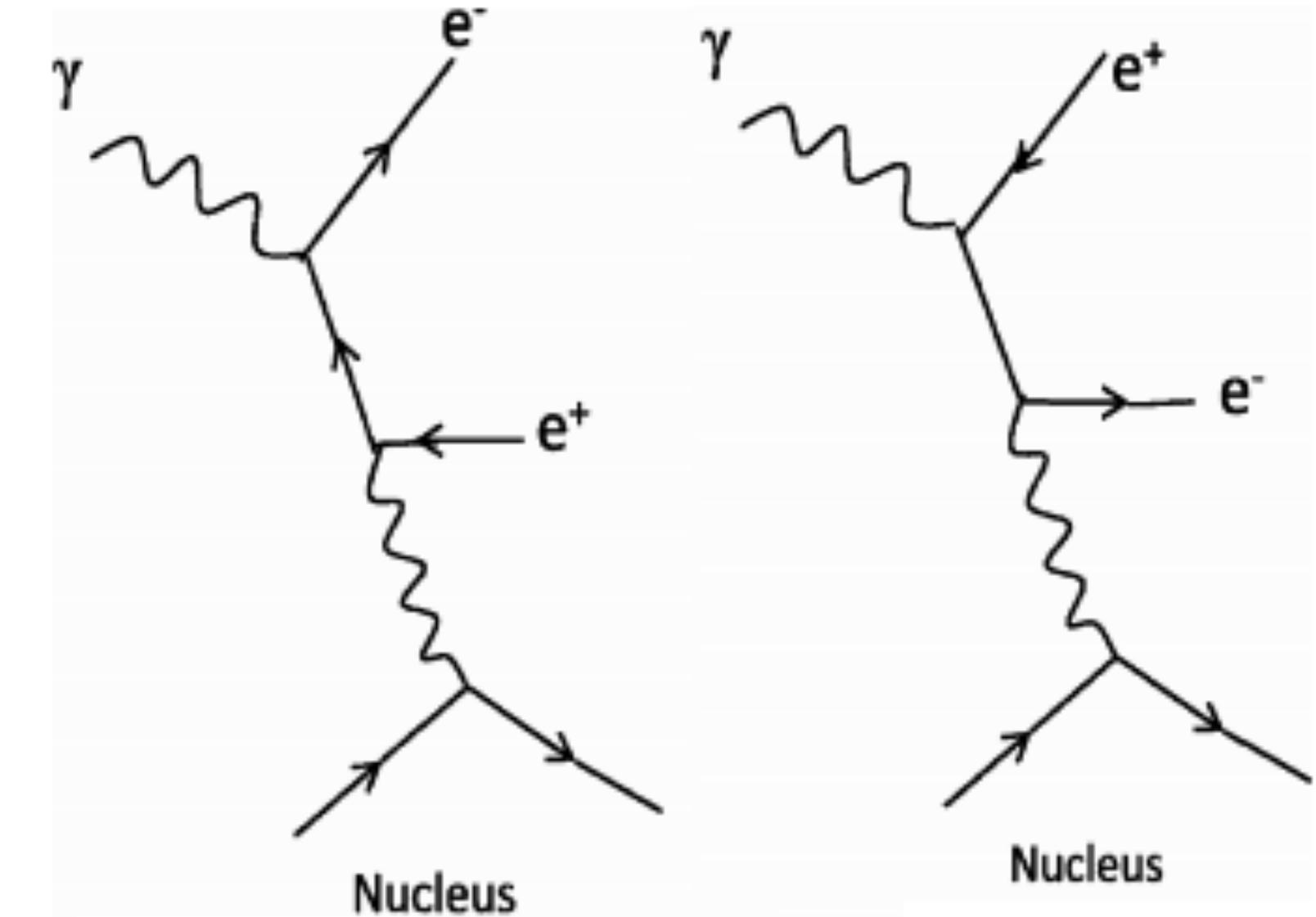
Bethe-Heitler pair production (i)



QED

$$\frac{d\sigma_{\text{pair}}}{du} = 4\alpha_{\text{em}} r_e^2 Z(Z+1) \left\{ \left[u^2 + (1-u)^2 + \frac{2}{3}u(1-u) \right] \ln(183Z^{-1/3}) - \frac{1}{9}u(1-u) \right\}$$

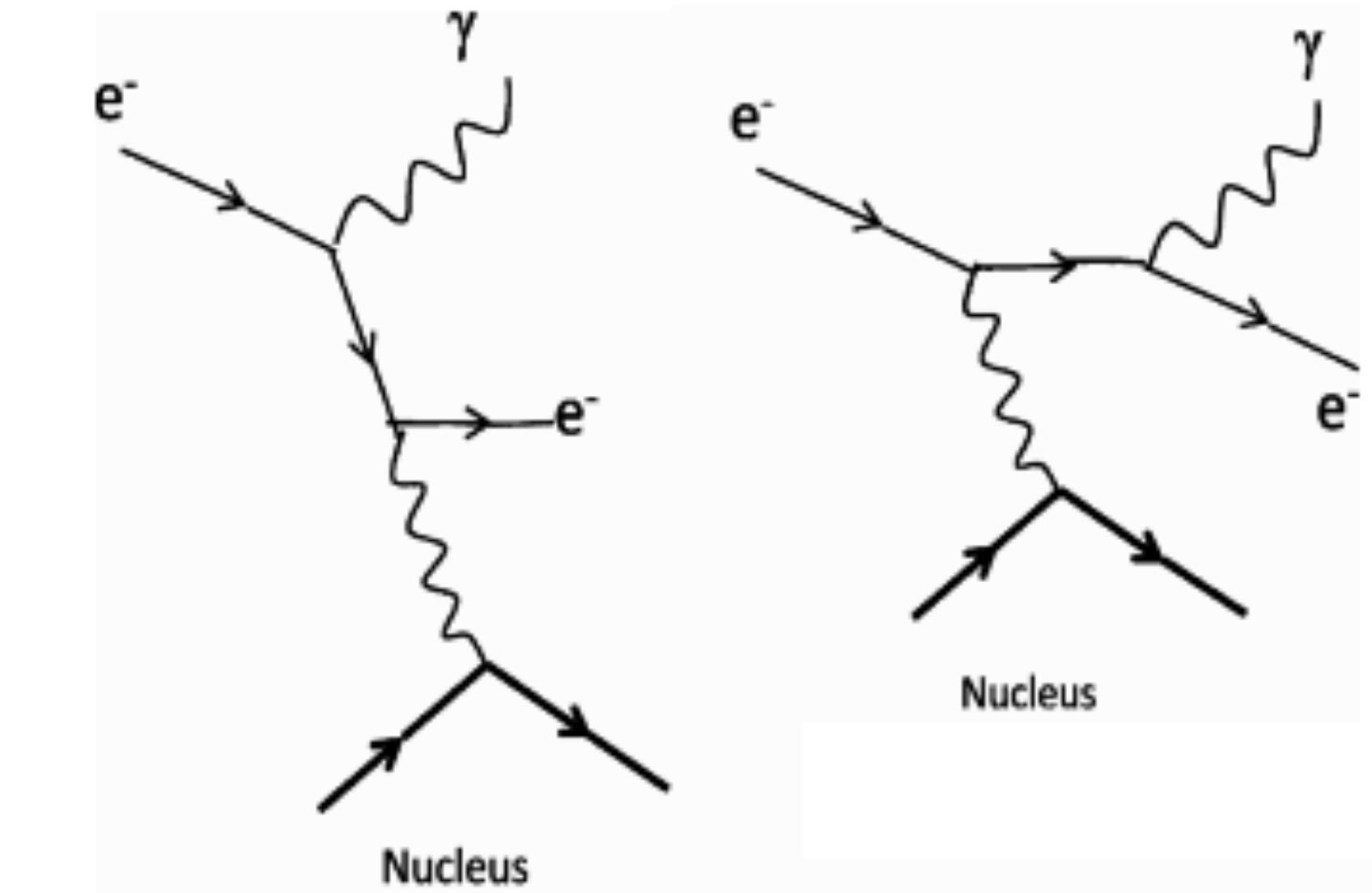
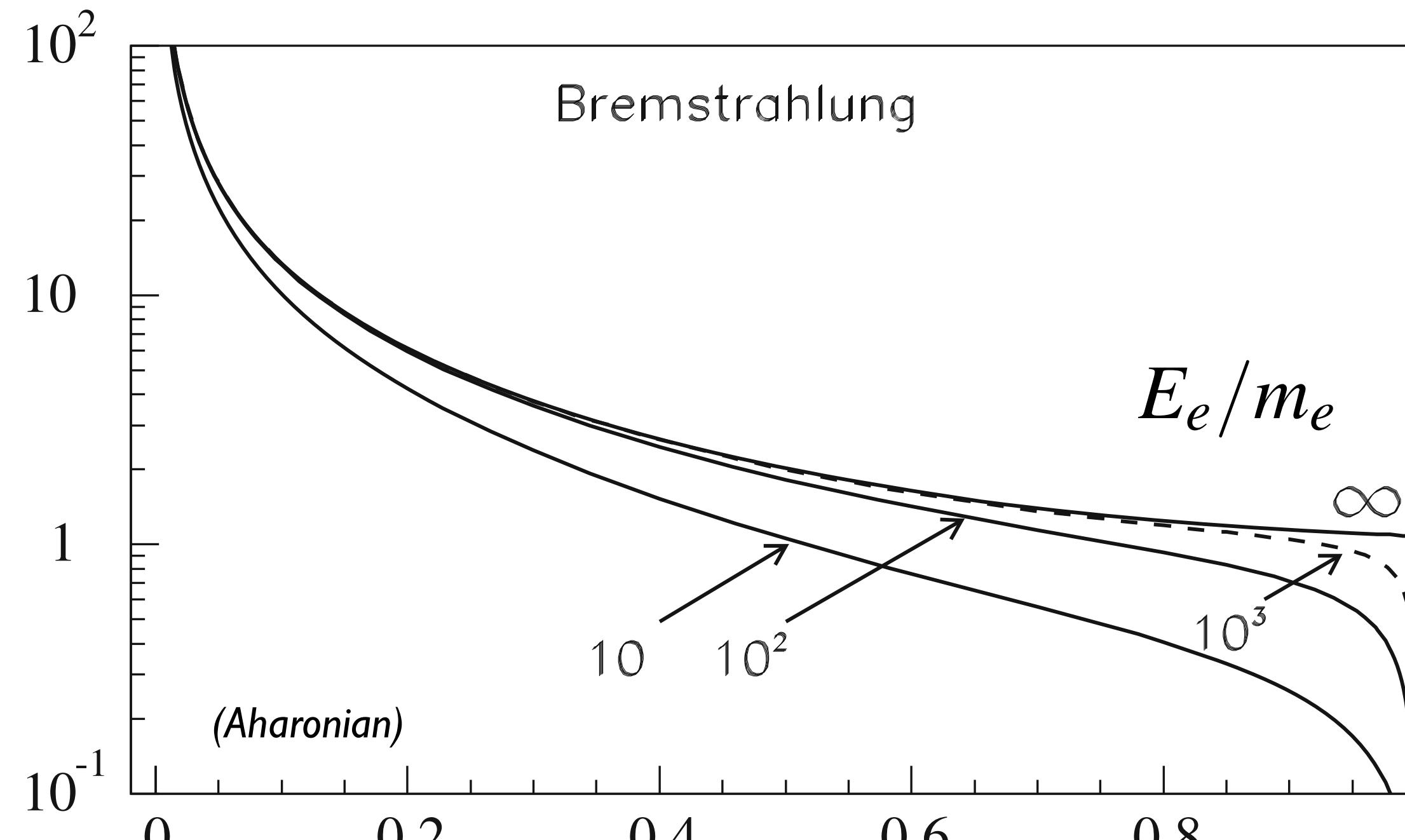
$$\sigma_{\text{pair,tot}} = \int \frac{d\sigma_{\text{pair}}}{du} du = 4\alpha_{\text{em}} r_e^2 Z(Z+1) \left[\frac{7}{9} \ln(183Z^{-1/3}) - \frac{1}{54} \right]$$



High-energy limit

$\sigma_{\text{pair,tot}} \sim 520 \text{ mb}$

Electron bremsstrahlung



QED

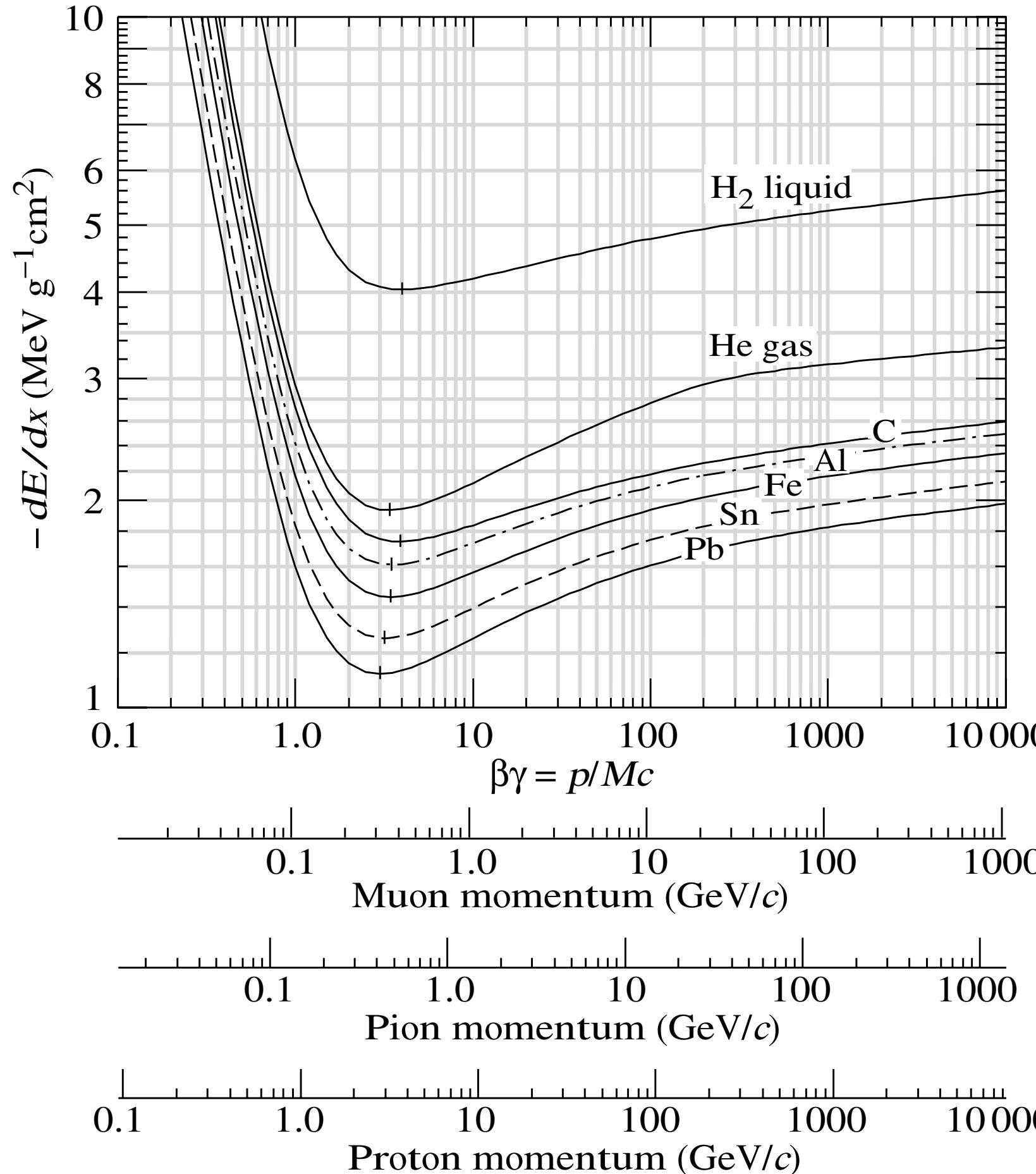
$$\frac{d\sigma_{\text{brem}}}{dv} = 4\alpha_{\text{em}} r_e^2 Z(Z+1) \frac{1}{v} \left\{ \left[1 + (1-v^2) - \frac{2}{3}(1-v) \right] \ln(183Z^{-1/3}) + \frac{1}{9}(1-v) \right\}$$

$$\sigma_{\text{brem,tot}} = \int \frac{d\sigma_{\text{brem}}}{dv} dv \rightarrow \infty$$

Cross section divergent (infrared catastrophe)

Ionization energy loss of charged particles

Ionization energy loss: Bethe-Bloch formula



$$-\frac{dE}{dx} = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 T_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$

Symbol	Definition	Units or Value
α	Fine structure constant $(e^2/4\pi\epsilon_0\hbar c)$	$1/137.035\,999\,11(46)$
M	Incident particle mass	MeV/c ²
E	Incident part. energy γMc^2	MeV
T	Kinetic energy	MeV
$m_e c^2$	Electron mass $\times c^2$	0.510 998 918(44) MeV
r_e	Classical electron radius $e^2/4\pi\epsilon_0 m_e c^2$	2.817 940 325(28) fm
N_A	Avogadro's number	$6.022\,1415(10) \times 10^{23}$ mol ⁻¹
ze	Charge of incident particle	
Z	Atomic number of absorber	
A	Atomic mass of absorber	g mol ⁻¹
K/A	$4\pi N_A r_e^2 m_e c^2 / A$	0.307 075 MeV g ⁻¹ cm ² for A = 1 g mol ⁻¹
I	Mean excitation energy	eV (<i>Nota bene!</i>)
$\delta(\beta\gamma)$	Density effect correction to ionization energy loss	

Total energy loss of charged particles

Ionization energy loss: Bethe-Bloch formula

Radiation energy loss: bremsstrahlung

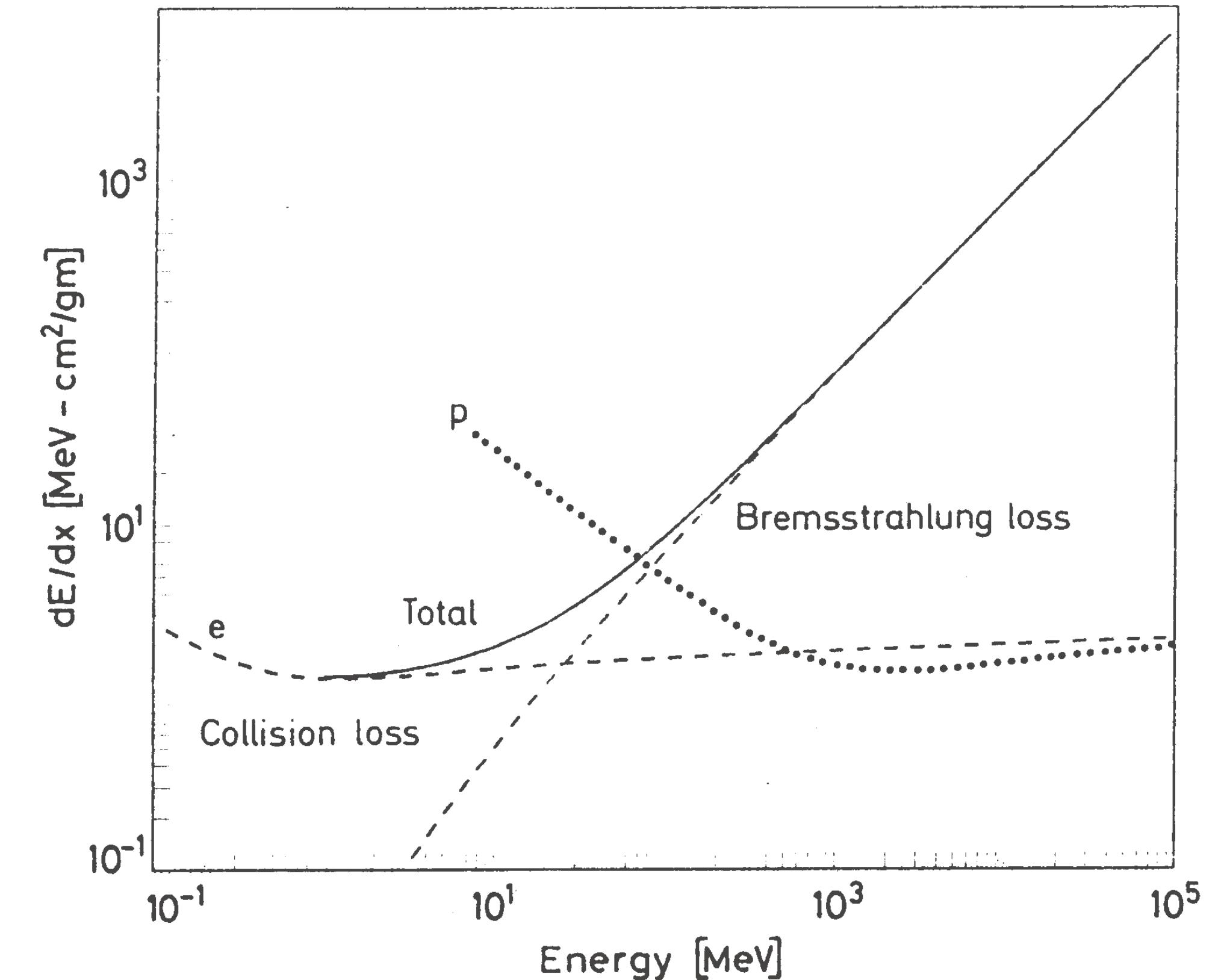
$$\int v \frac{d\sigma_{\text{brem}}}{dv} dv = 4\alpha_{\text{em}} r_e^2 Z(Z+1) \left[\ln(183Z^{-1/3}) + \frac{1}{18} \right] = \frac{\langle m_{\text{target}} \rangle}{X_0}$$

Radiation length X_0

$$X_0 \sim 36 \text{ g/cm}^2$$

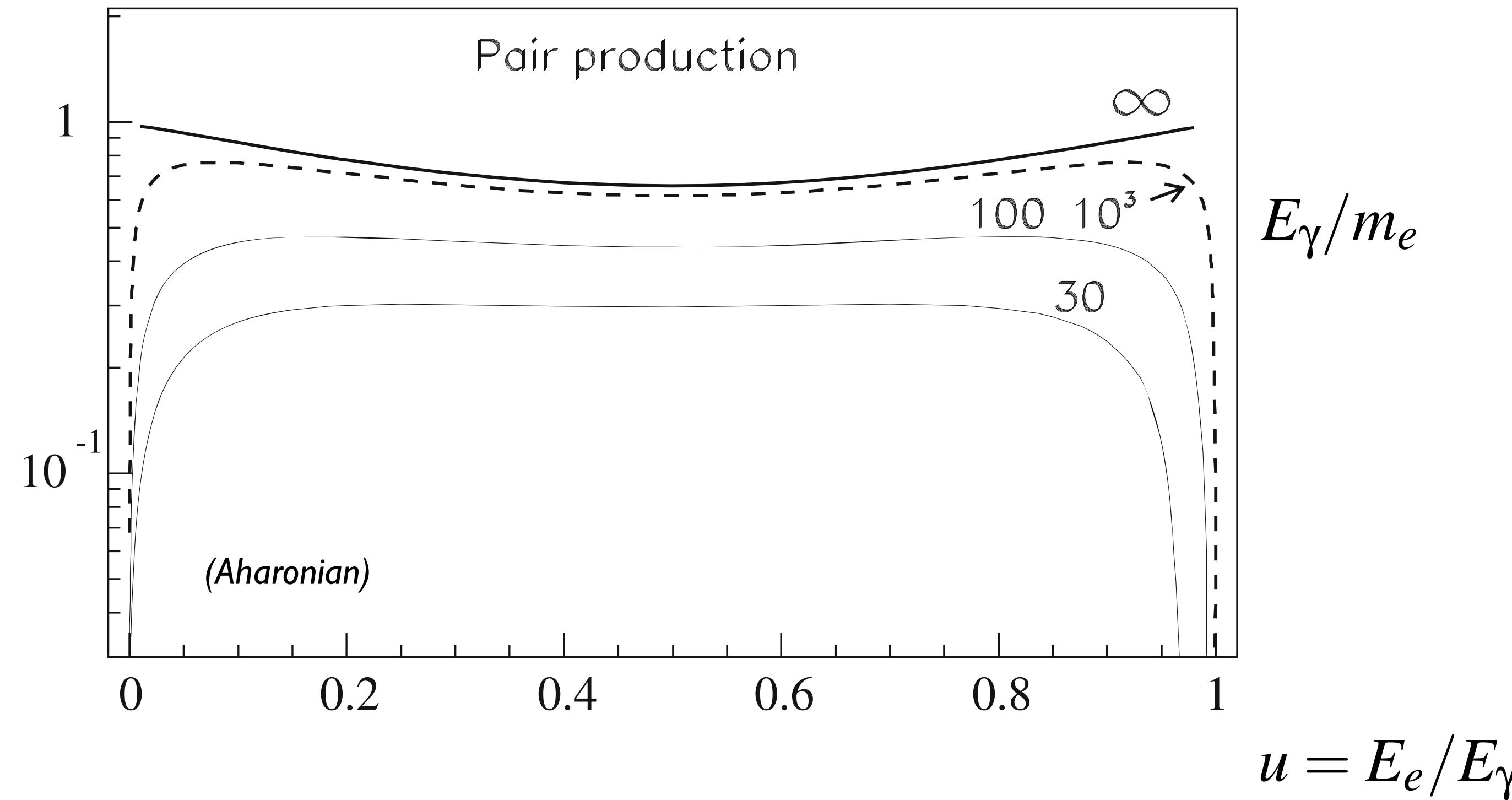
$$\frac{dE}{dX} = -\alpha(E) - \frac{E}{X_0}$$

Critical energy E_c defined as
energy at which both losses are equal



$$E_c = \alpha X_0 \sim 85 \text{ MeV}$$

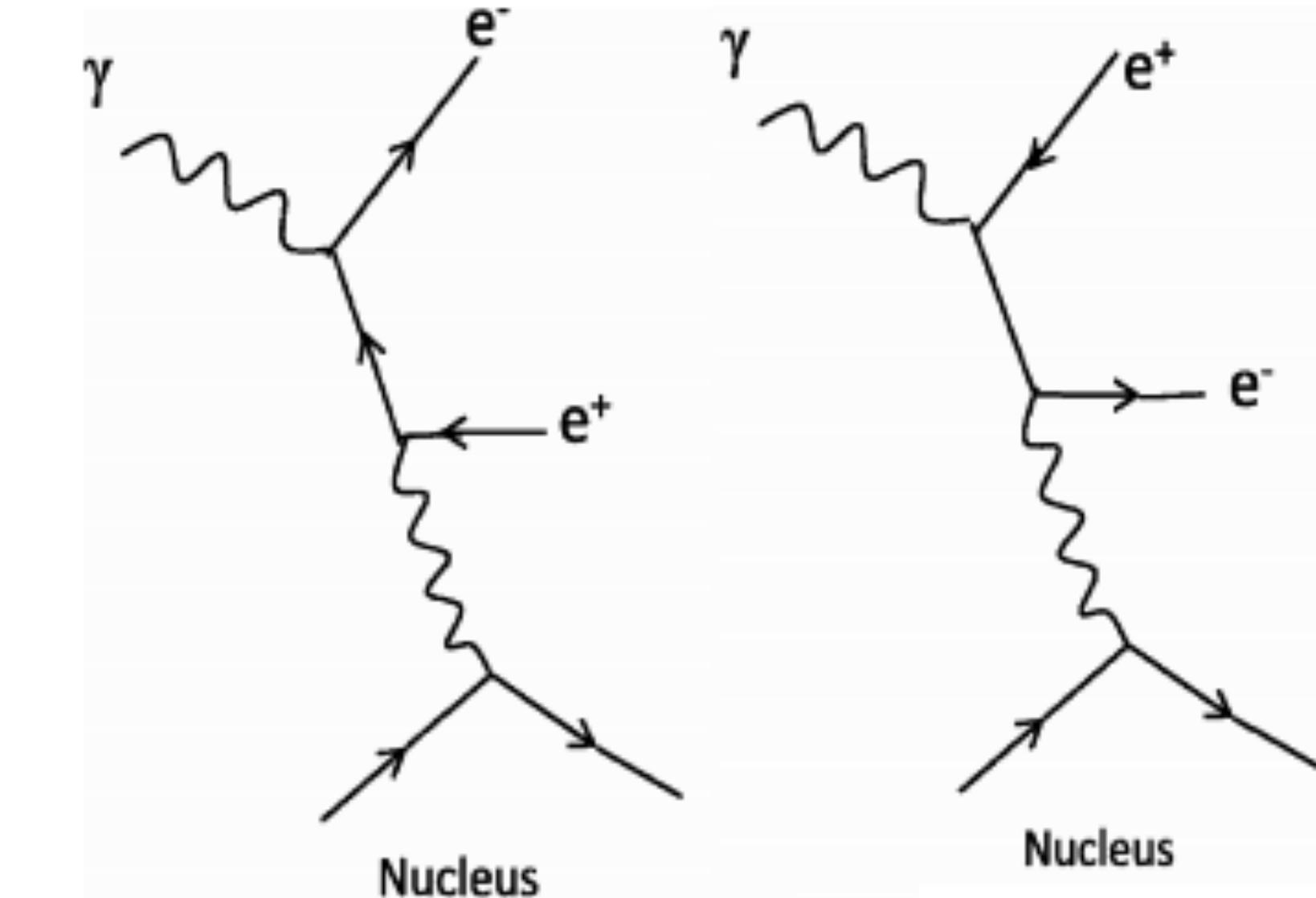
Bethe-Heitler pair production (ii)



QED

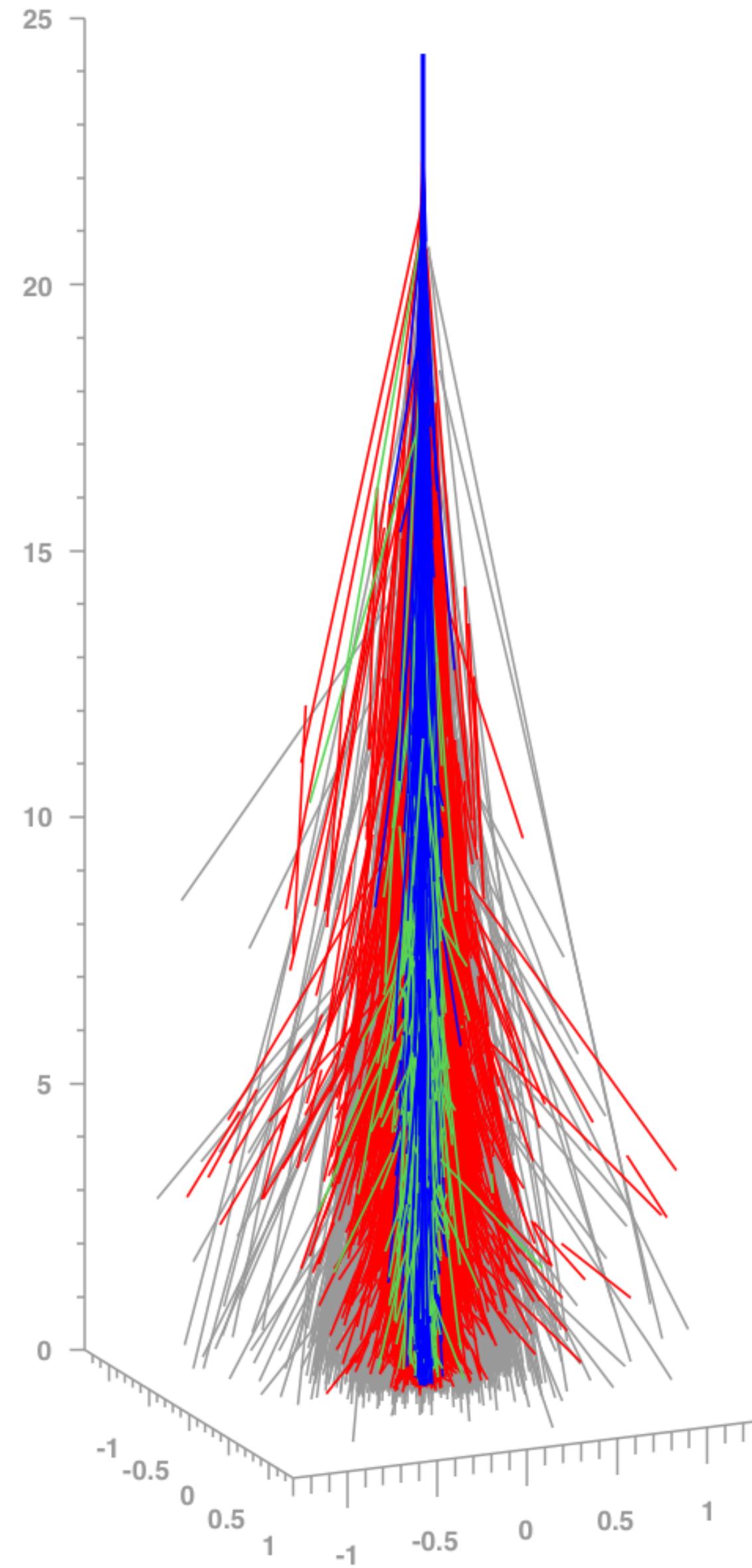
High-energy limit

$$\sigma_{\text{pair,tot}} \sim 520 \text{ mb}$$



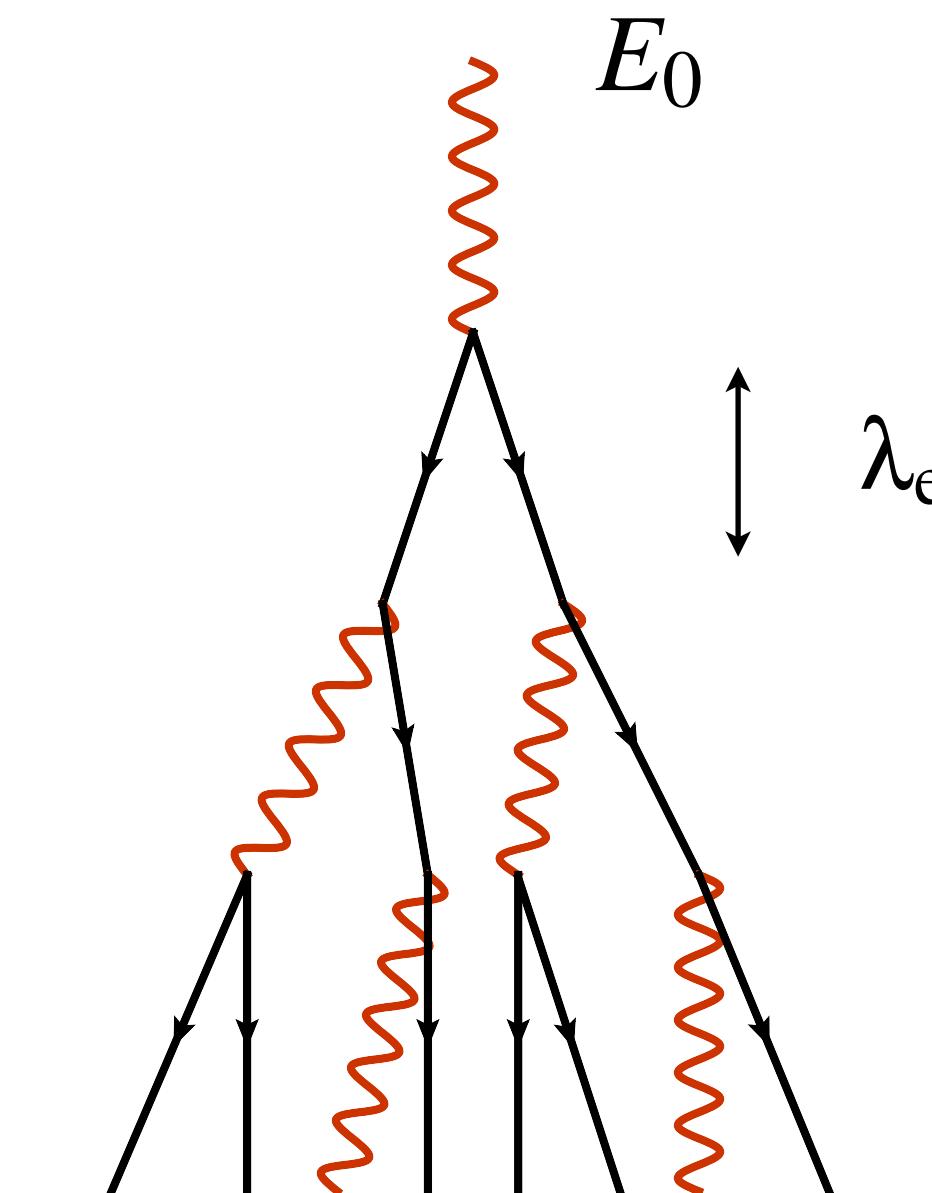
$$\lambda_{\text{pair}} = \frac{\langle m_{\text{air}} \rangle}{\sigma_{\text{pair,tot}}} = \frac{9}{7} X_0$$

Qualitative approach: Heitler model



Number of charged particles

Depth X (g/cm^2)



Shower maximum: $E = E_c$

$$N_{\max} = E_0/E_c$$

$$X_{\max} \sim \lambda_{\text{em}} \ln(E_0/E_c)$$

Cascade equations

Energy loss
of electron:

$$\frac{dE}{dX} = -\alpha - \frac{E}{X_0}$$

Critical energy: $E_c = \alpha X_0 \sim 85 \text{ MeV}$

Radiation length: $X_0 \sim 36 \text{ g/cm}^2$

Cascade equations

$$\frac{d\Phi_e(E)}{dX} = -\frac{\sigma_e}{\langle m_{\text{air}} \rangle} \Phi_e(E) + \int_E^\infty \frac{\sigma_e}{\langle m_{\text{air}} \rangle} \Phi_e(\tilde{E}) P_{e \rightarrow e}(\tilde{E}, E) d\tilde{E}$$

$$+ \int_E^\infty \frac{\sigma_\gamma}{\langle m_{\text{air}} \rangle} \Phi_\gamma(\tilde{E}) P_{\gamma \rightarrow e}(\tilde{E}, E) d\tilde{E} + \alpha \frac{\partial \Phi_e(E)}{\partial E}$$

$$X_{\max} \approx X_0 \ln \left(\frac{E_0}{E_c} \right)$$

$$N_{\max} \approx \frac{0.31}{\sqrt{\ln(E_0/E_c) - 0.33}} \frac{E_0}{E_c}$$

Shower age and Greisen formula

Longitudinal profile

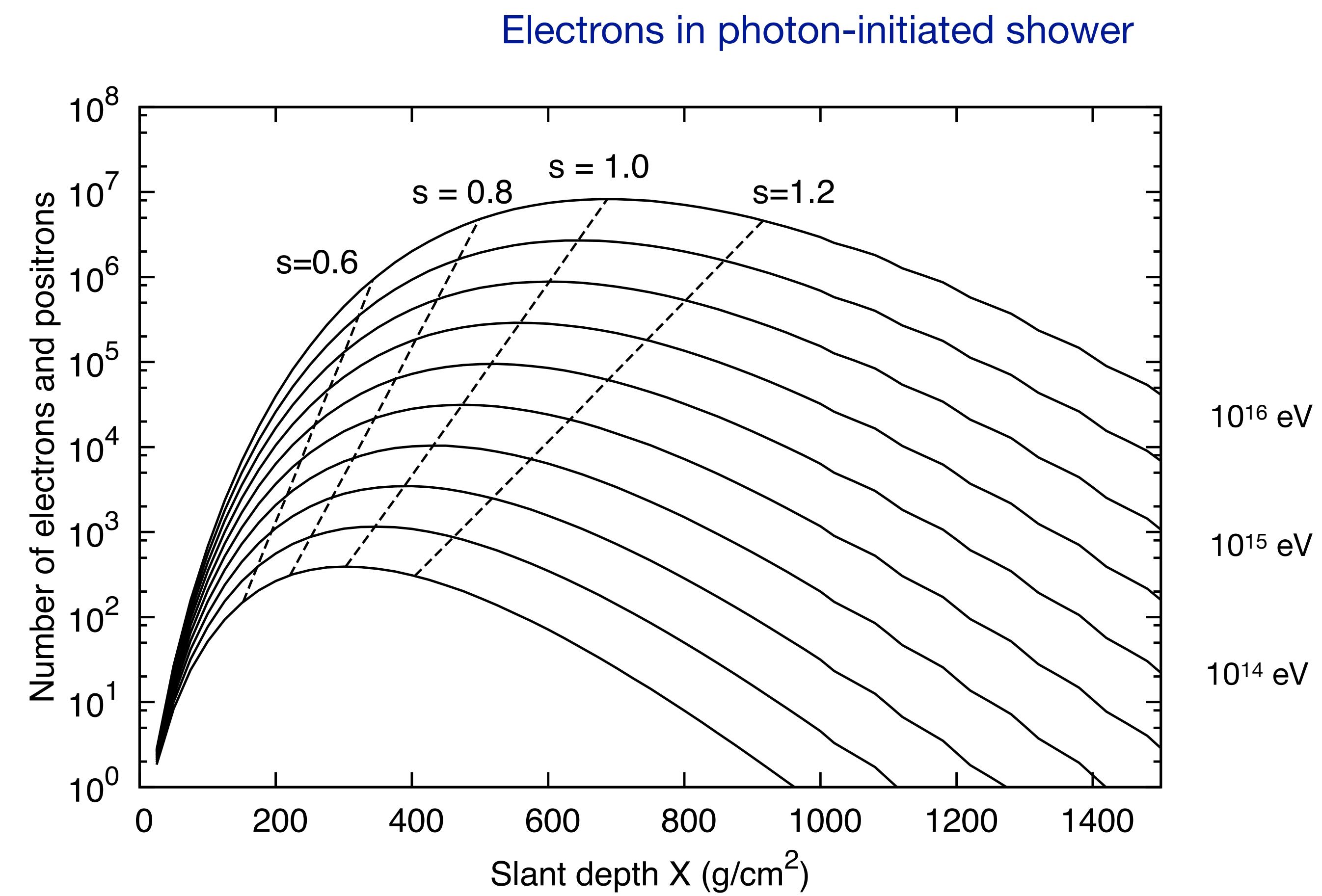
$$N_e(X) \approx \frac{0.31}{[\ln E_0/E_c]^{1/2}} \exp \left\{ \frac{X}{X_0} \left(1 - \frac{3}{2} \ln s \right) \right\}$$

Shower age

$$s = \frac{3X}{X + 2X_{\max}}$$

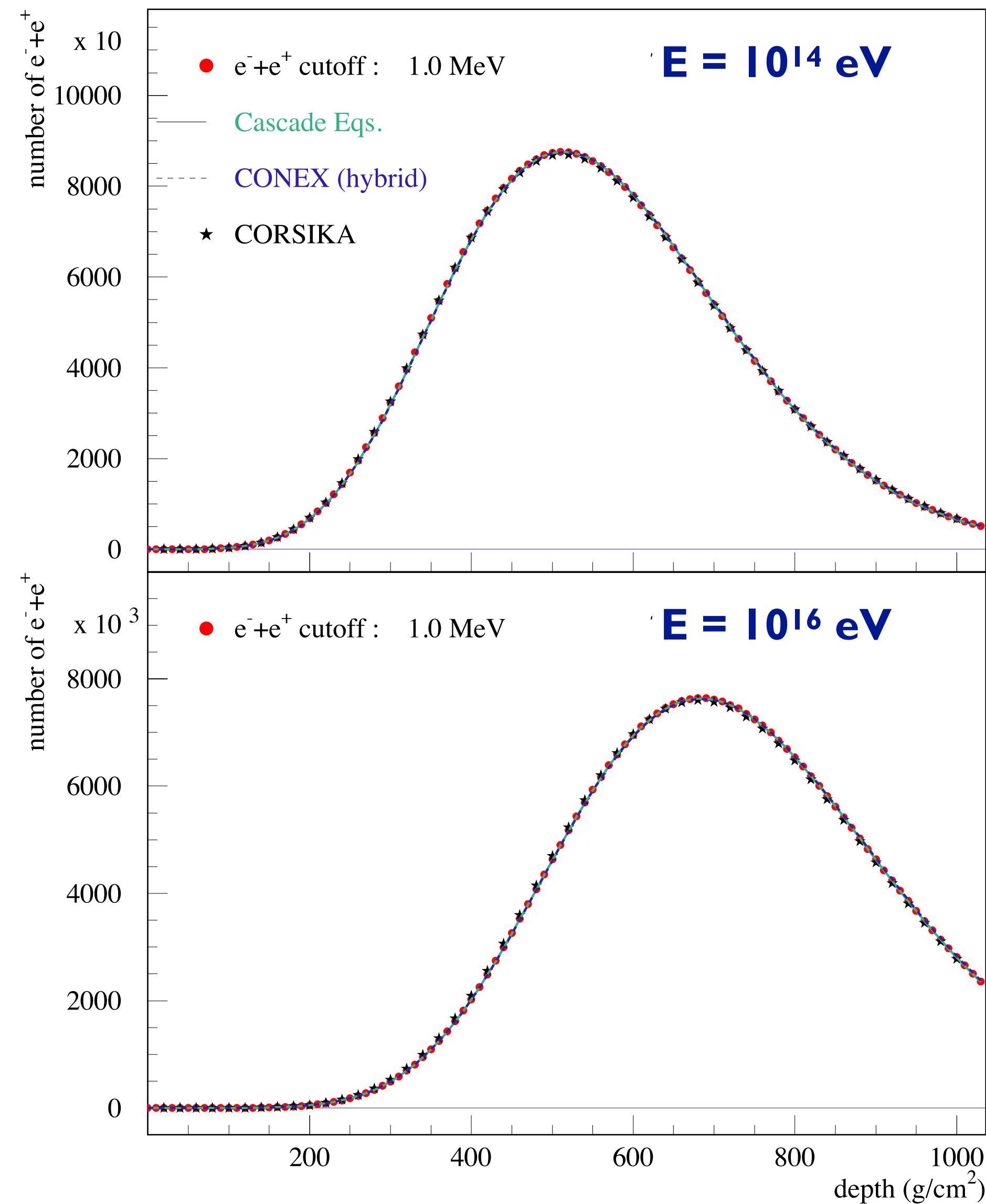
Energy spectrum particles

$$\frac{dN_e}{dE} \sim \frac{1}{E^{1+s}}$$



(Greisen 1956, see also Lipari PRD 2009)

Mean longitudinal shower profile



Calculation with cascade Eqs.

Photons

- Pair production
- Compton scattering

Electrons

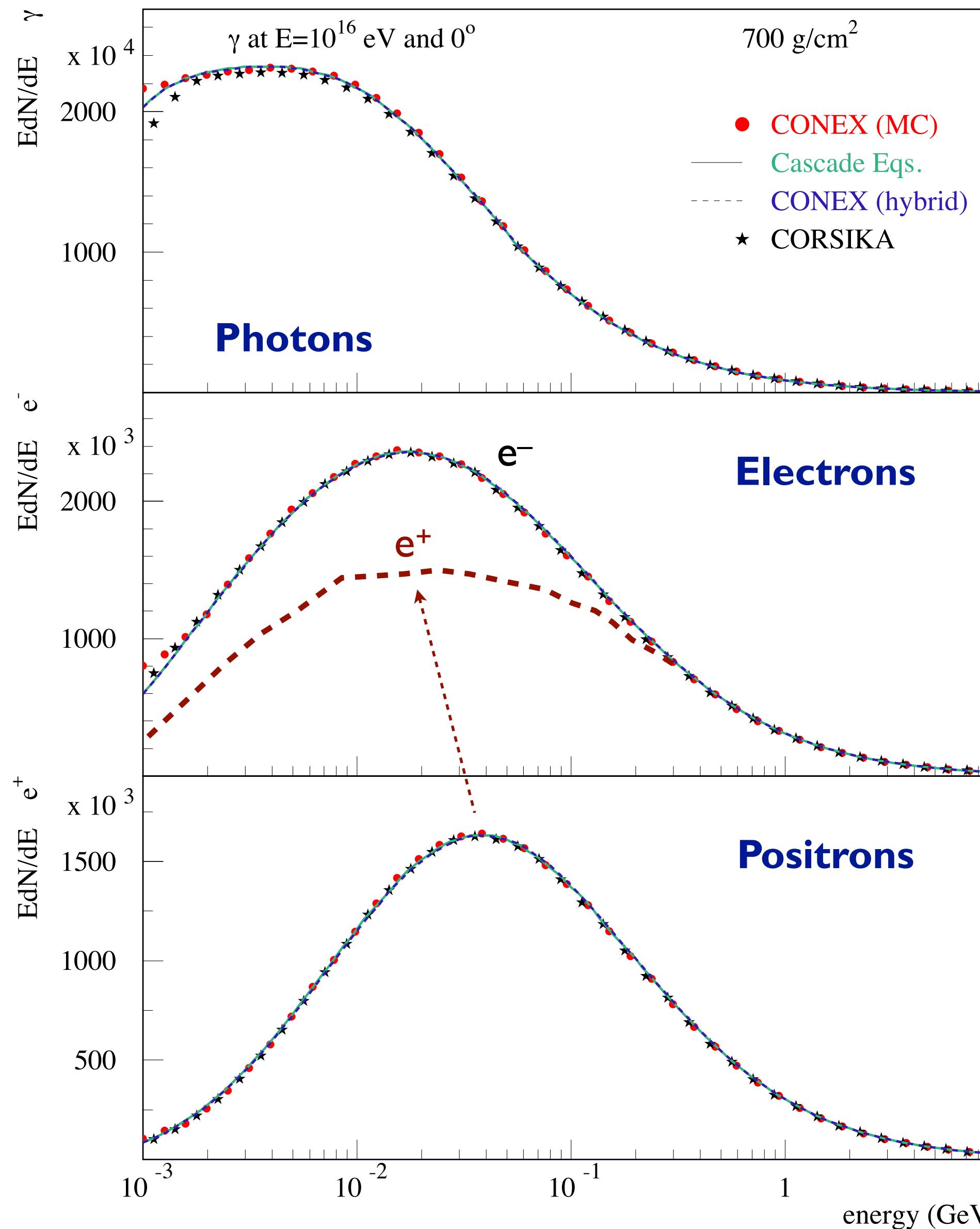
- Bremsstrahlung
- Moller scattering

Positrons

- Bremsstrahlung
- Bhabha scattering

(Bergmann et al., Astropart.Phys. 26 (2007) 420)

Energy spectra of secondary particles



Number of photons divergent,
energy threshold applied in calculation

- Typical energy of electrons and positrons $E_c \sim 80$ MeV
- Electron excess of 20 - 30%
- Pair production symmetric
- Excess of electrons in target

Lateral distribution of shower particles

Coulomb scattering

$$\frac{dN}{d\Omega} = \frac{1}{64\pi} \frac{1}{\ln(191Z^{-1/3})} \left(\frac{E_s}{E}\right)^2 \frac{1}{\sin^4 \theta/2} \quad E_s \approx 21 \text{ MeV}$$

Expectation value

$$\int \theta^2 \frac{dN}{d\Omega} d\Omega$$

$$\langle \theta^2 \rangle \sim \left(\frac{E_s}{E}\right)^2$$

Displacement of particle

$$r \sim \left(\frac{E_s}{E}\right) \frac{X_0}{\rho_{\text{air}}}$$

$$r_1 = r_M = \left(\frac{E_s}{E_c}\right) \frac{X_0}{\rho_{\text{air}}}$$

$$\frac{dN_e}{dE} \sim \frac{E_c}{E^{1+s}}$$

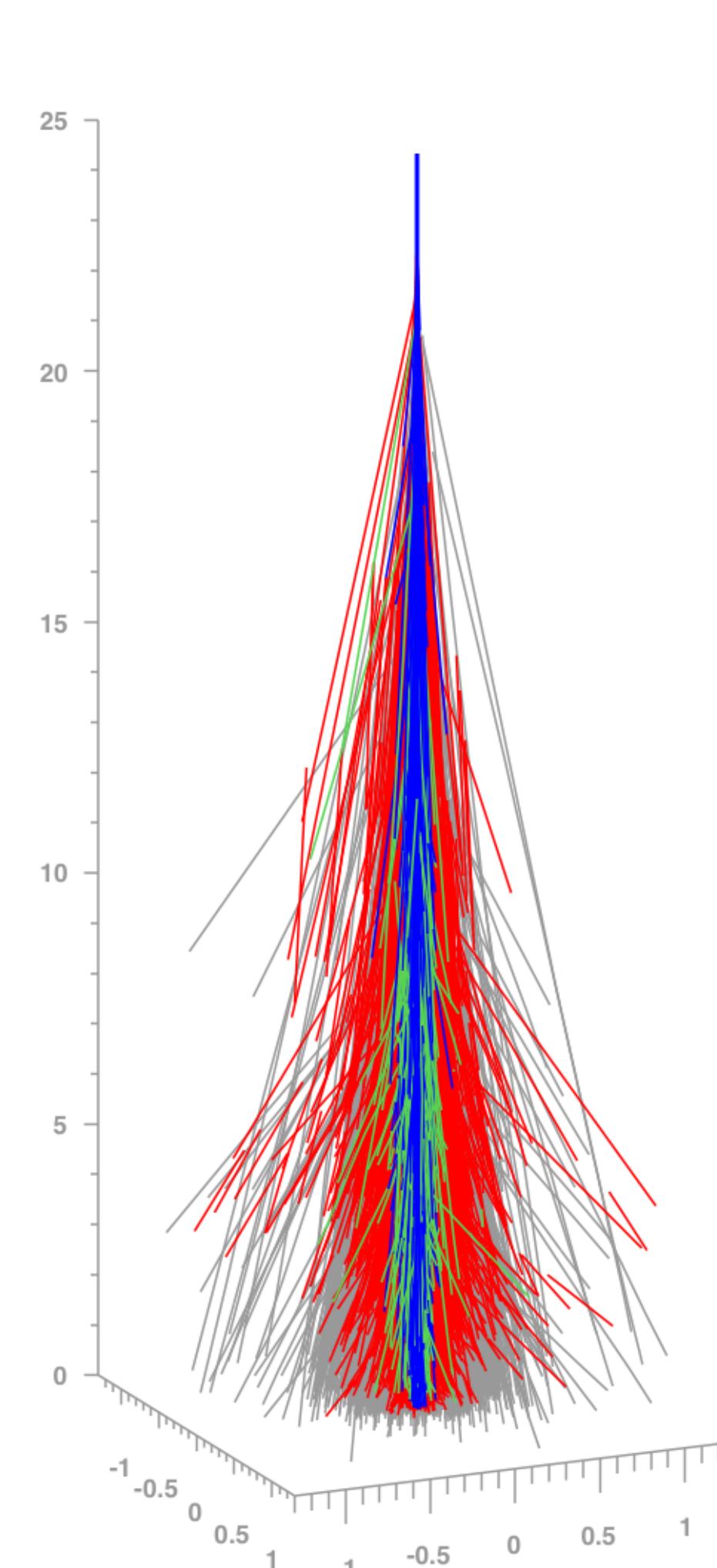
$$\frac{dN_e}{r dr} \sim \left(\frac{r}{r_1}\right)^{s-2} \left(1 + \frac{r}{r_1}\right)^{s-4.5}$$

**Moliere unit
(78 m at sea level)**

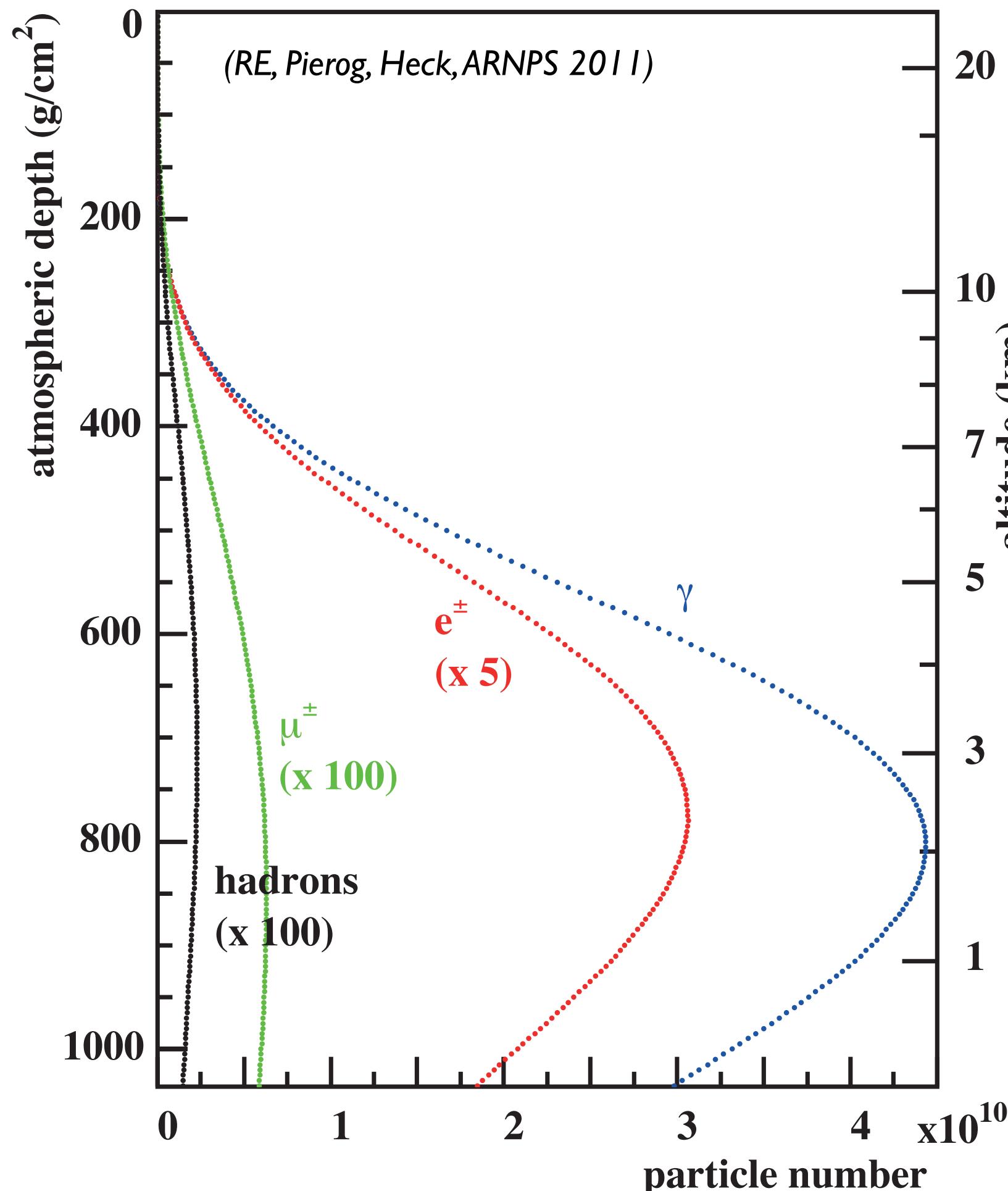
**Nishimura-Kamata-Greisen (NKG)
lateral distribution function**

Hadronic showers

Expectation from simulations



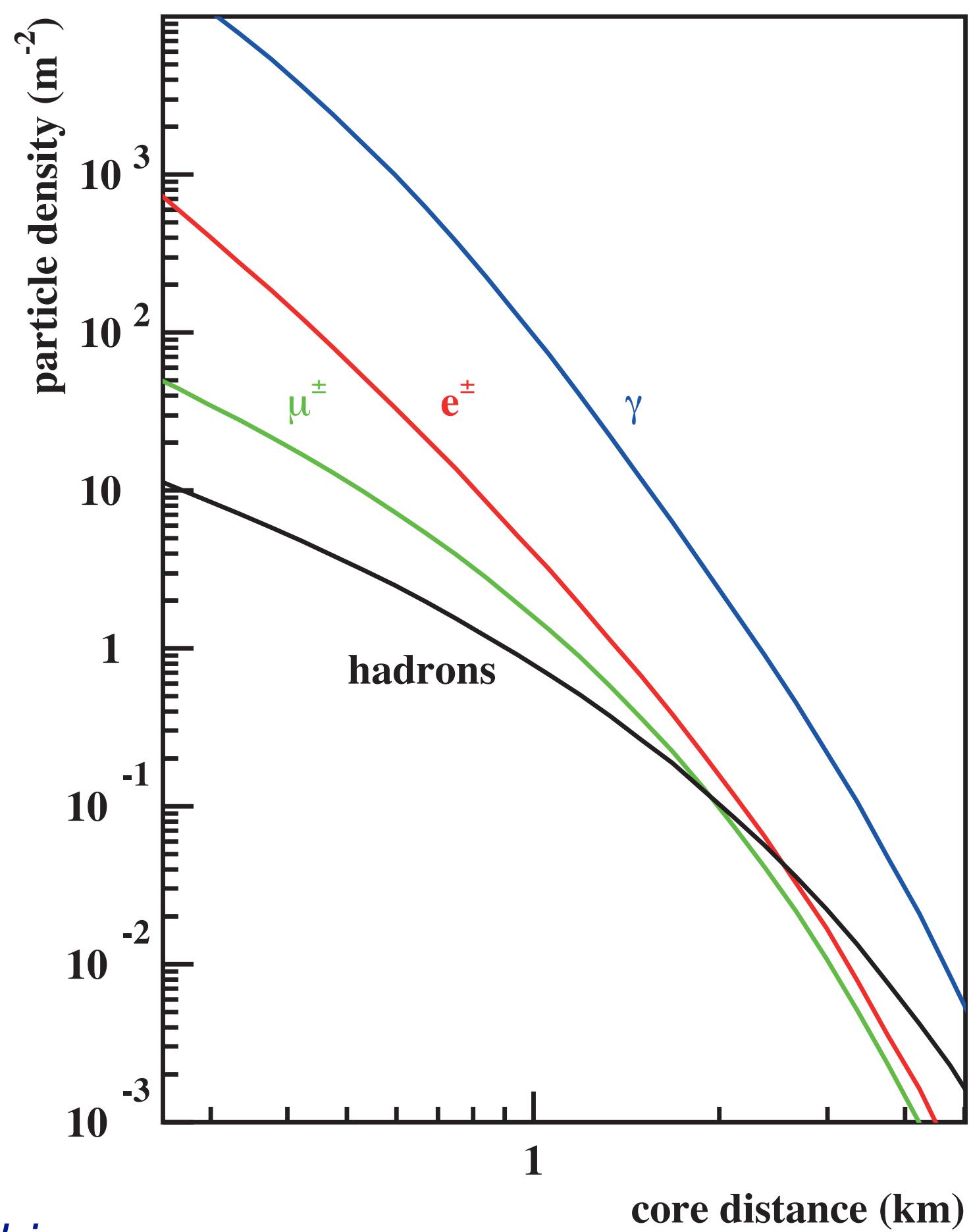
Longitudinal profile:
Cherenkov light
Fluorescence light
(bulk of particles measured)



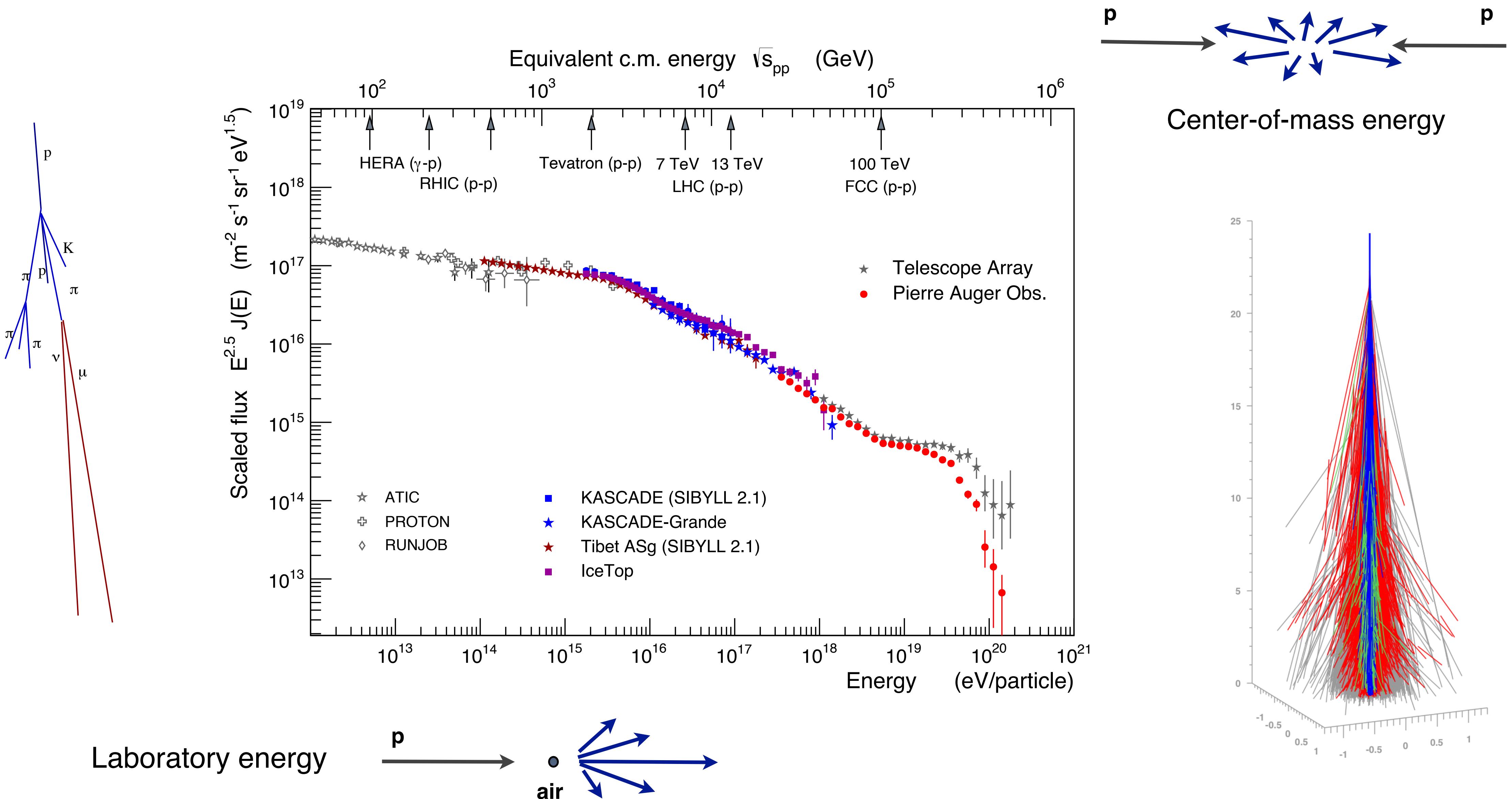
(RE, Pierog, Heck, ARNPS 2011)

See talk by Piera Ghia

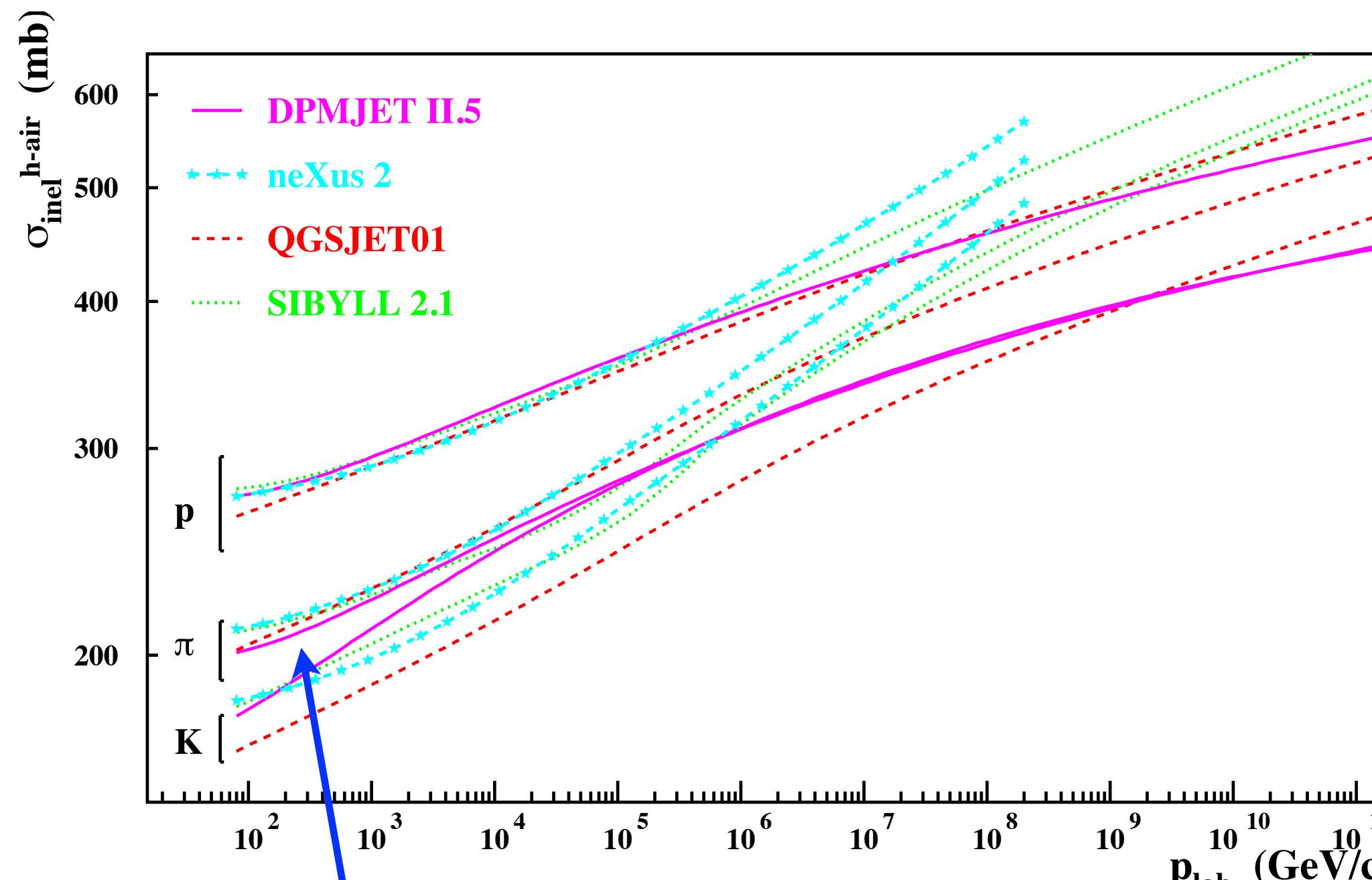
Lateral profiles:
particle detectors at ground
(very small fraction of particles sampled)



Cosmic ray flux and interaction energies



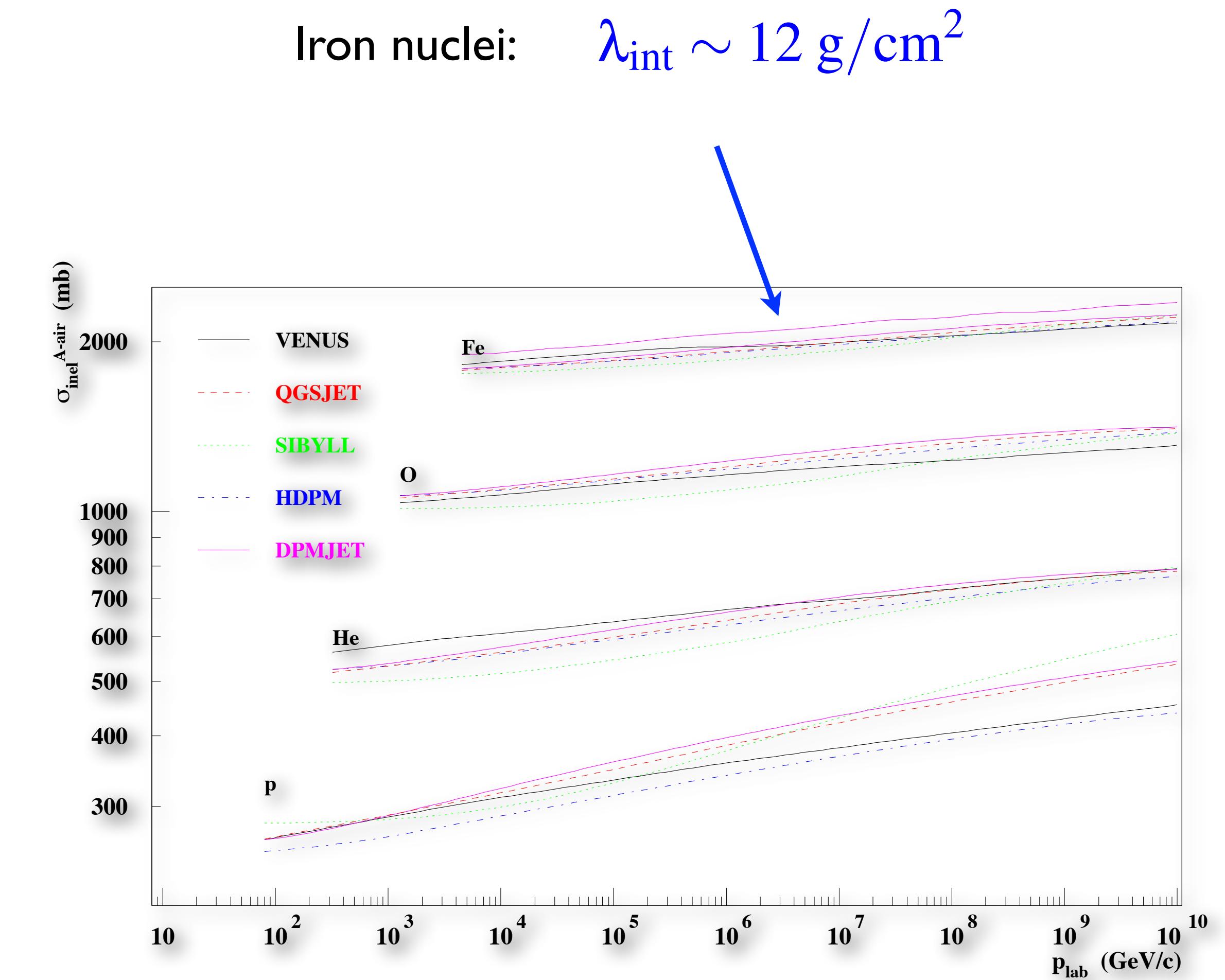
Interaction cross sections: mesons and nuclei



Pions: $\lambda_{\text{int}} \sim 120 \text{ g/cm}^2$

Protons: $\lambda_{\text{int}} \sim 75 \text{ g/cm}^2$

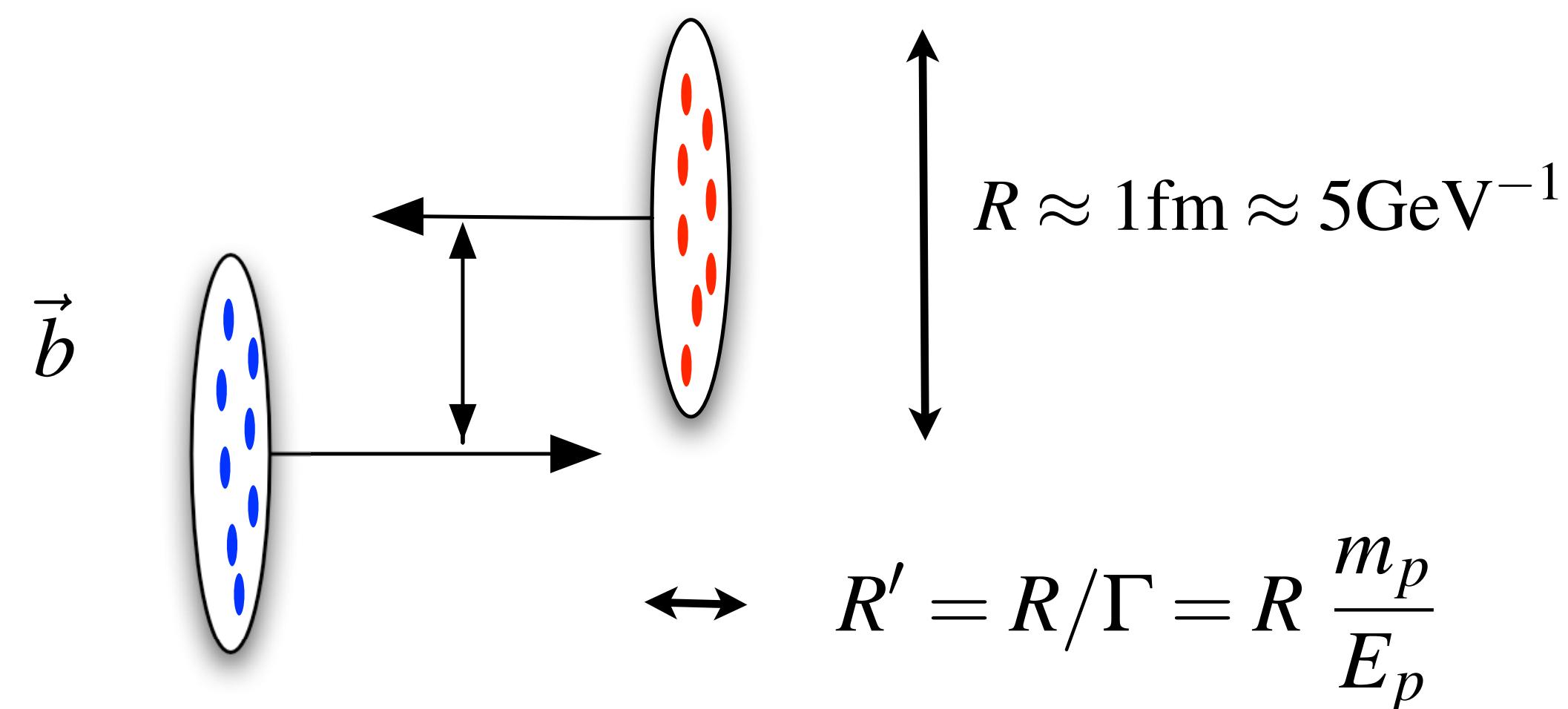
Photons: $\lambda_{\gamma,\text{pair}} = \frac{9}{7} X_0 \sim 50 \text{ g/cm}^2$



Expectations from uncertainty relation

Assumptions:

- hadrons built up of partons
- partons deflected/liberated in collision process, small momentum
- partons fragment into hadrons (pions, kaons,...) after interaction
- interaction viewed in c.m. system (other systems equally possible)



Heisenberg uncertainty relation

$$\Delta x \Delta p_x \simeq 1$$

Longitudinal momenta of secondaries

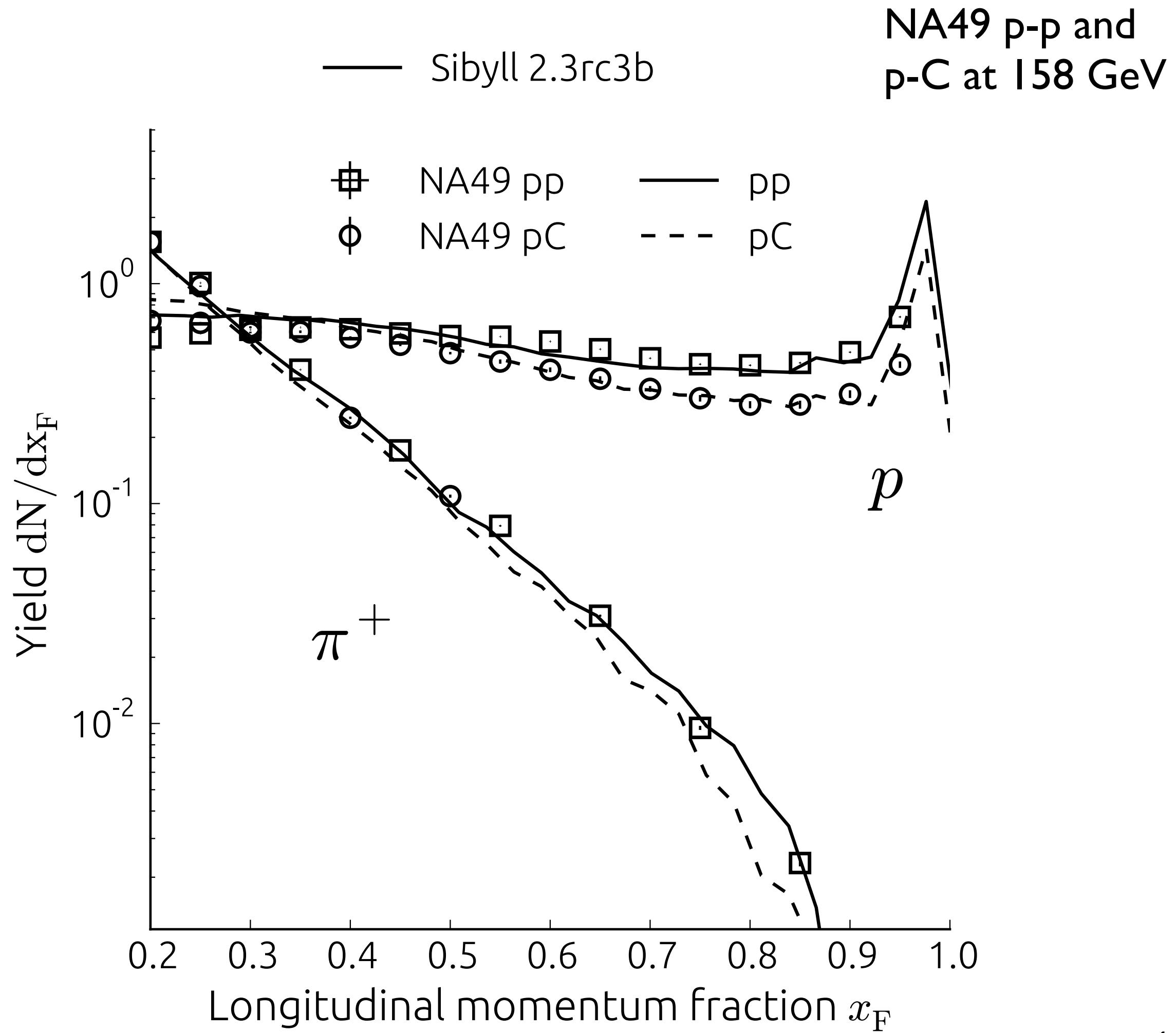
$$\langle p_{\parallel} \rangle \sim \Delta p_{\parallel} \approx \frac{1}{R'} \approx \frac{1}{5} E_p$$

$$\Gamma = E_p/m_p$$

Transverse momenta of secondaries

$$\langle p_{\perp} \rangle \sim \Delta p_{\perp} \sim \frac{1}{R} \approx 200 \text{ MeV}$$

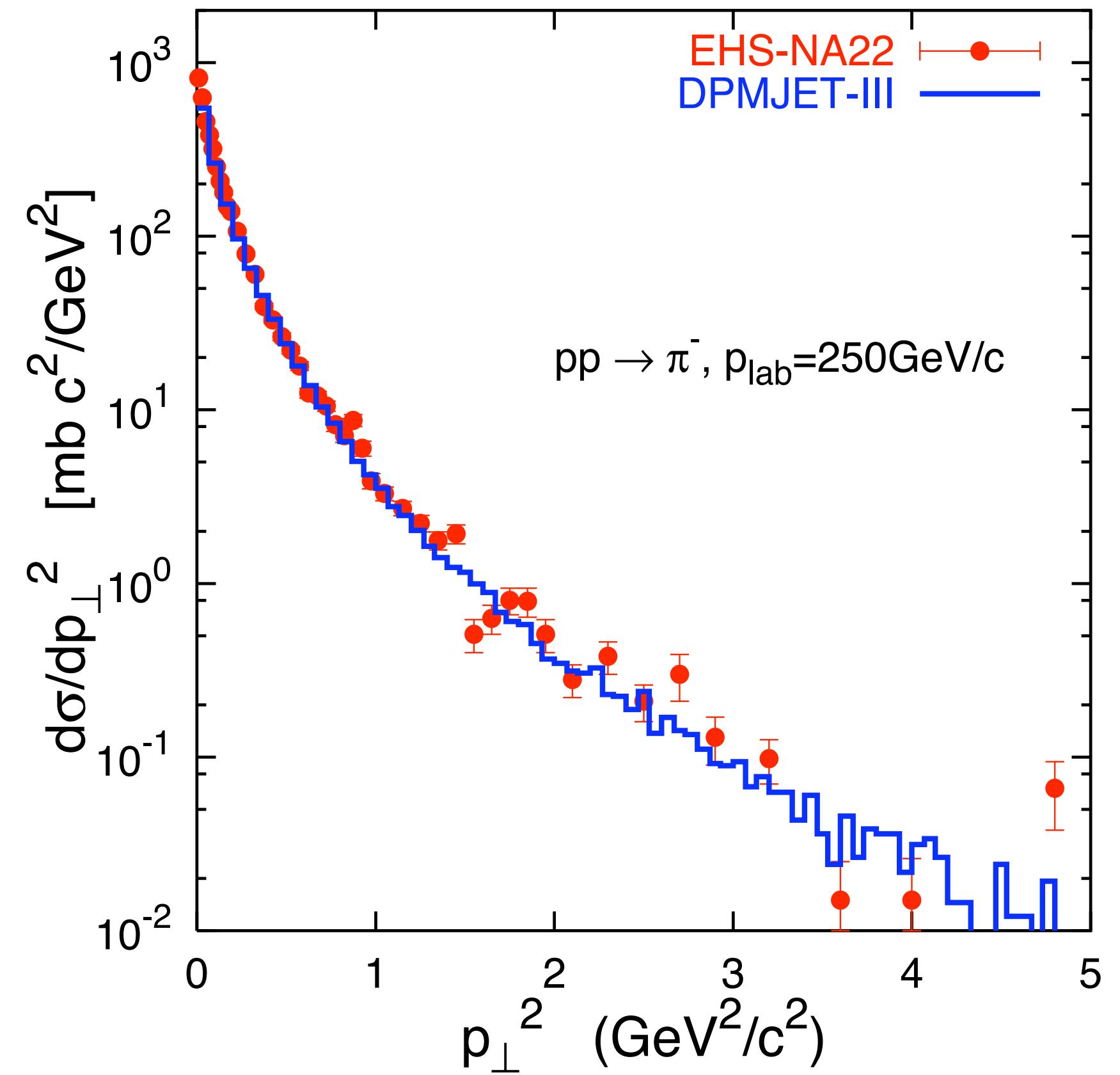
Typical hadronic final states



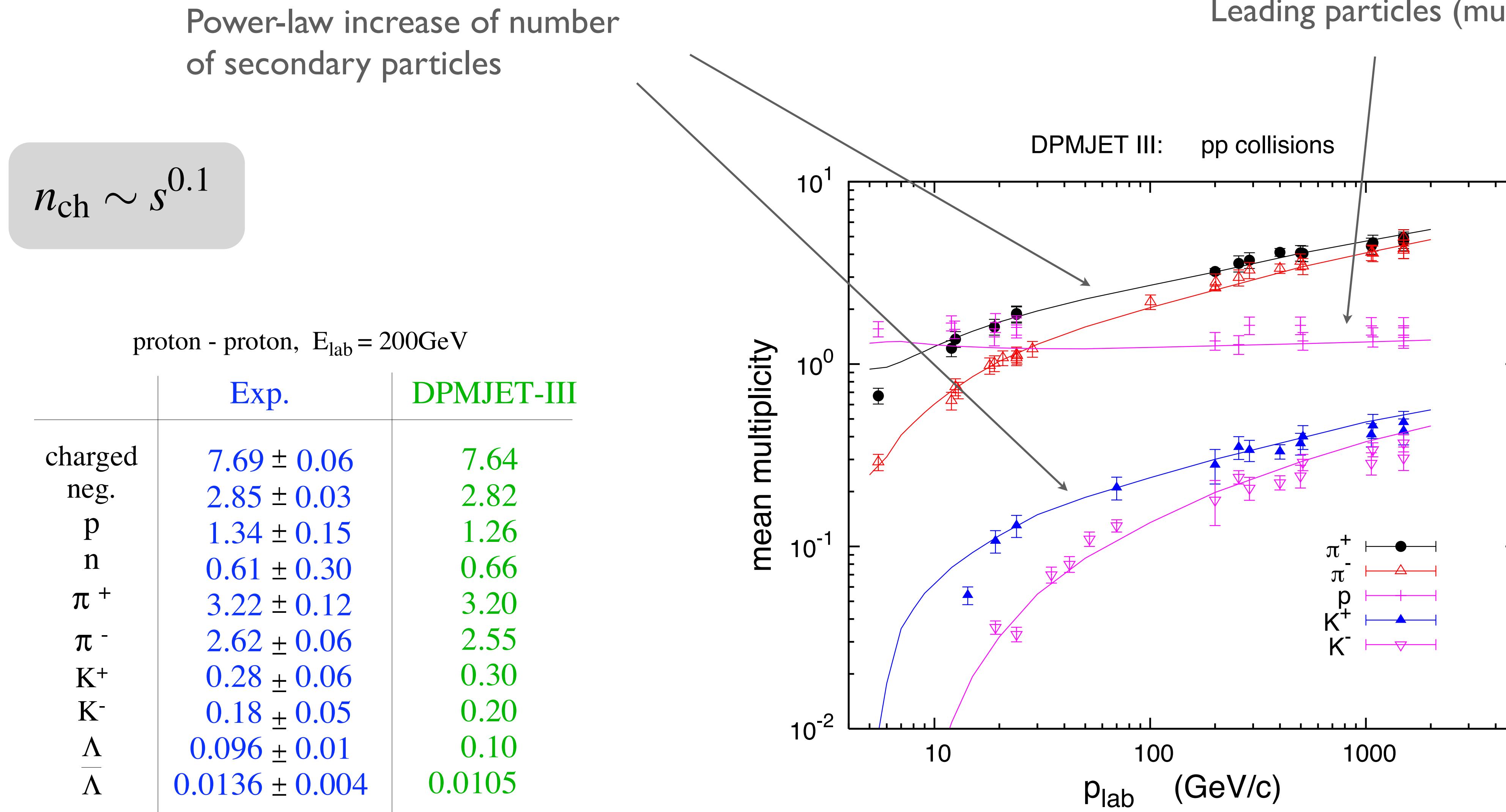
(Riehn et al. ICRC 2017)

Feynman-x

$$x_F = \left(\frac{p_{||}}{p_{\max}} \right)_{\text{CMS}}$$

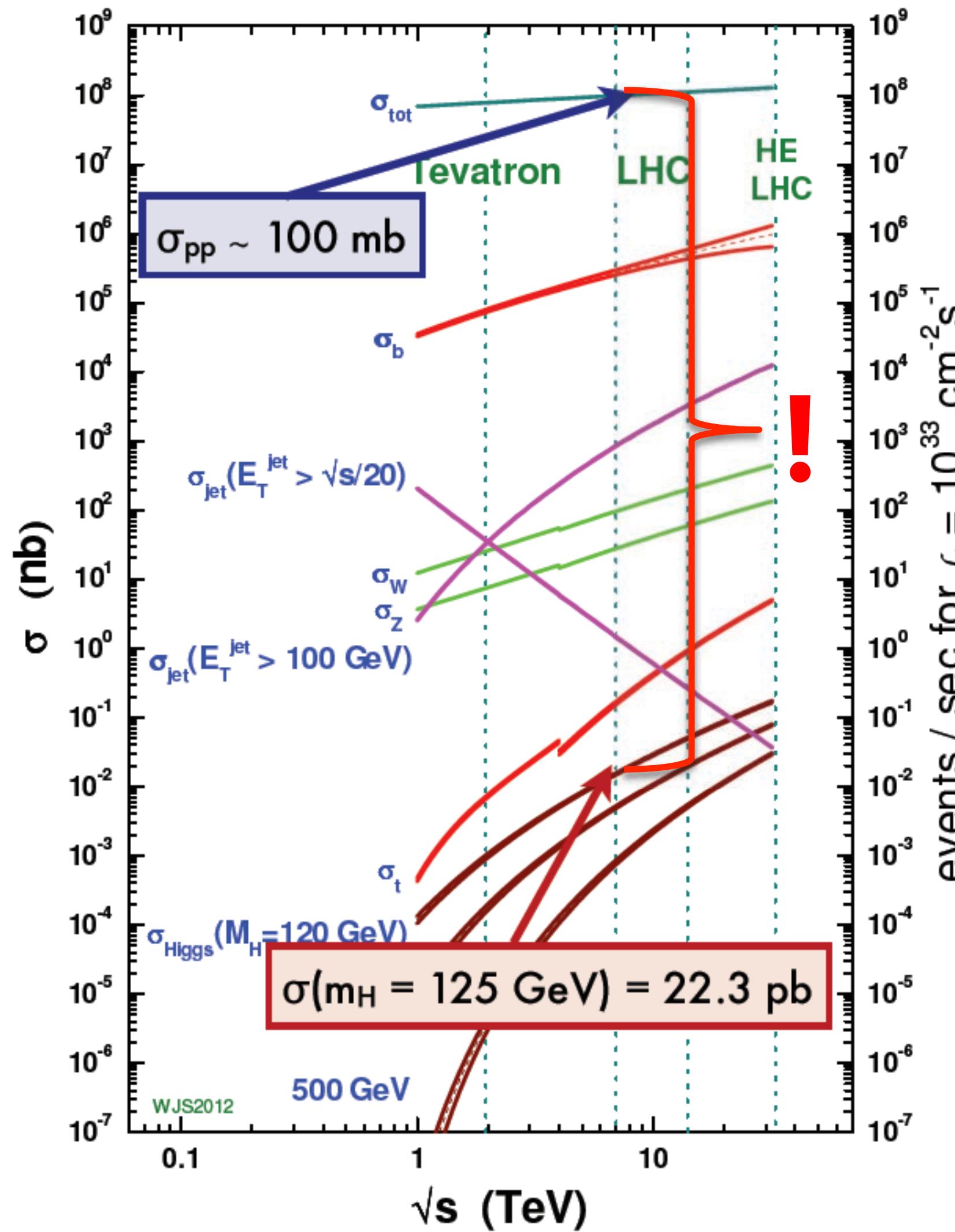


Secondary particle multiplicities



Competing processes of interaction and decay

proton - (anti)proton cross sections



Interaction length

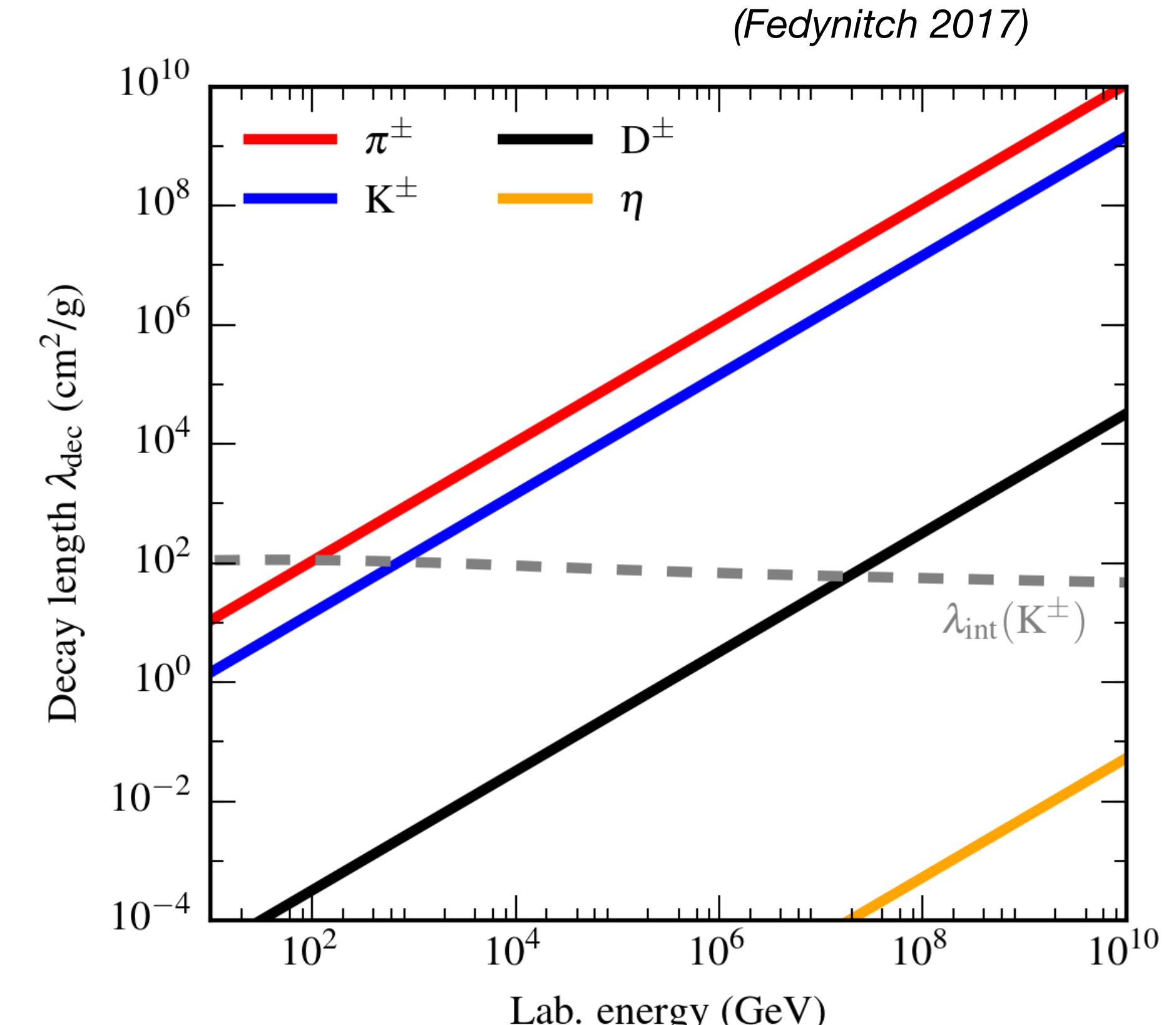
$$\lambda_{\text{int}} = \frac{\langle m_{\text{air}} \rangle}{\sigma_{\text{int}}}$$

$$\lambda_\pi \approx \lambda_K \approx 120 \text{ g/cm}^2$$

Decay length

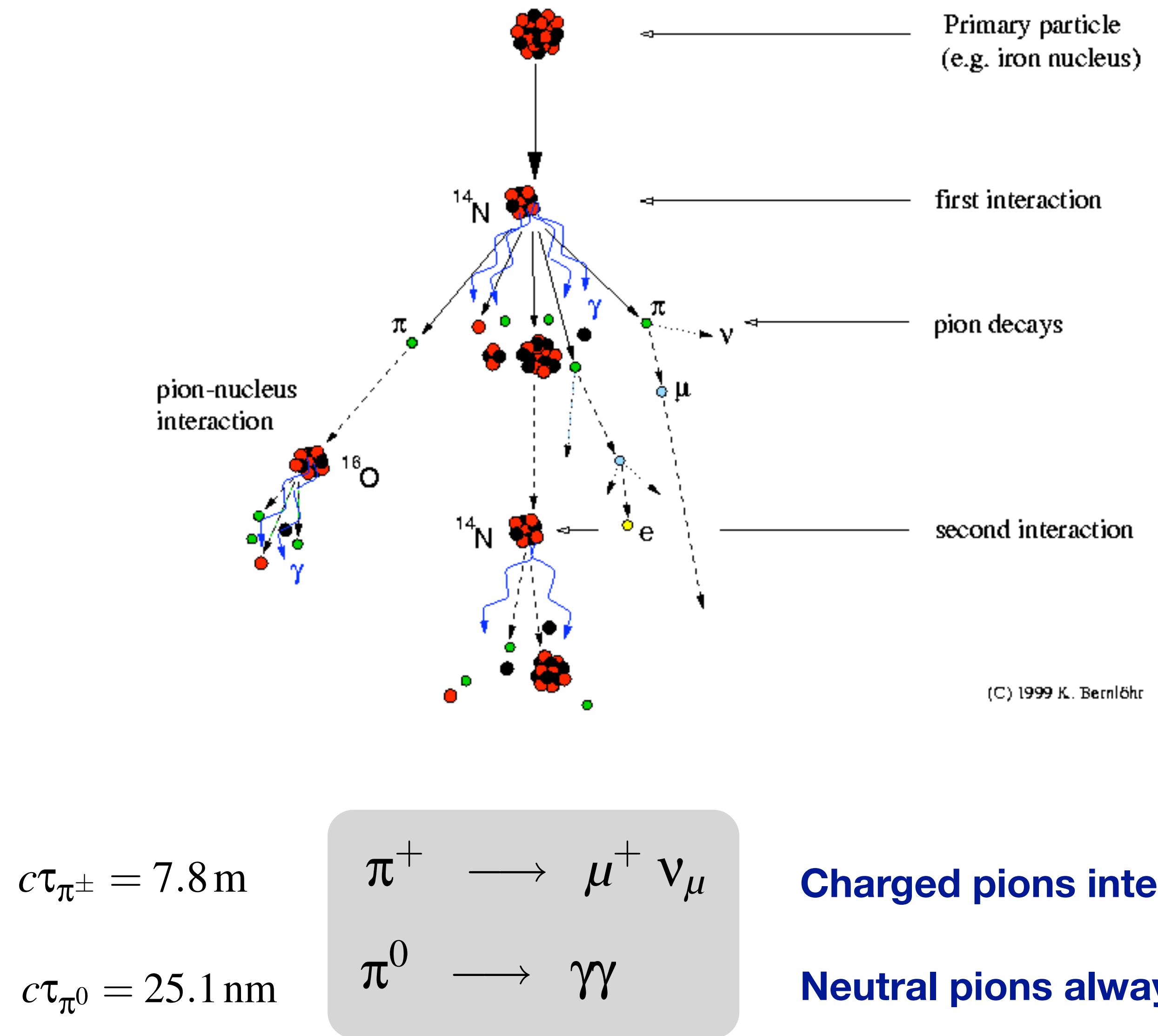
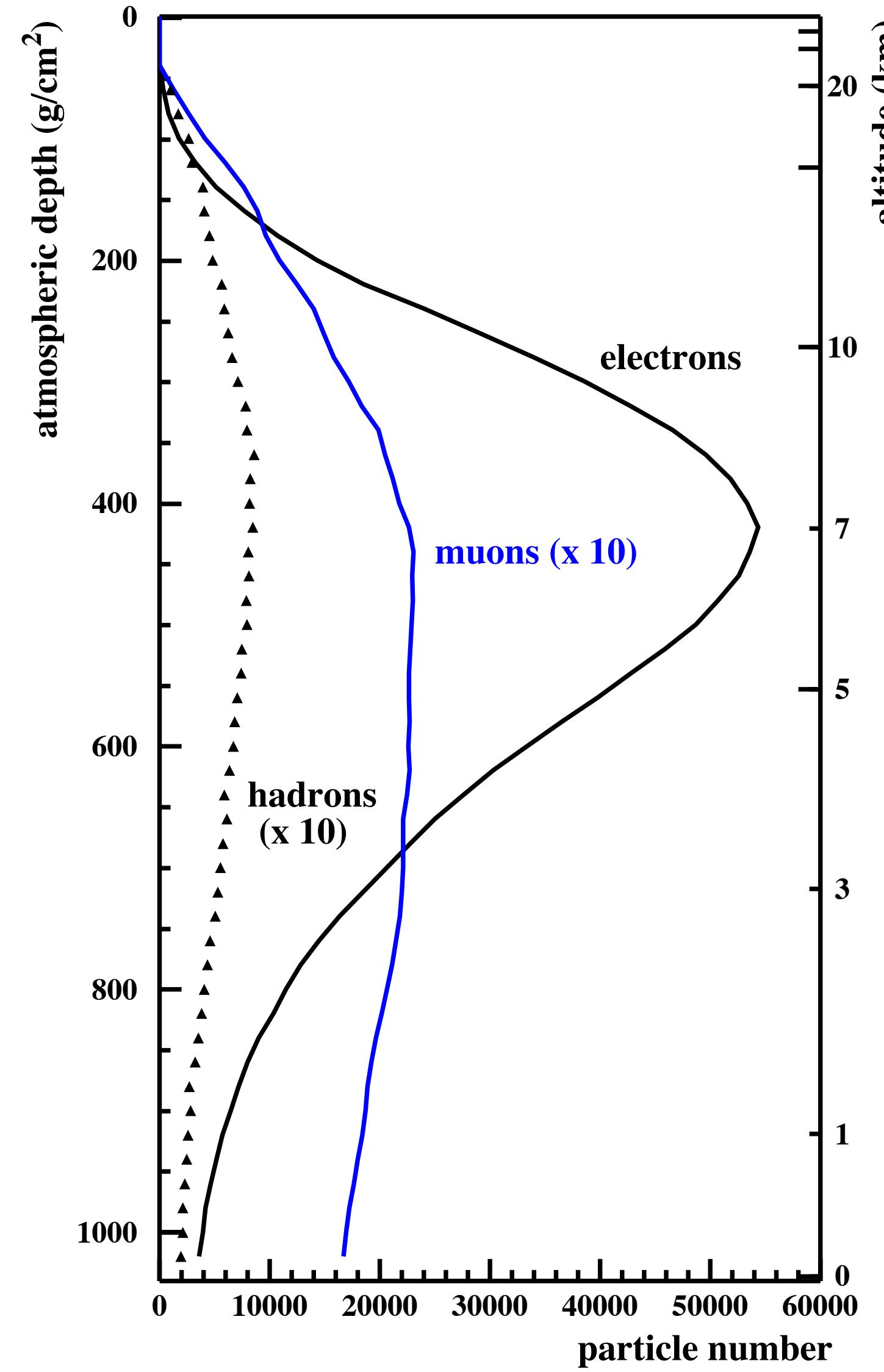
$$l_{\text{dec}} = \beta c \tau \Gamma \approx c \tau \frac{E}{m}$$

$$\lambda_{\text{dec}} = \rho l_{\text{dec}} \approx c \tau \rho \frac{E}{m}$$



air density

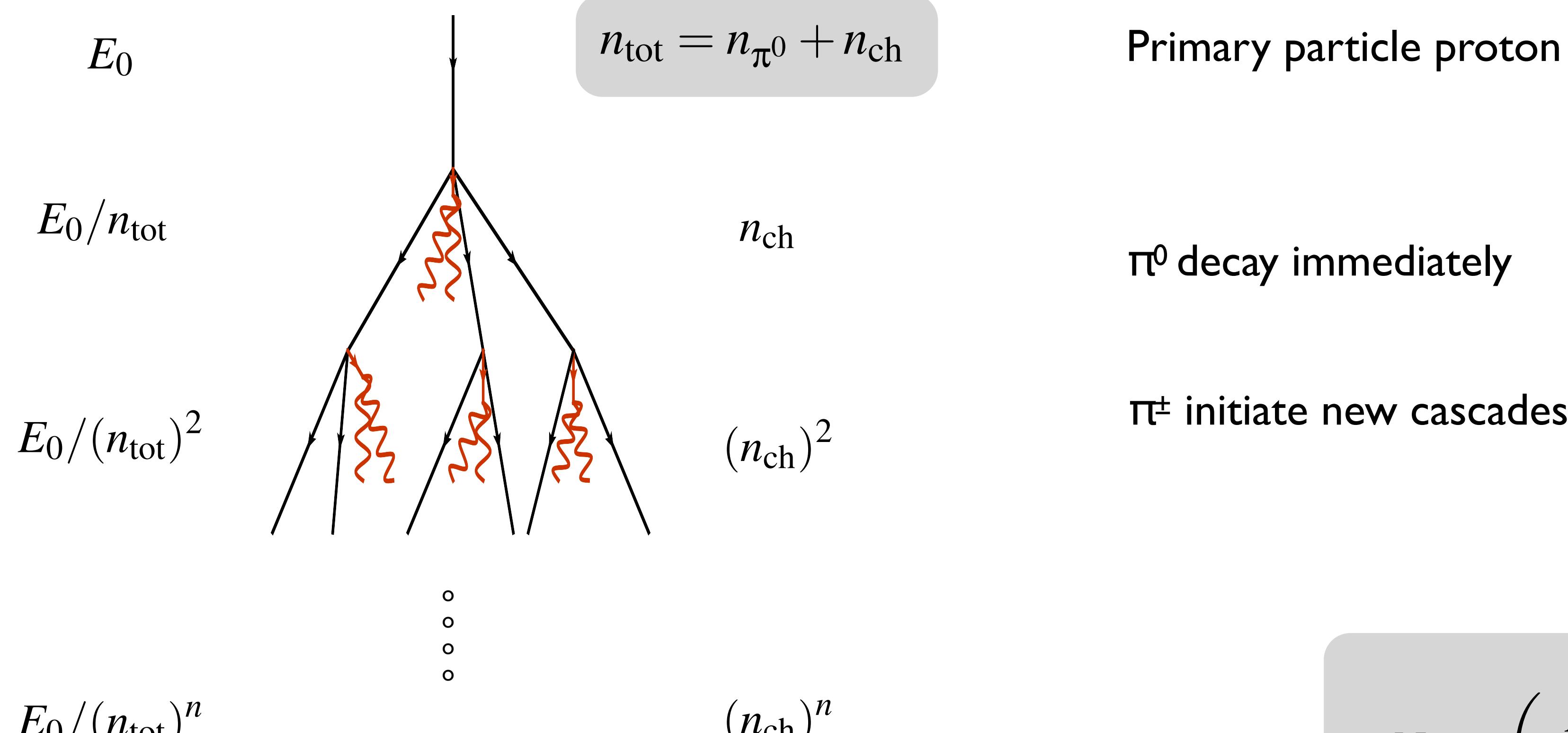
Hadron-induced showers



Charged pions interact $E > 30 \text{ GeV}$

Neutral pions always decay

Qualitative approach: Heitler-Matthews model



Assumptions:

- cascade stops at $E_{\text{part}} = E_{\text{dec}}$
- each hadron produces one muon

(Matthews, *Astropart.Phys.* 22, 2005)

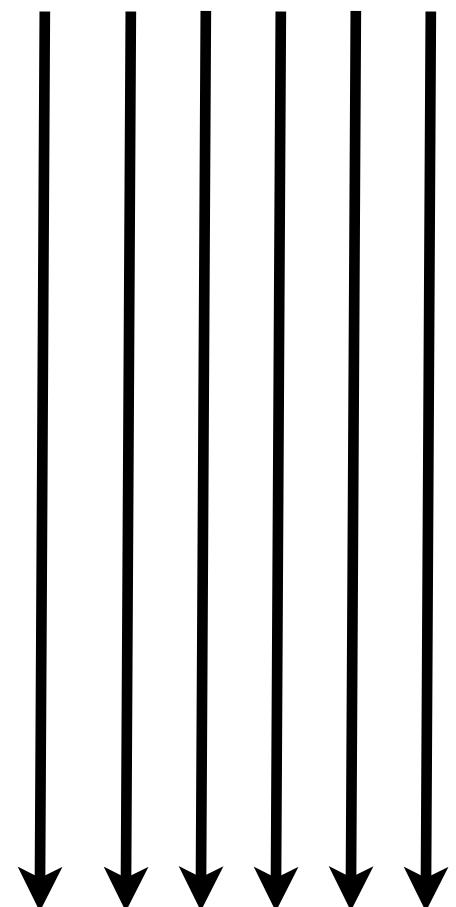
$$N_\mu = \left(\frac{E_0}{E_{\text{dec}}} \right)^\alpha$$

$$\alpha = \frac{\ln n_{\text{ch}}}{\ln n_{\text{tot}}} \approx 0.82 \dots 0.95$$

Superposition model

Nucleus

$$E_i = E_0/A$$



Target

$$N_{\max}^A \sim A \left(\frac{E_0}{AE_c} \right) = N_{\max}$$

Proton-induced shower

$$N_{\max} \sim E_0/E_c$$

$$X_{\max} \sim \lambda_{\text{eff}} \ln(E_0)$$

$$N_{\mu} = \left(\frac{E_0}{E_{\text{dec}}} \right)^{\alpha}$$

$$\alpha \approx 0.9$$

Assumption: nucleus of mass A and energy E_0 corresponds to A nucleons (protons) of energy $E_n = E_0/A$

$$X_{\max}^A \sim \lambda_{\text{eff}} \ln(E_0/A)$$

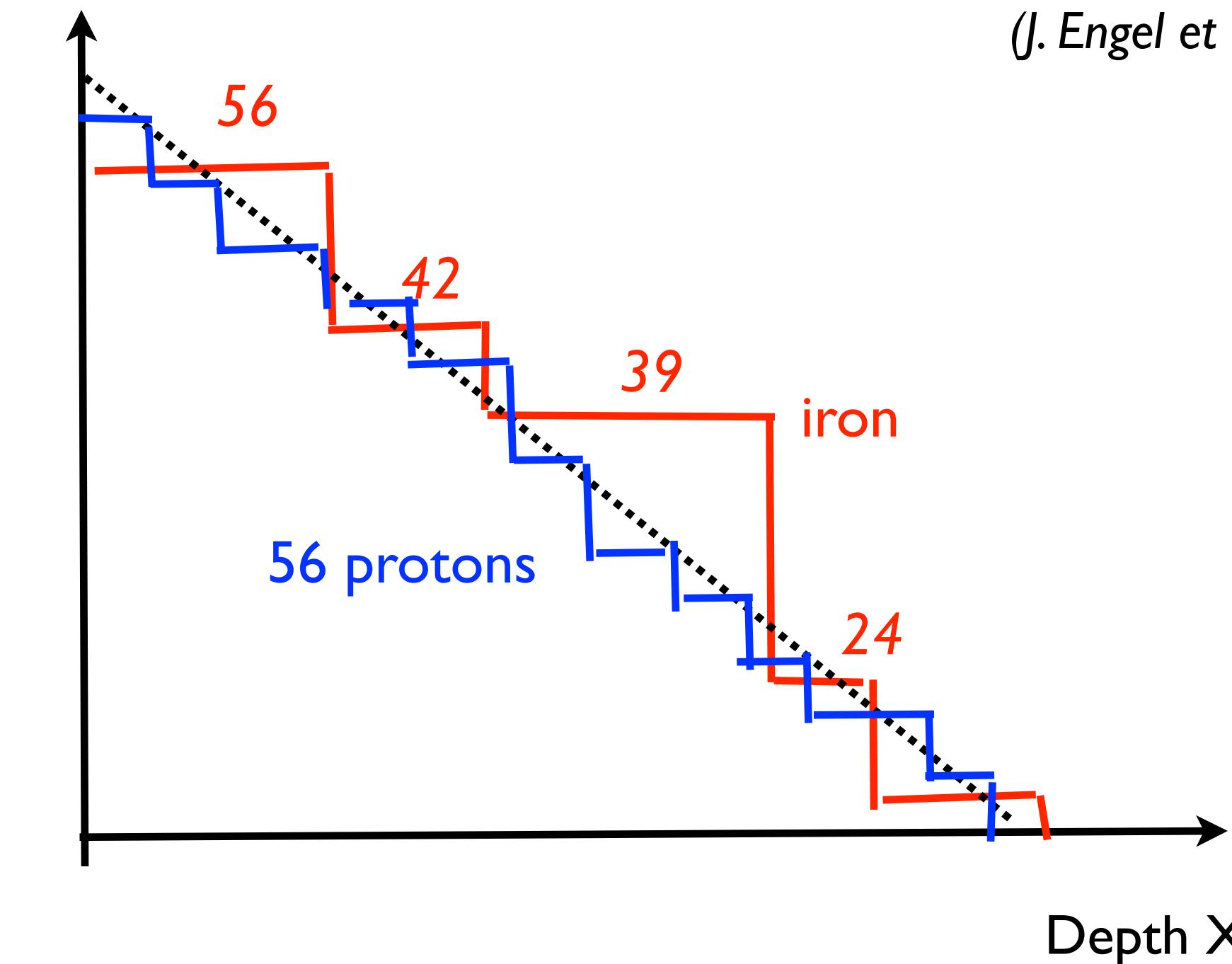
$$N_{\mu}^A = A \left(\frac{E_0}{AE_{\text{dec}}} \right)^{\alpha} = A^{1-\alpha} N_{\mu}$$

Superposition model: correct prediction of mean X_{max}

iron nucleus



Number of
nucleons without
interaction

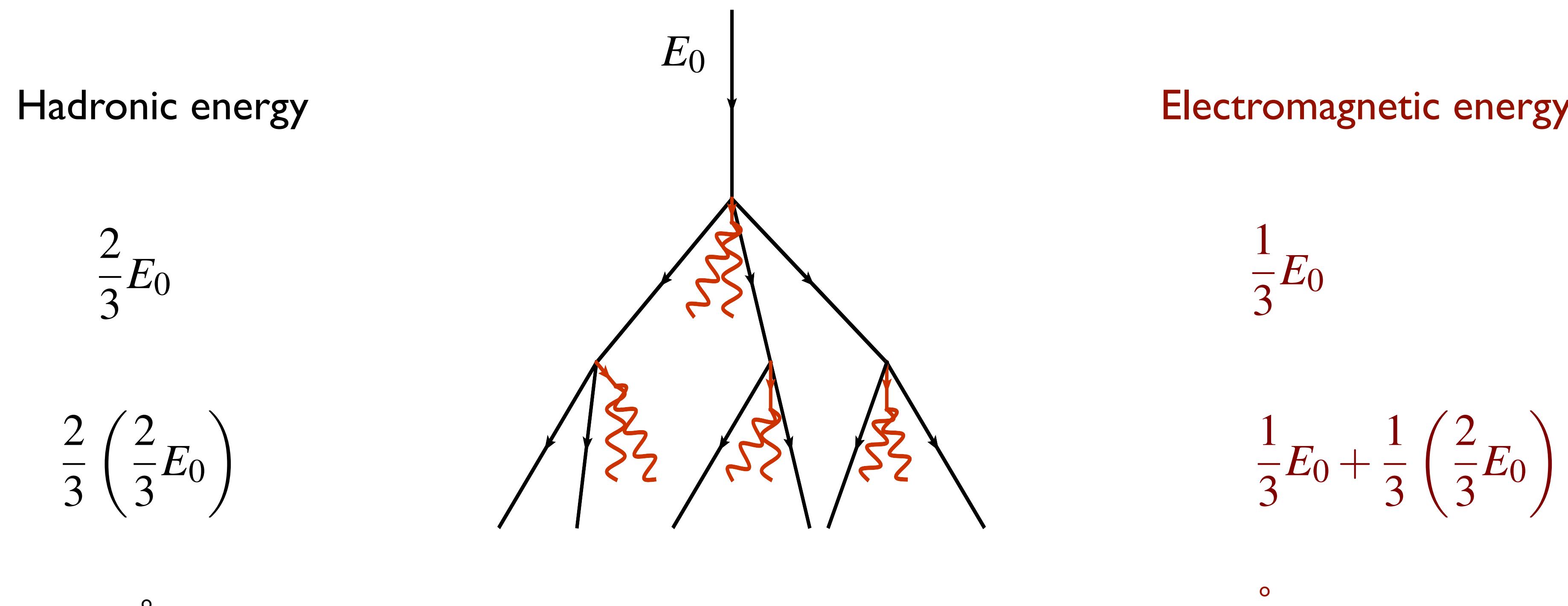


Glauber approximation (unitarity)

$$n_{\text{part}} = \frac{\sigma_{\text{Fe-air}}}{\sigma_{\text{p-air}}}$$

Superposition and semi-superposition models
applicable to inclusive (averaged) observables

Electromagnetic energy and energy transfer



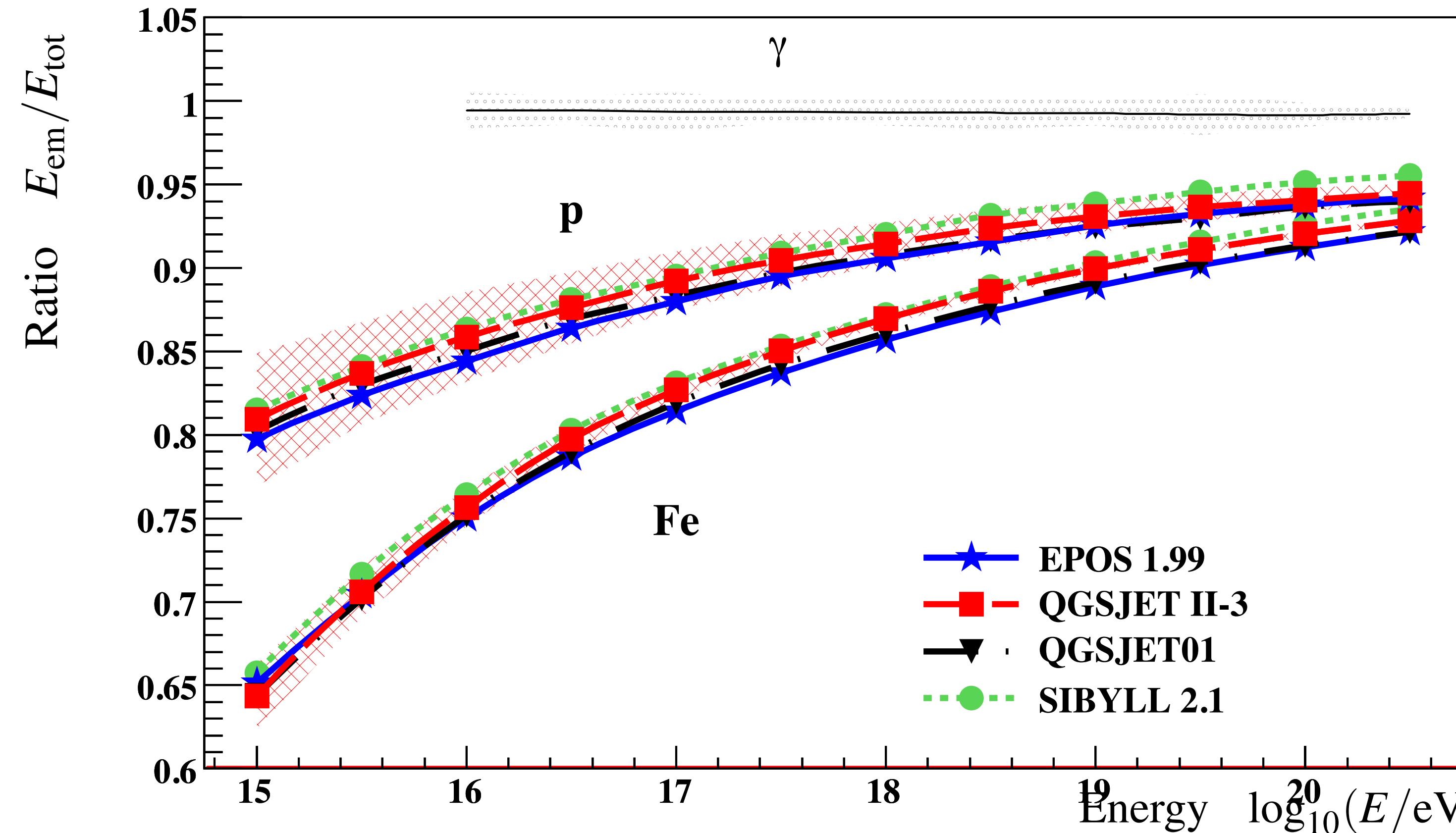
$$E_{\text{had}} = \left(\frac{2}{3}\right)^n E_0$$

$$\begin{aligned} n = 5, \quad & E_{\text{had}} \sim 12\% \\ n = 6, \quad & E_{\text{had}} \sim 8\% \end{aligned}$$

$$E_{\text{em}} = \left[1 - \left(\frac{2}{3}\right)^n\right] E_0$$

Energy transferred to electromagnetic component

(RE, Pierog, Heck, ARNPS 2011)



Ratio of em. to total shower energy

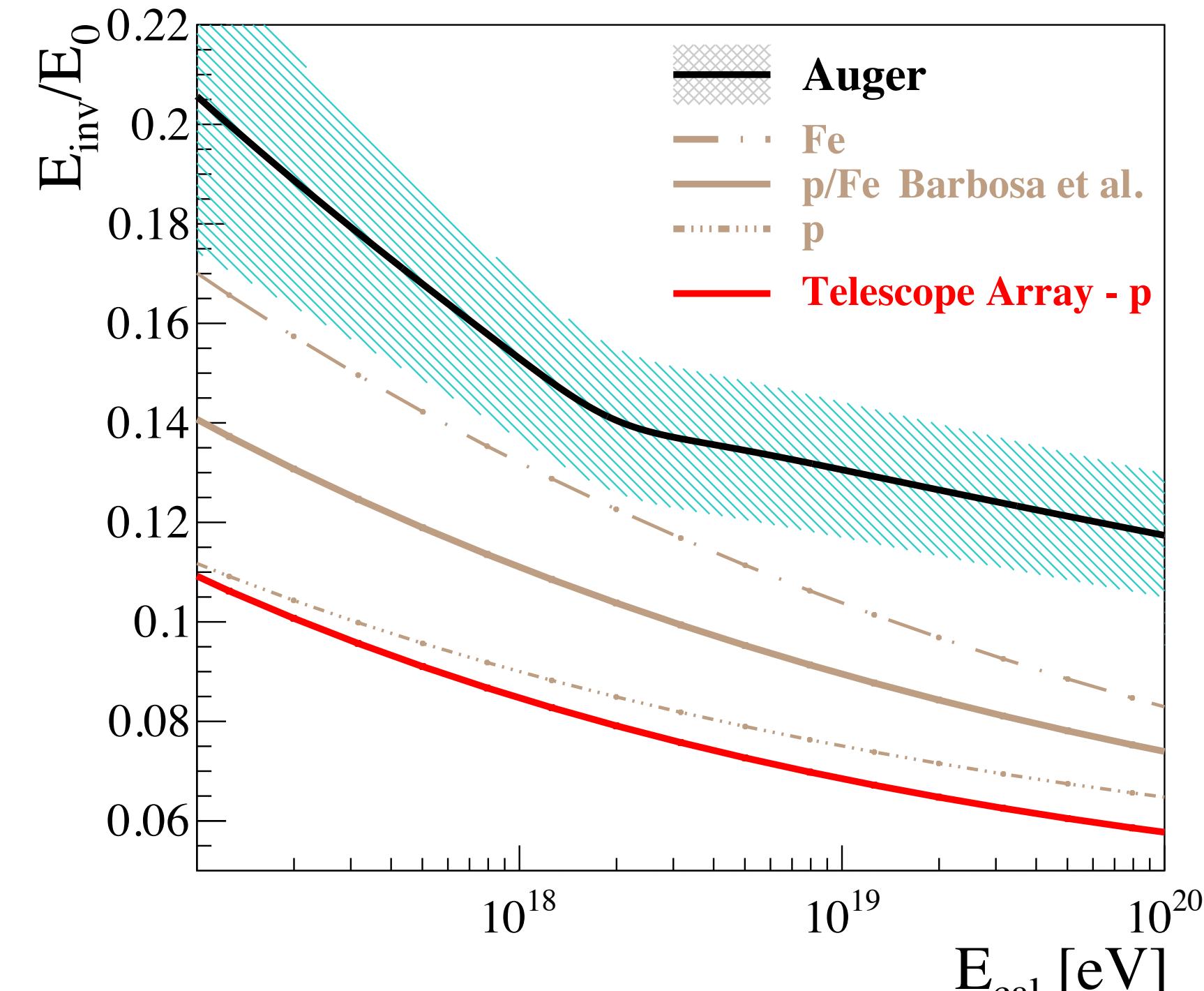
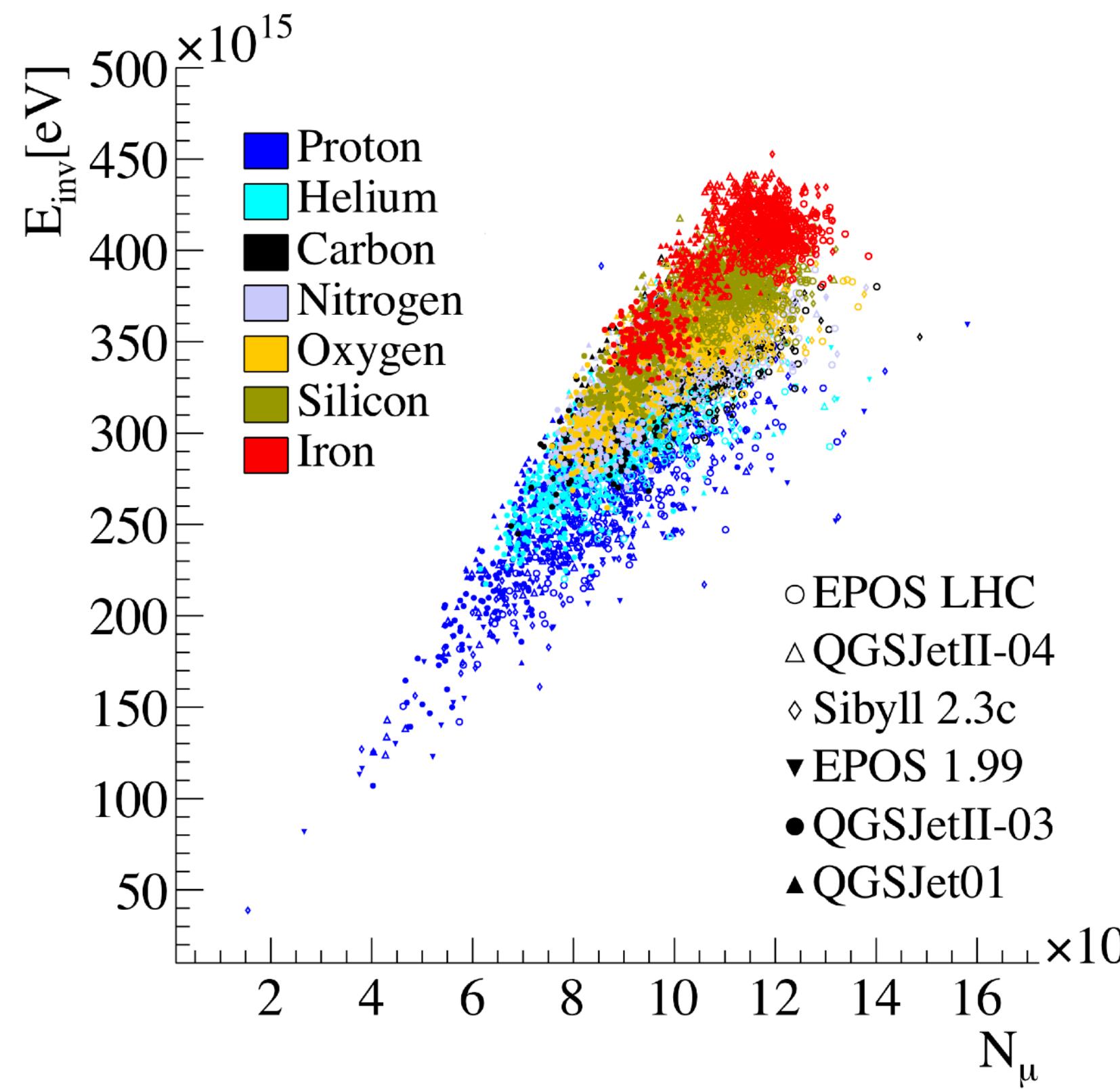
Detailed Monte Carlo simulation with CONEX

$$E_{\text{inv}} = E_{\text{tot}} - E_{\text{em}}$$

At high energy: model dependence of correction to obtain total energy small

Muons as tracers of the hadronic core

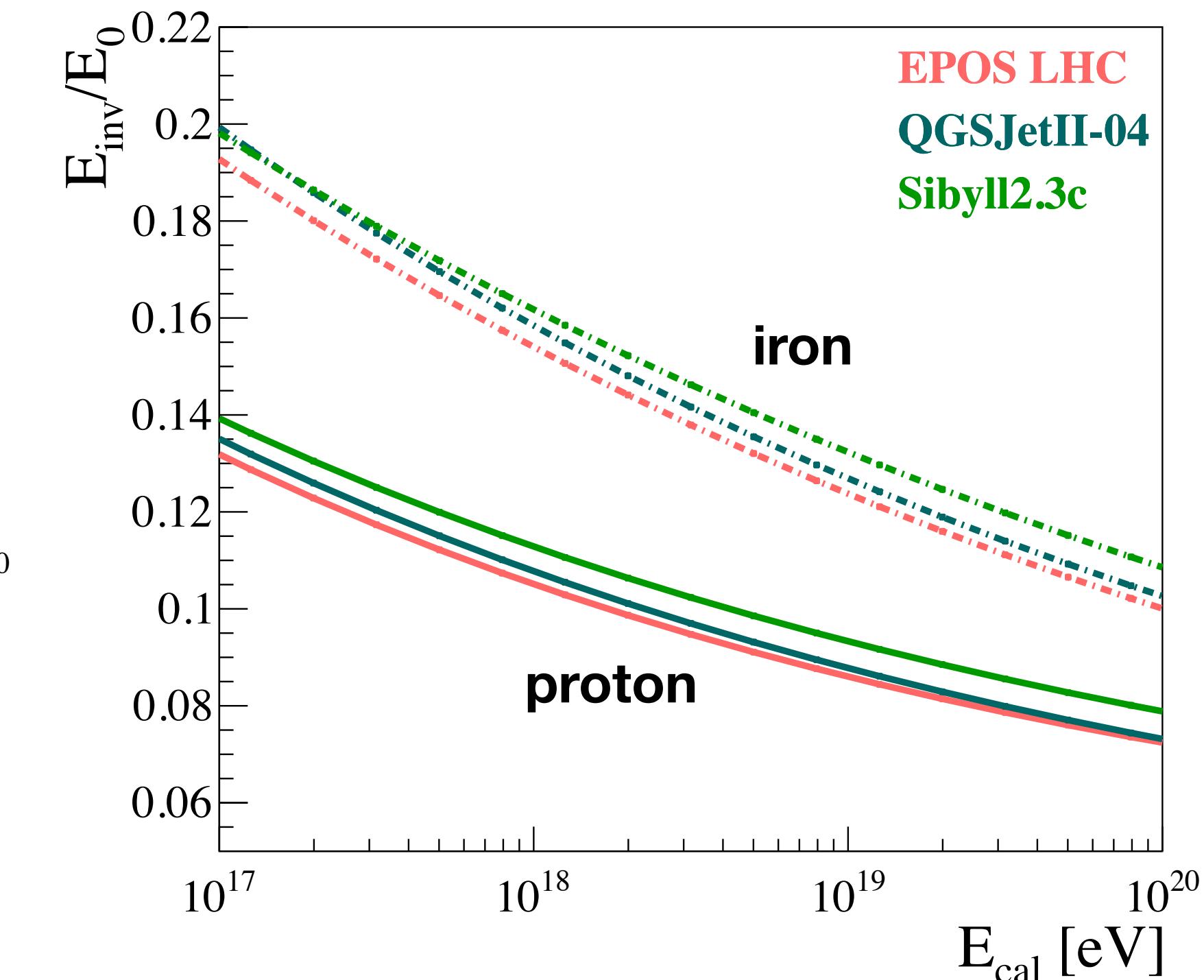
**Very good correlation
between muon number
and invisible energy**



**Muon-data based correction for
invisible energy used in Auger**

$$E_{\text{inv}} = E_{\text{tot}} - E_{\text{em}}$$

Most recent model predictions

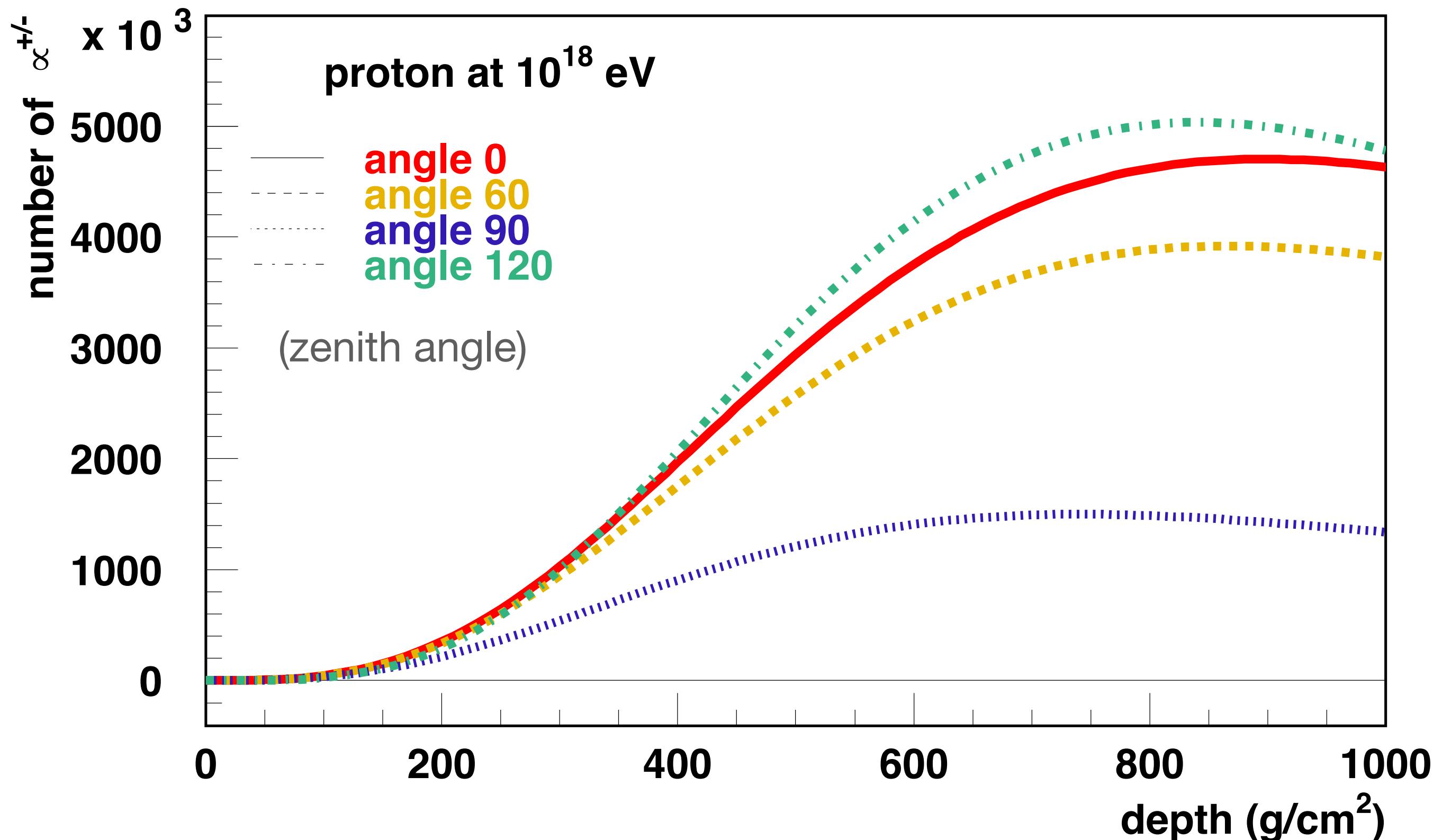


(Auger, PRD 2019)

Effect of air density (number of generations)

(Bergmann et al,
APP 26, 2007)

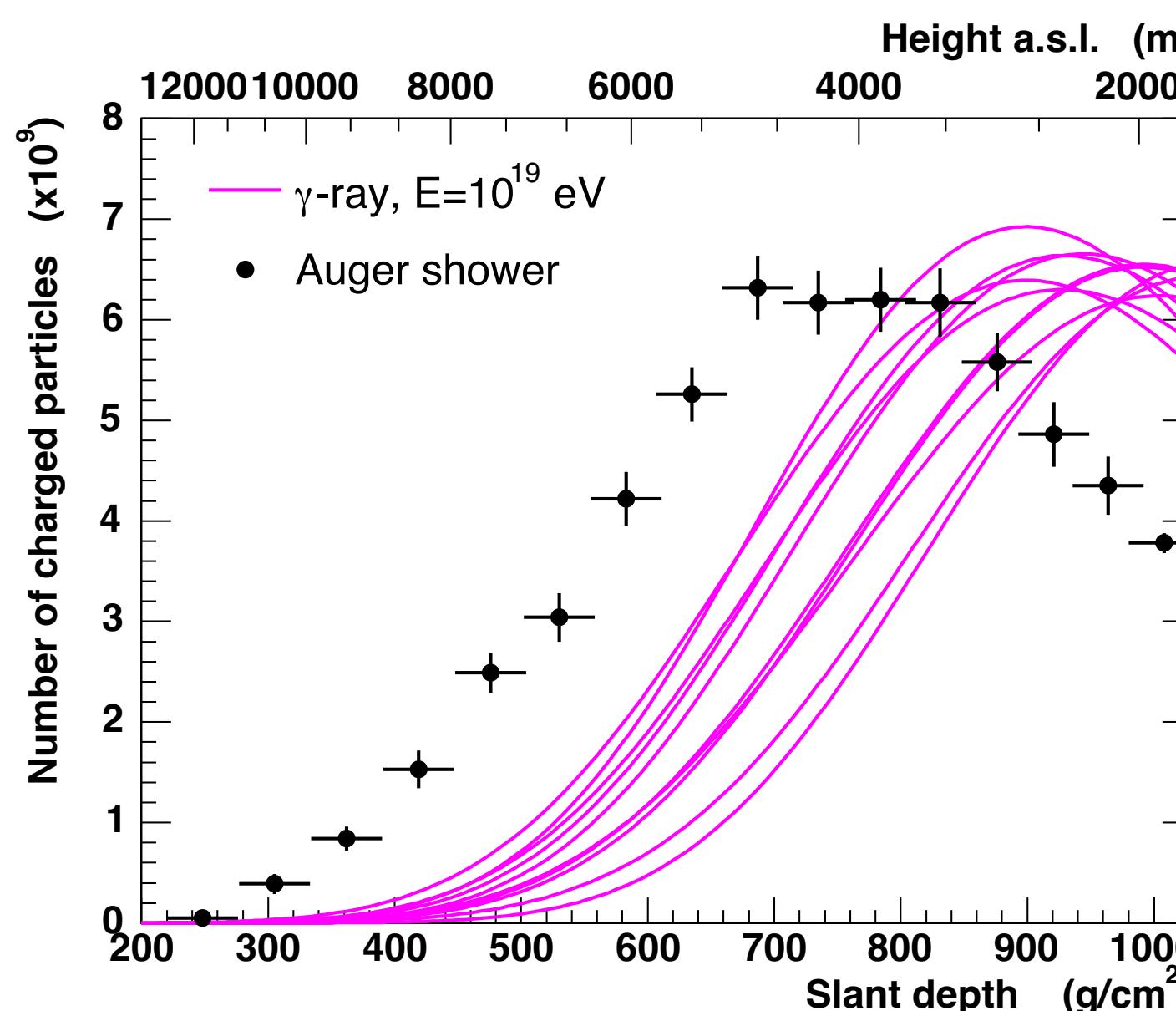
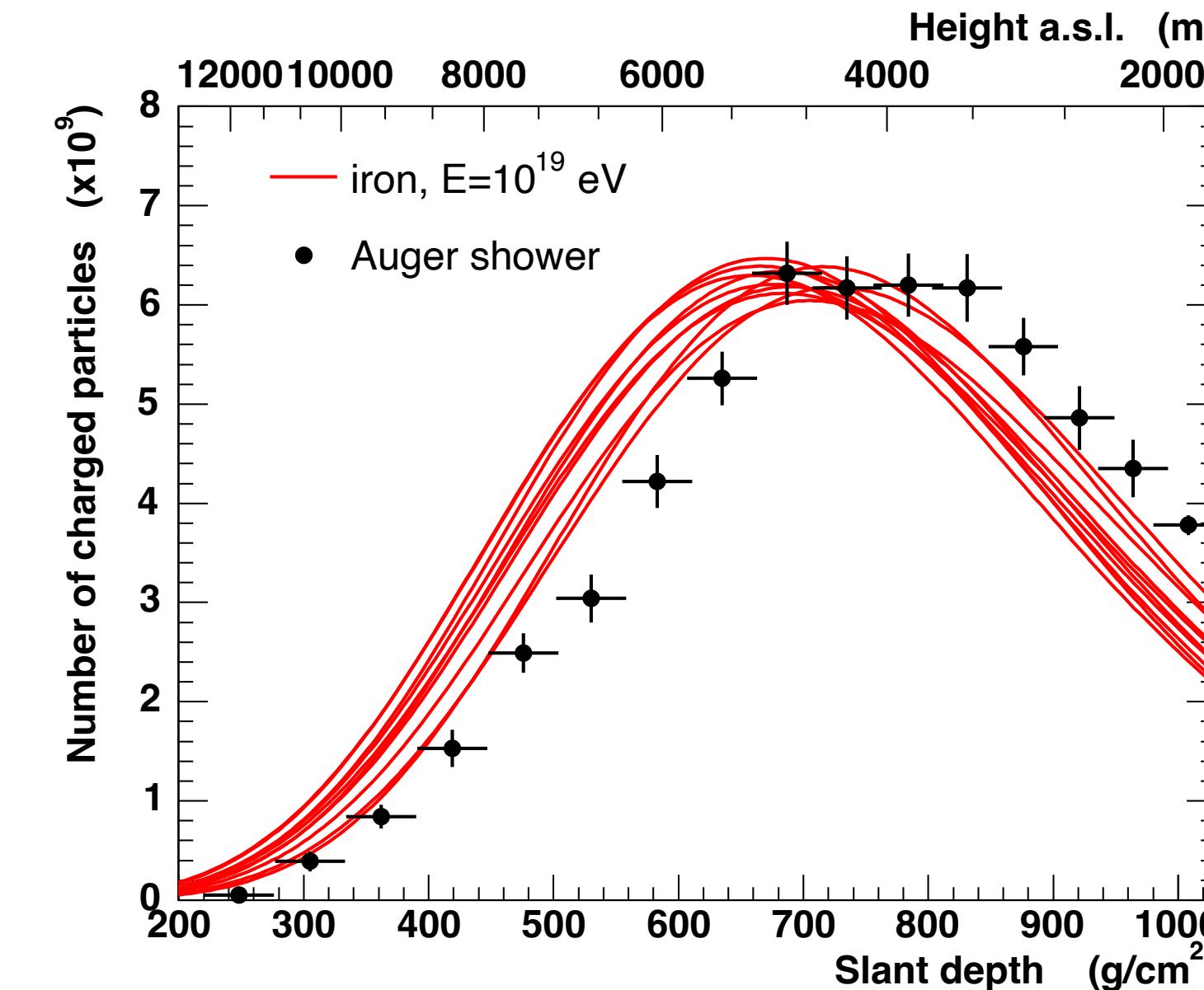
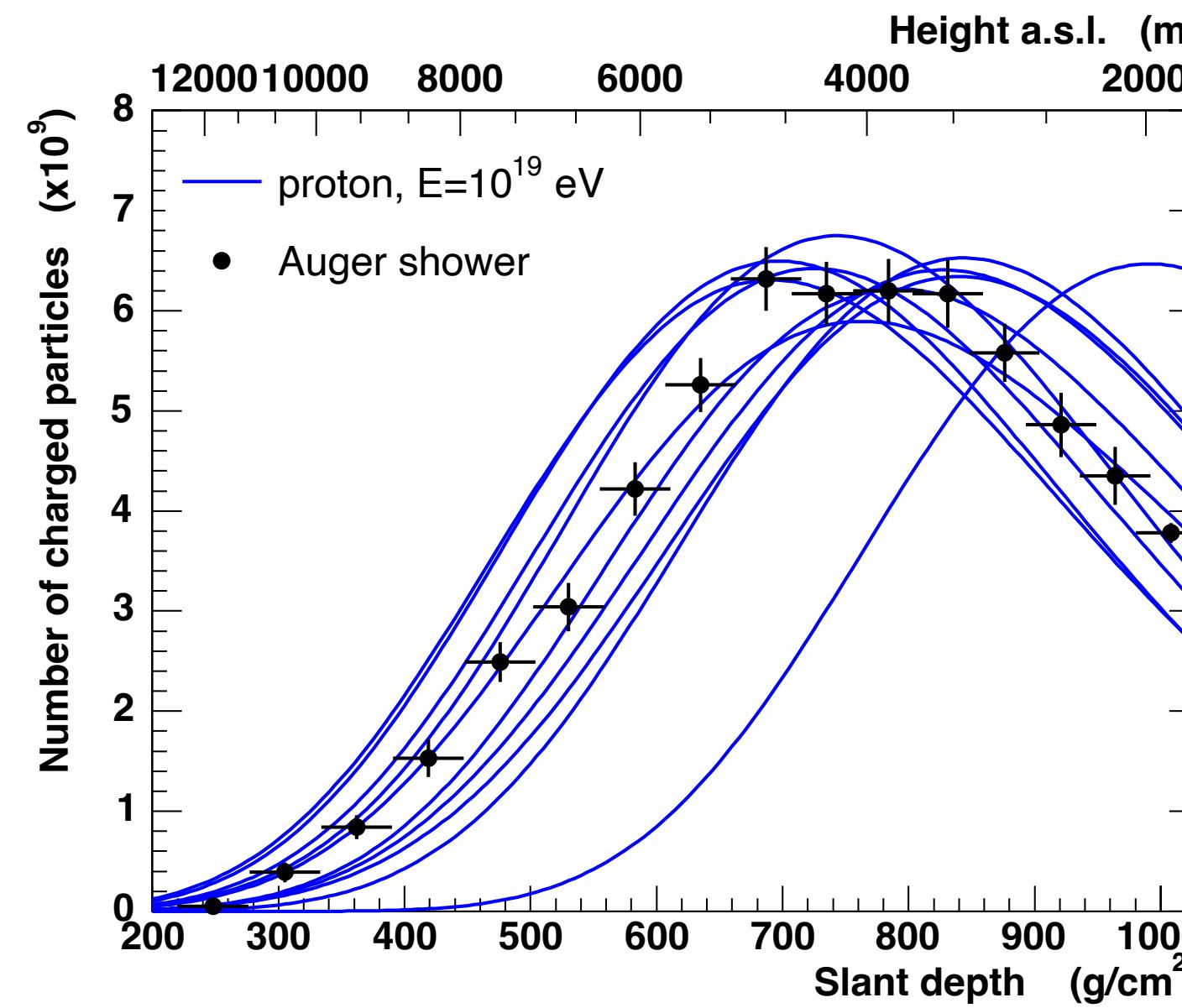
$$N_\mu = \left(\frac{E_0}{E_{\text{dec}}} \right)^\alpha$$



Pion decay energy depends on air density,
low density corresponds to large E_{dec}

**Electromagnetic showers are independent
of air density, hadronic showers not**

Longitudinal shower profiles: simulations and data



$$N_{\max} = E_0/E_c$$

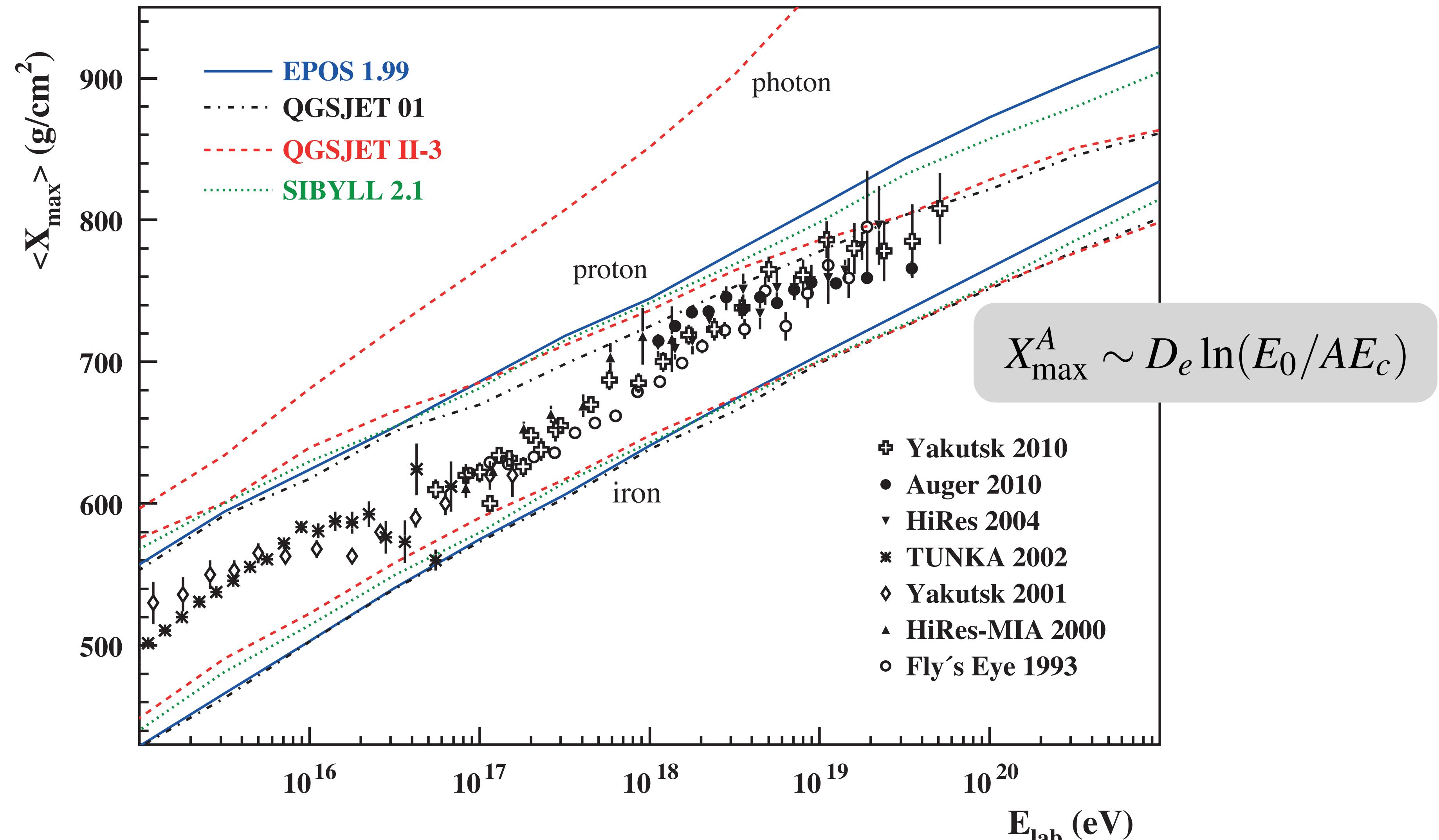
$$X_{\max} \sim D_e \ln(E_0/E_c)$$

Superposition model:

$$X_{\max}^A \sim D_e \ln(E_0/AE_c)$$

Mean depth of shower maximum

Note: old data and
model predictions
(just for clarity)

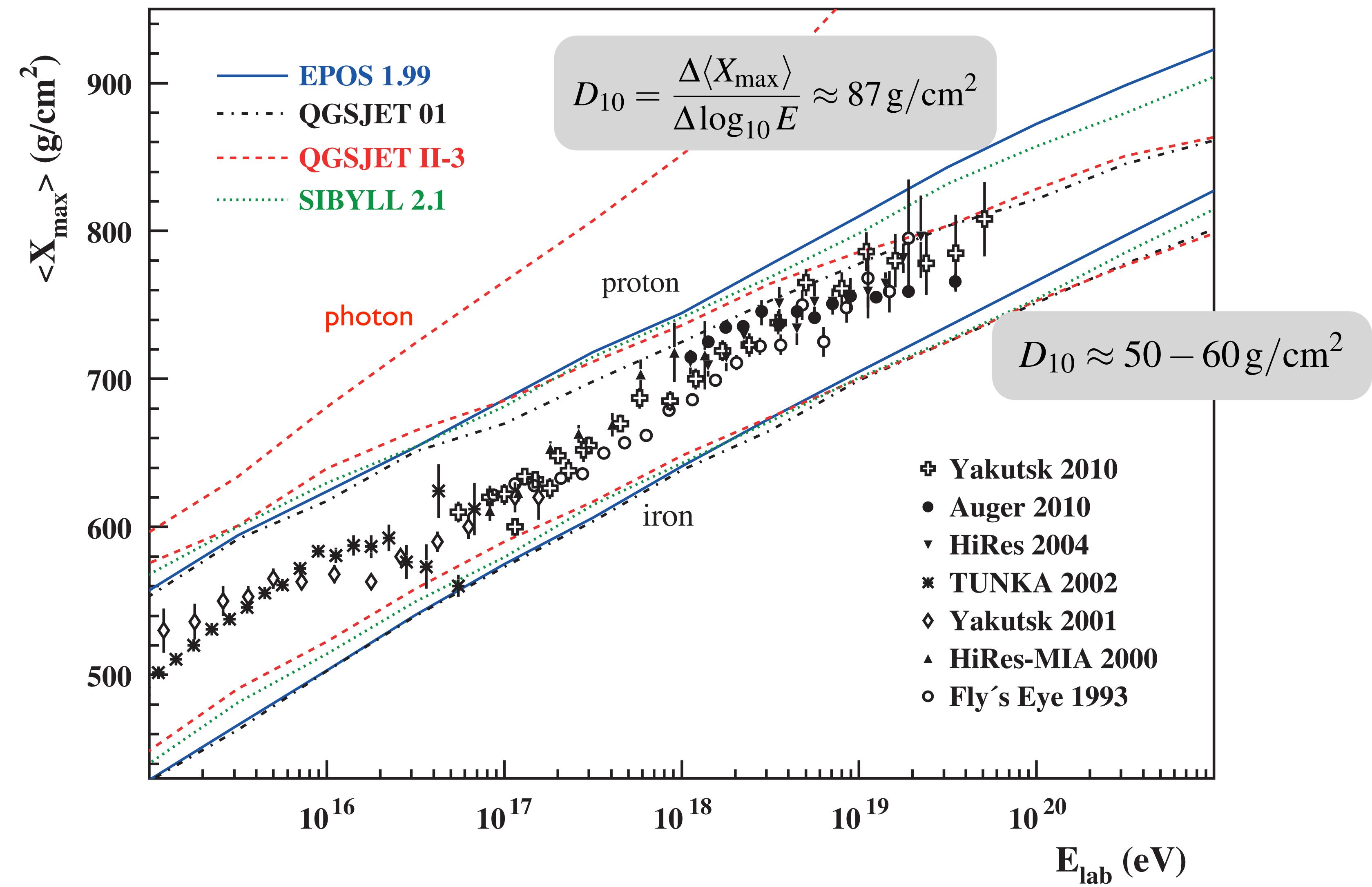


Different slopes for em. and hadronic showers

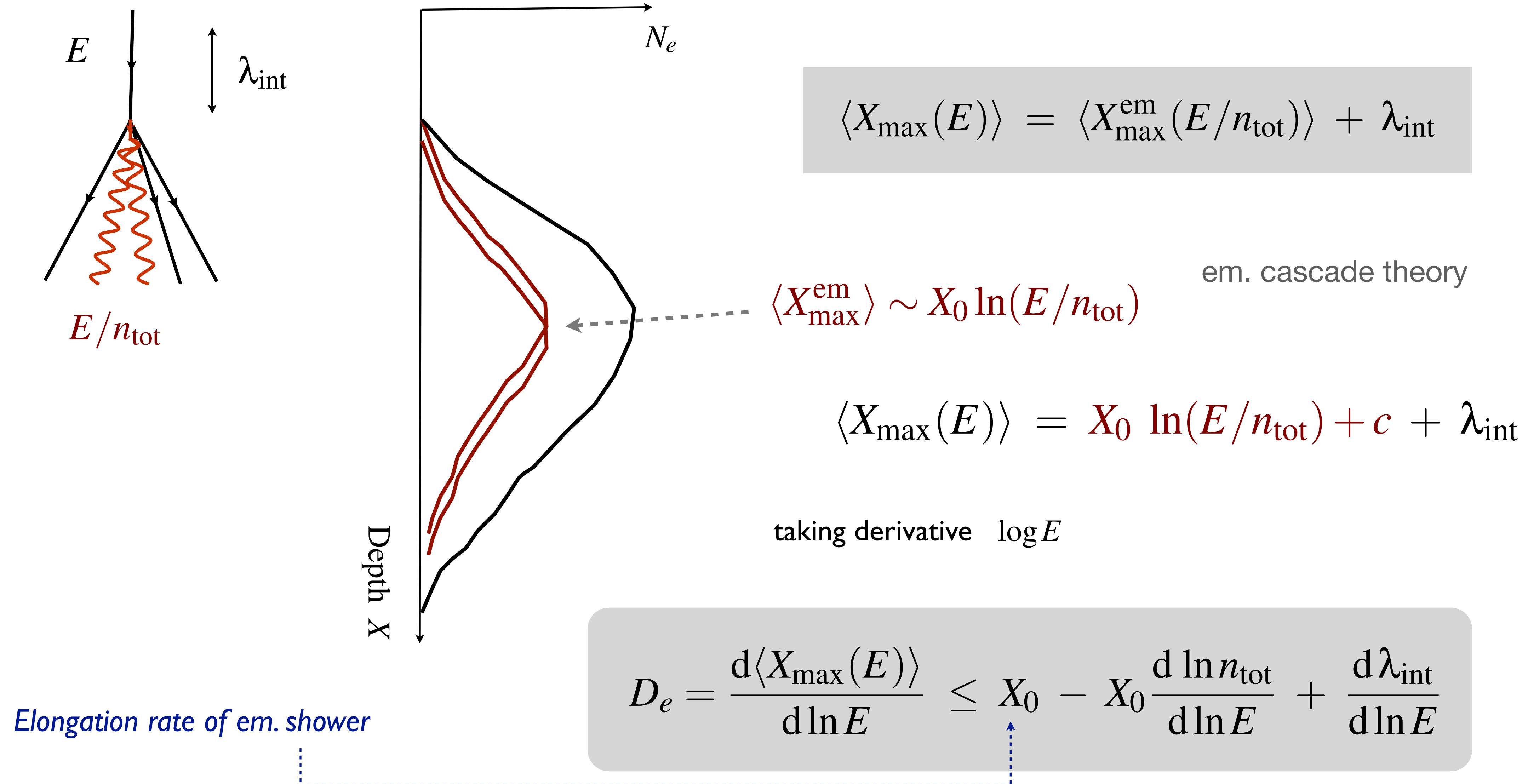
$$D_{10} = \frac{\Delta \langle X_{\max} \rangle}{\Delta \log_{10} E}$$

$$D_e = \frac{\Delta \langle X_{\max} \rangle}{\Delta \ln E}$$

$$D_{10} = \log(10) D_e$$



Derivation of elongation rate theorem



Elongation rate theorem

$$X_0 = 36 \text{ g/cm}^2$$

$$D_e^{\text{had}} = X_0(1 - B_n - B_\lambda)$$

(Linsley, Watson PRL46, 1981)

$$B_n = \frac{d \ln n_{\text{tot}}}{d \ln E}$$

Large if multiplicity of high energy particles rises very fast, **zero in case of scaling**

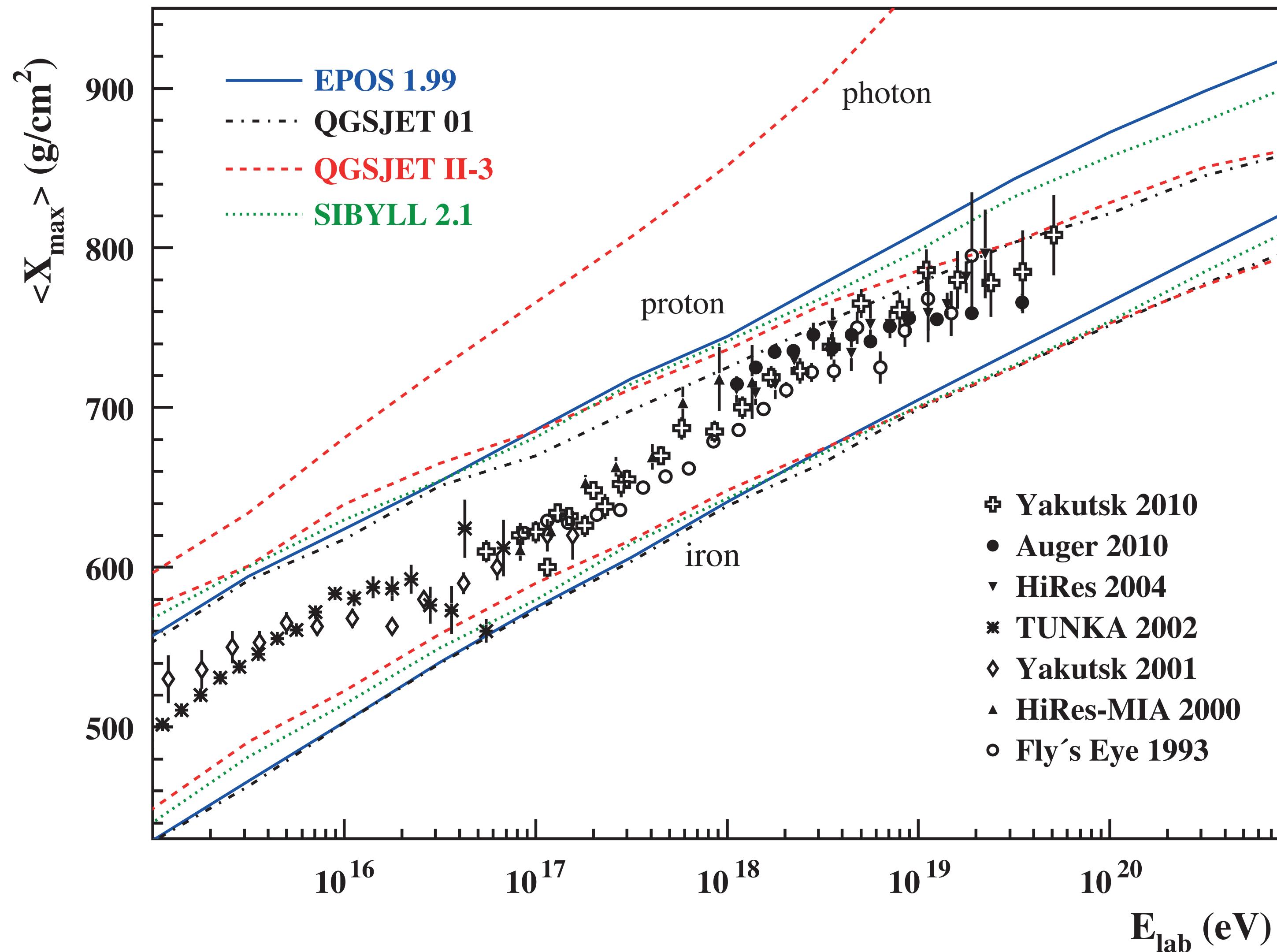
$$B_\lambda = -\frac{1}{X_0} \frac{d \lambda_{\text{int}}}{d \ln E}$$

Large if cross section rises rapidly with energy

Note:

$$D_{10} = \log(10) D_e$$

Mean depth of shower maximum



Elongation rates and model features

Elongation rate theorem

$$D_{10}^{\text{had}} = \ln 10 X_0 (1 - B_n - B_\lambda)$$

(Linsley, Watson PRL 46, 1981)

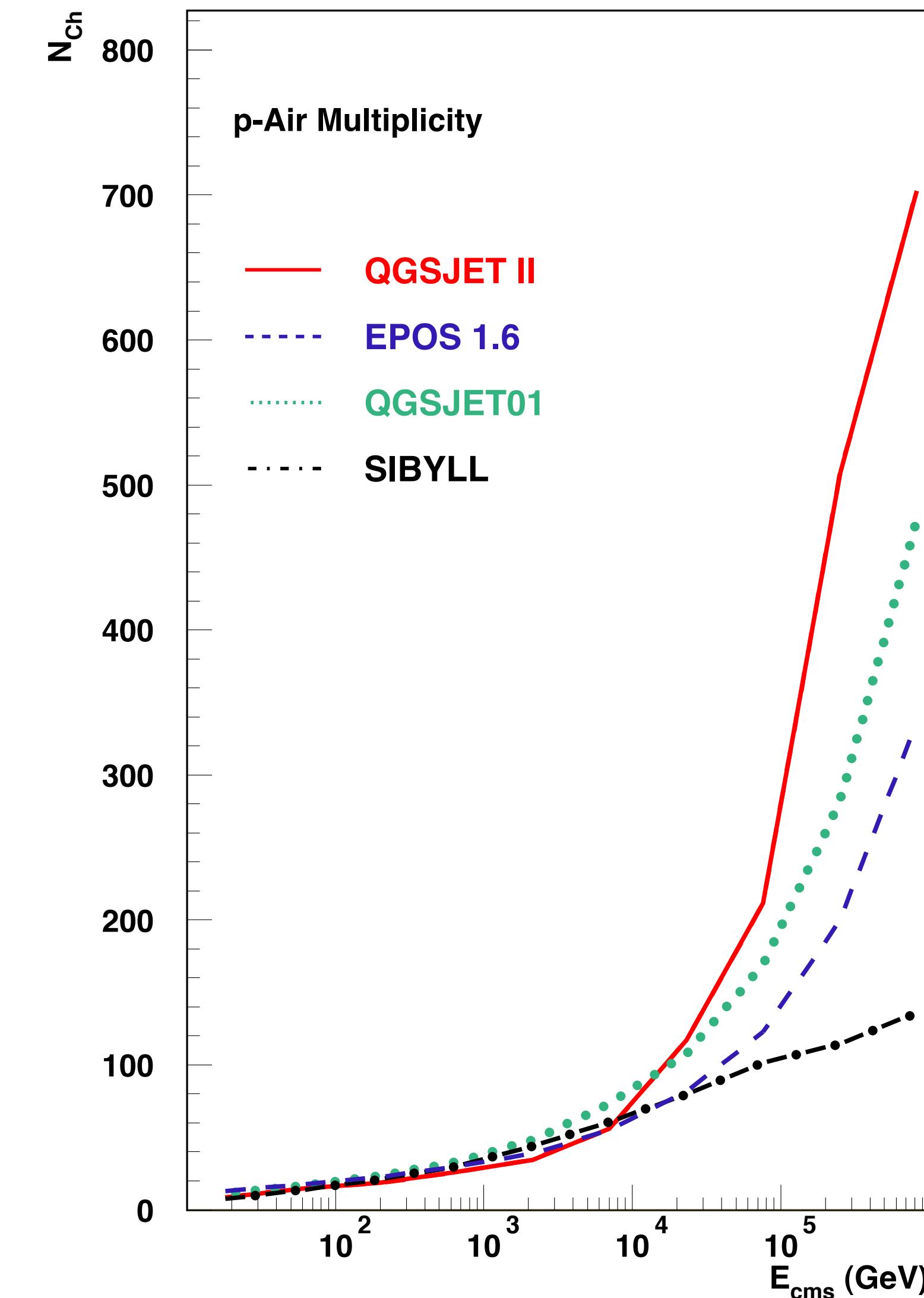
factor $\sim 87 \text{ g/cm}^2$

$$B_n = \frac{d \ln n_{\text{tot}}}{d \ln E}$$

Large if multiplicity of high energy particles rises very fast, **zero in case of scaling**

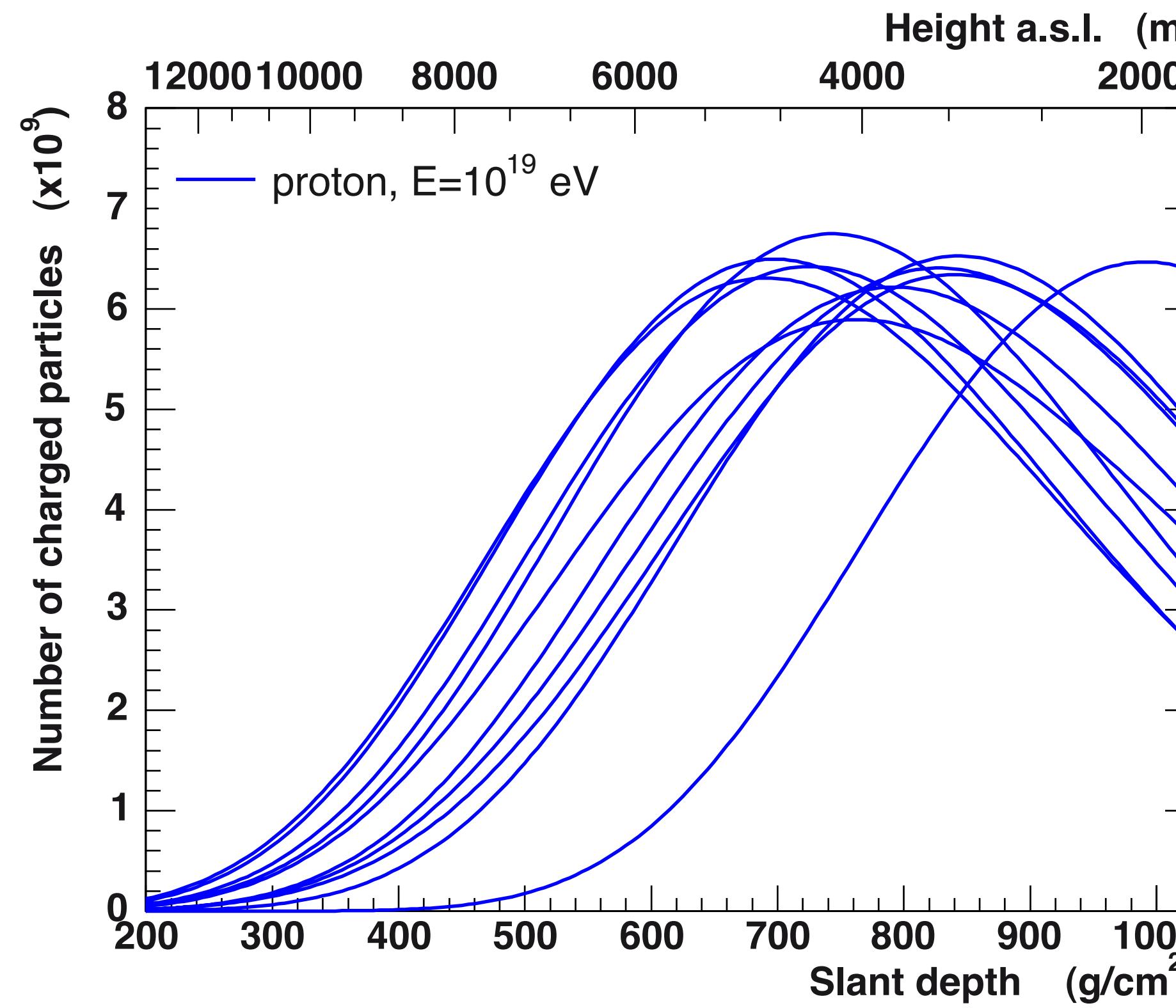
$$B_\lambda = -\frac{1}{X_0} \frac{d \lambda_{\text{int}}}{d \ln E}$$

Large if cross section rises rapidly with energy

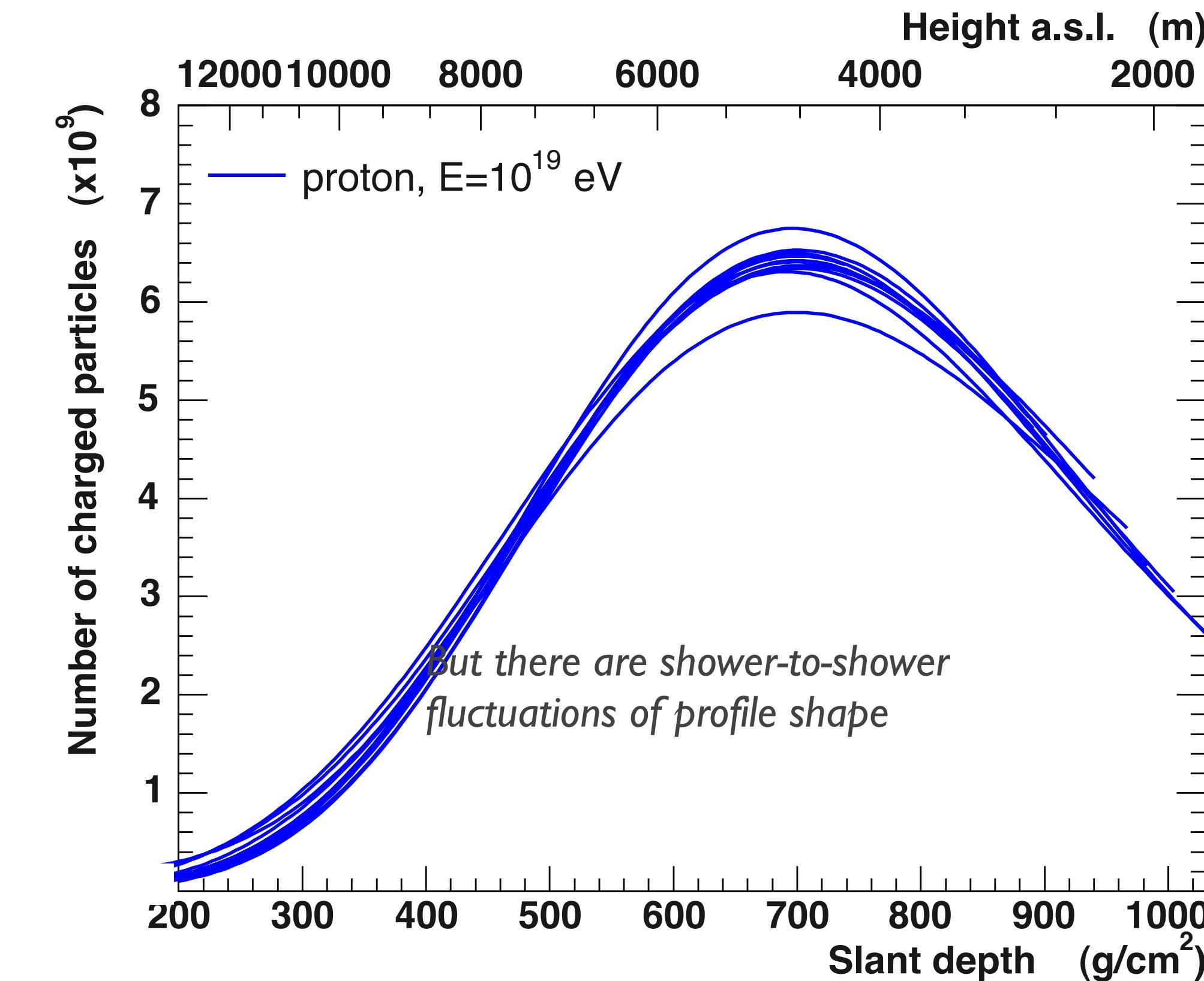


Universality features of high-energy shower profiles

Simulated shower profiles



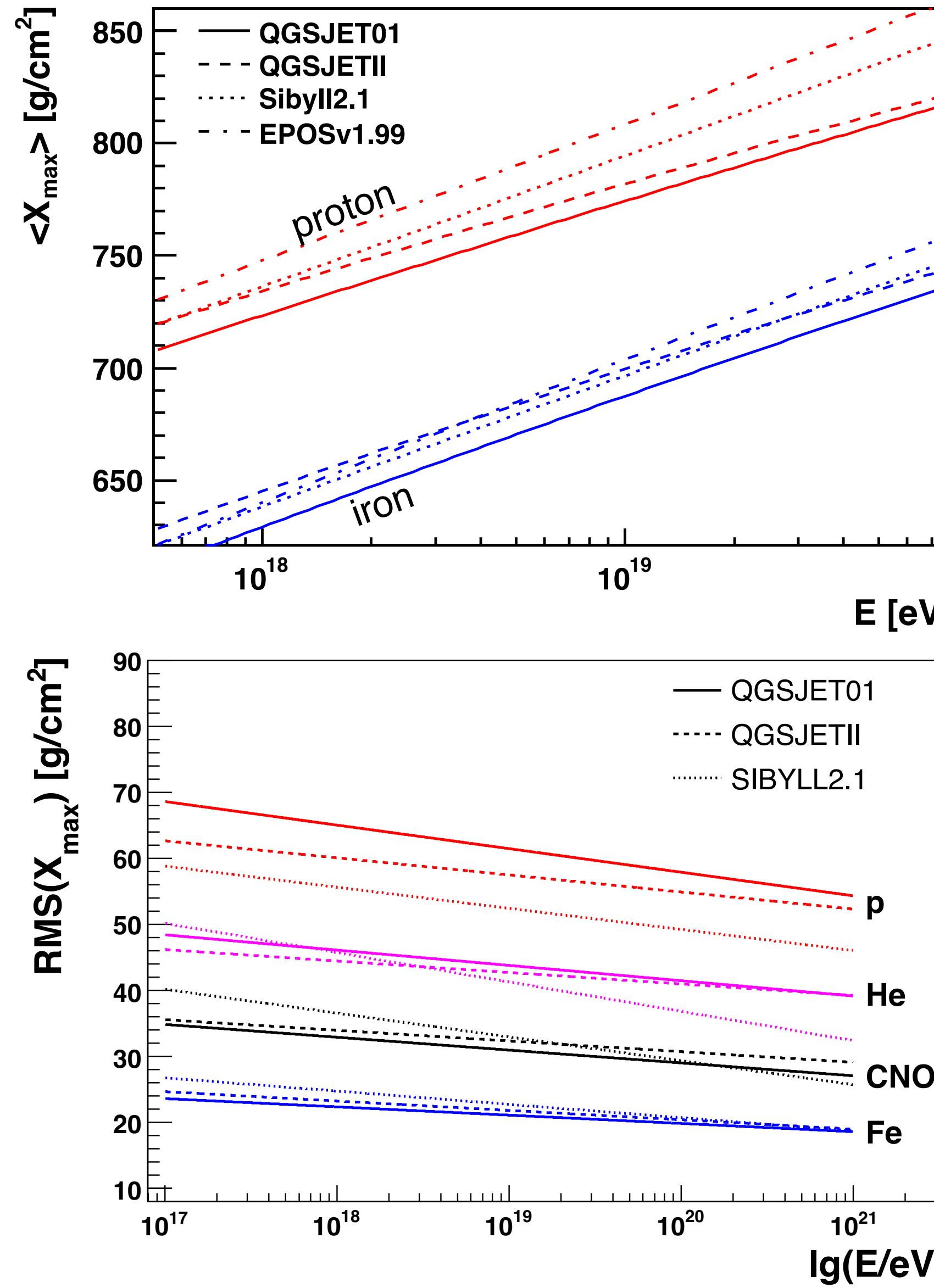
Profiles shifted in depth



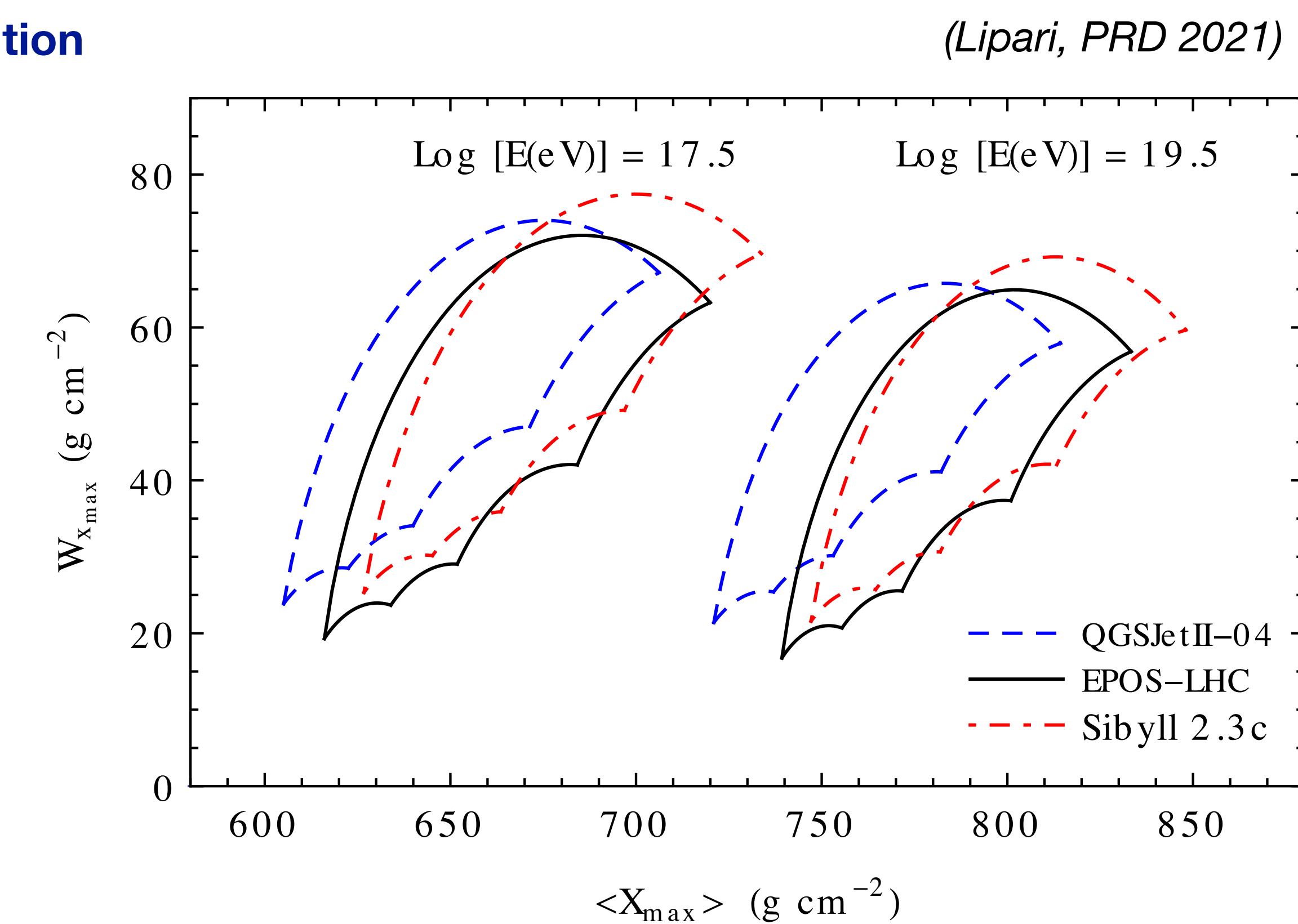
Depth of first interaction X_I and X_{max} strongly correlated, use X_{max} for analysis

Applications: mass composition and cross section

Information provided by Xmax fluctuations

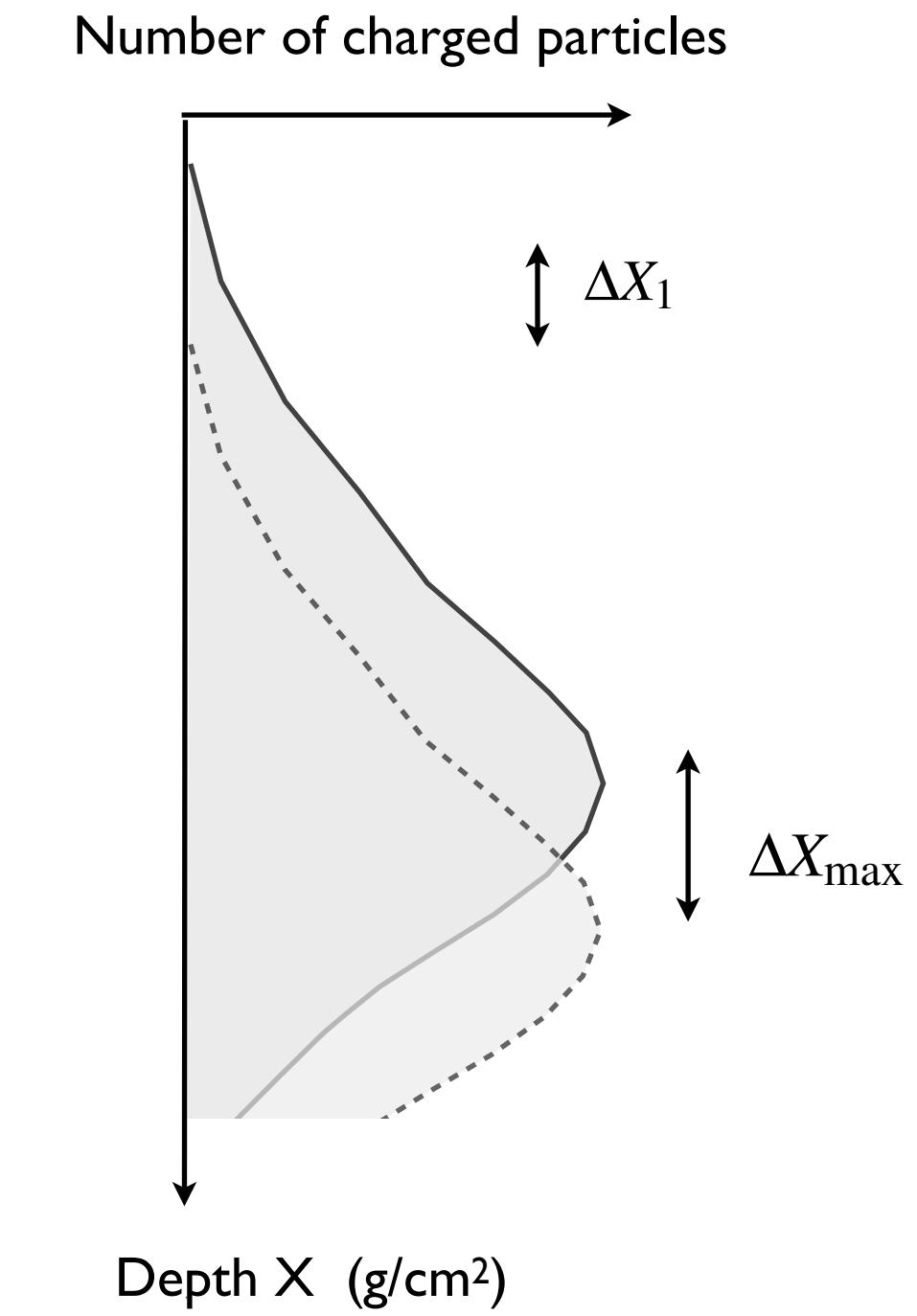
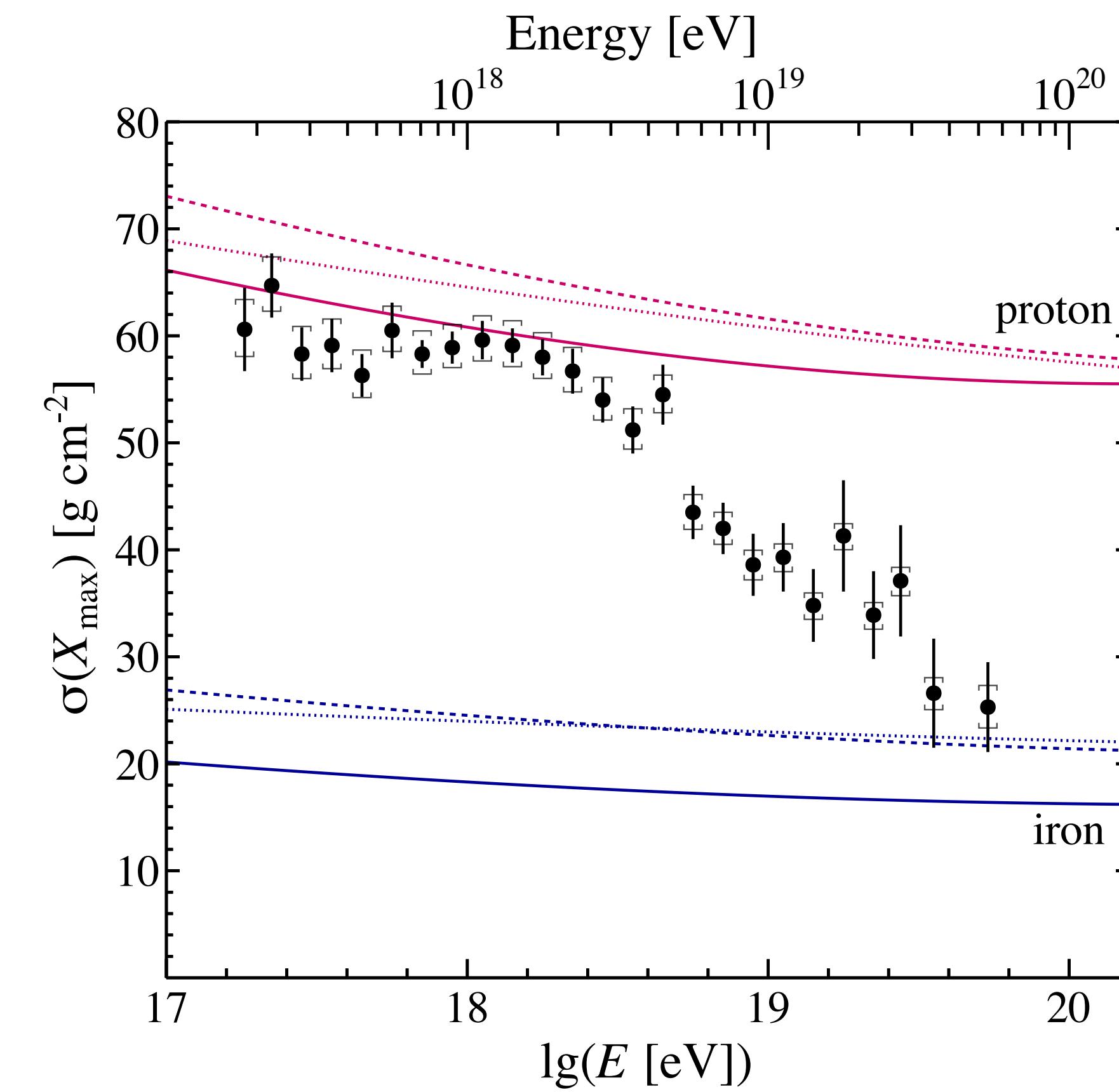
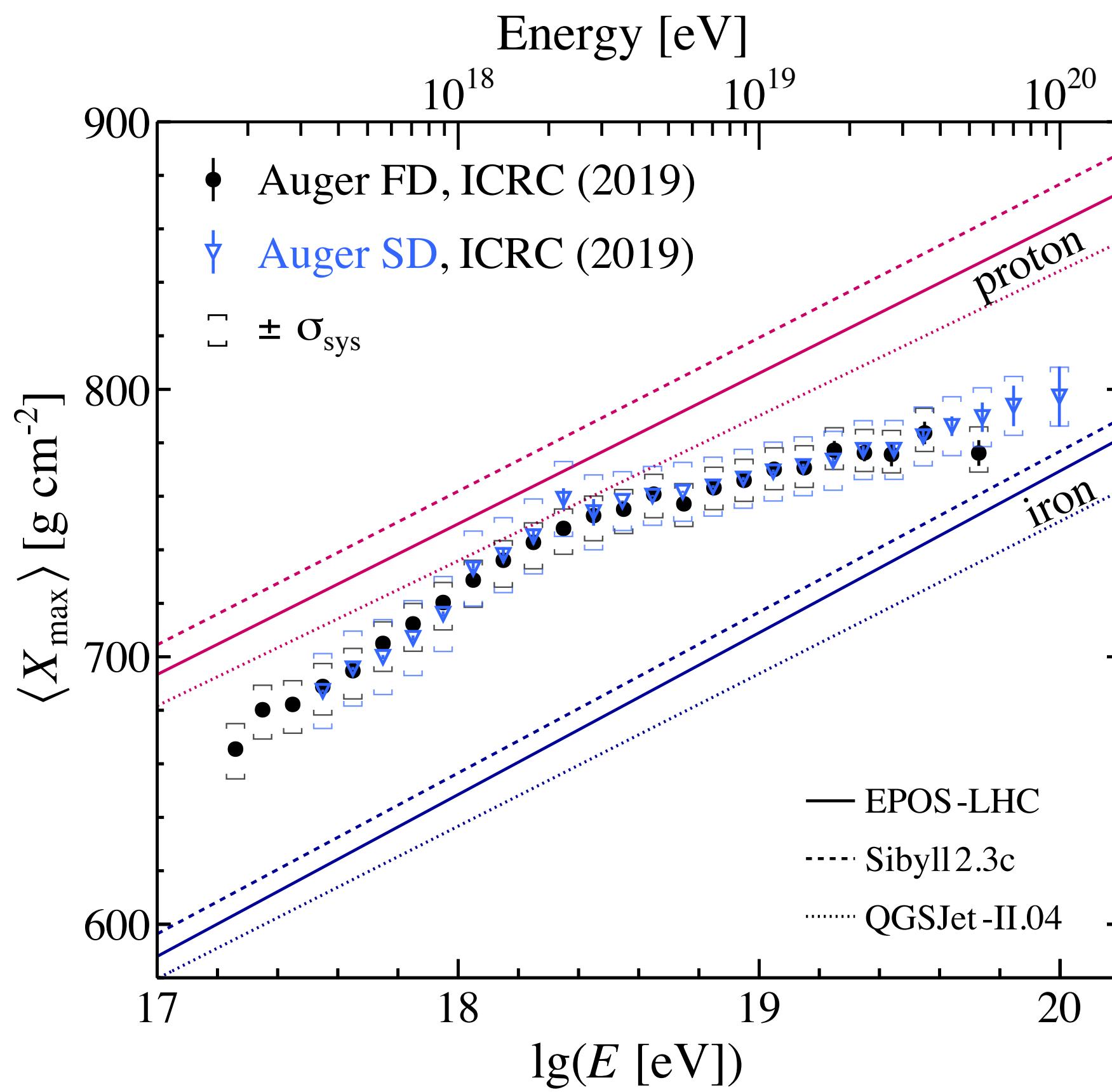


superposition
model



Model dependence of fluctuations
seems small (for mixed composition)

Mass composition results – Auger Observatory

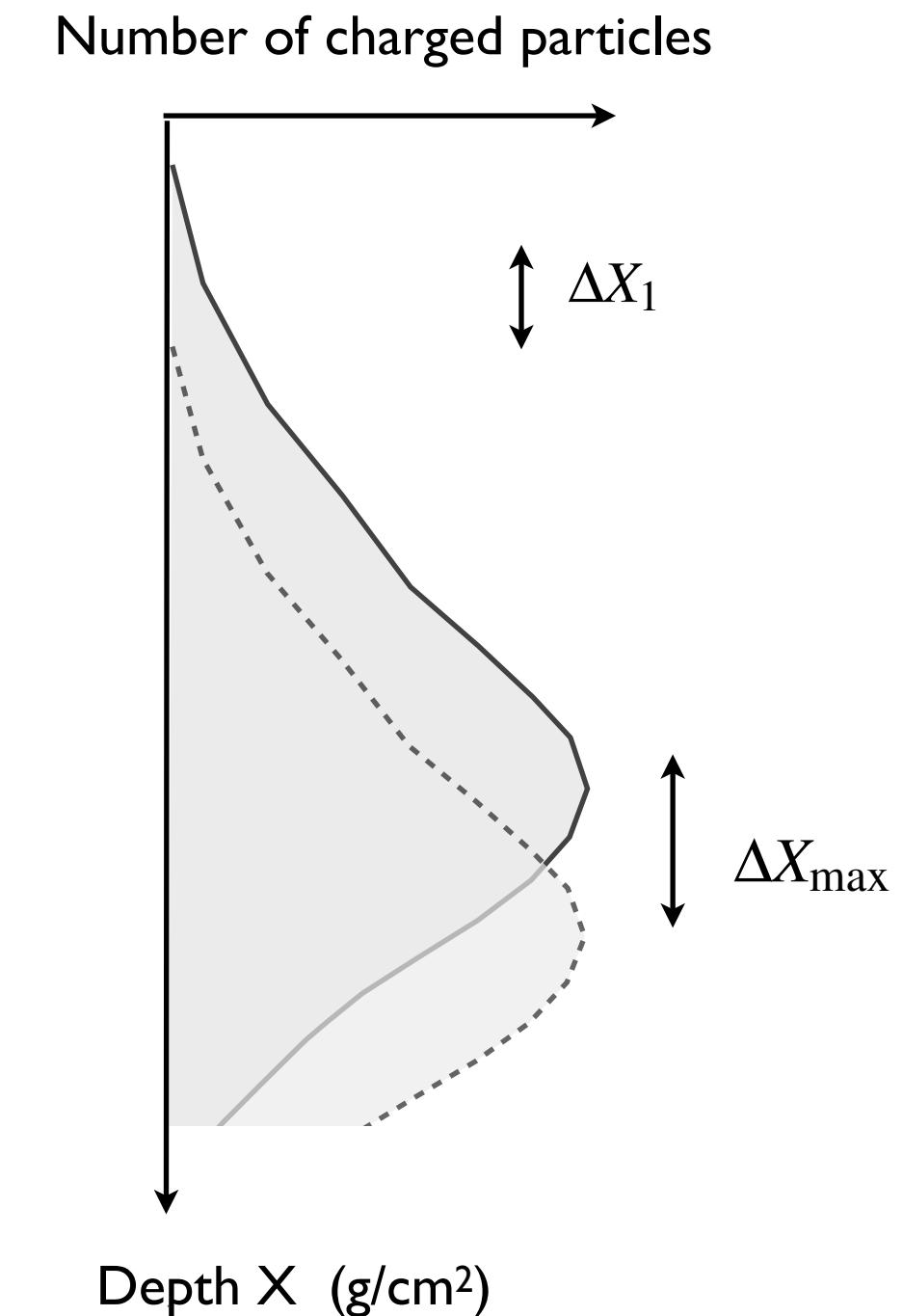
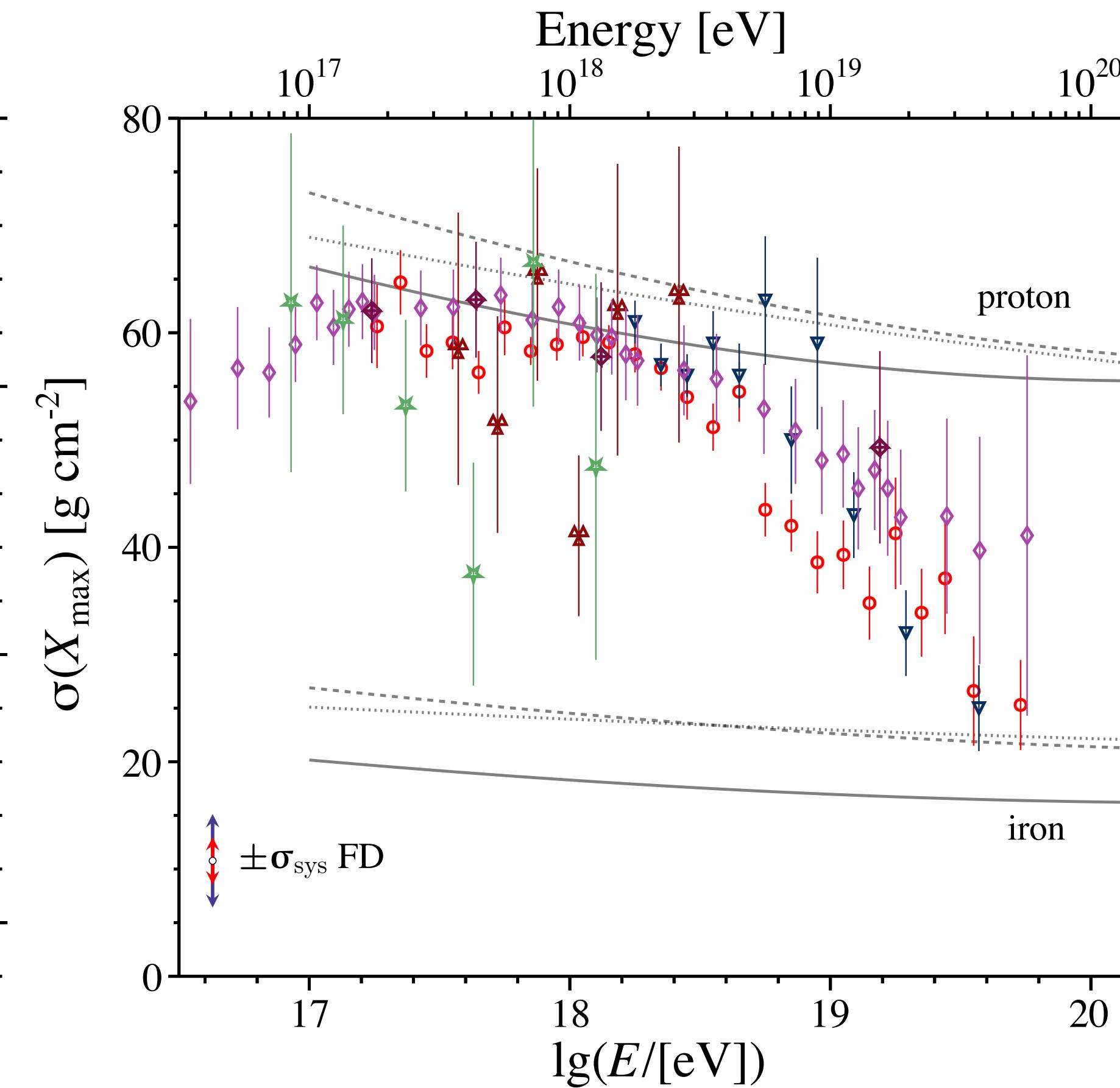
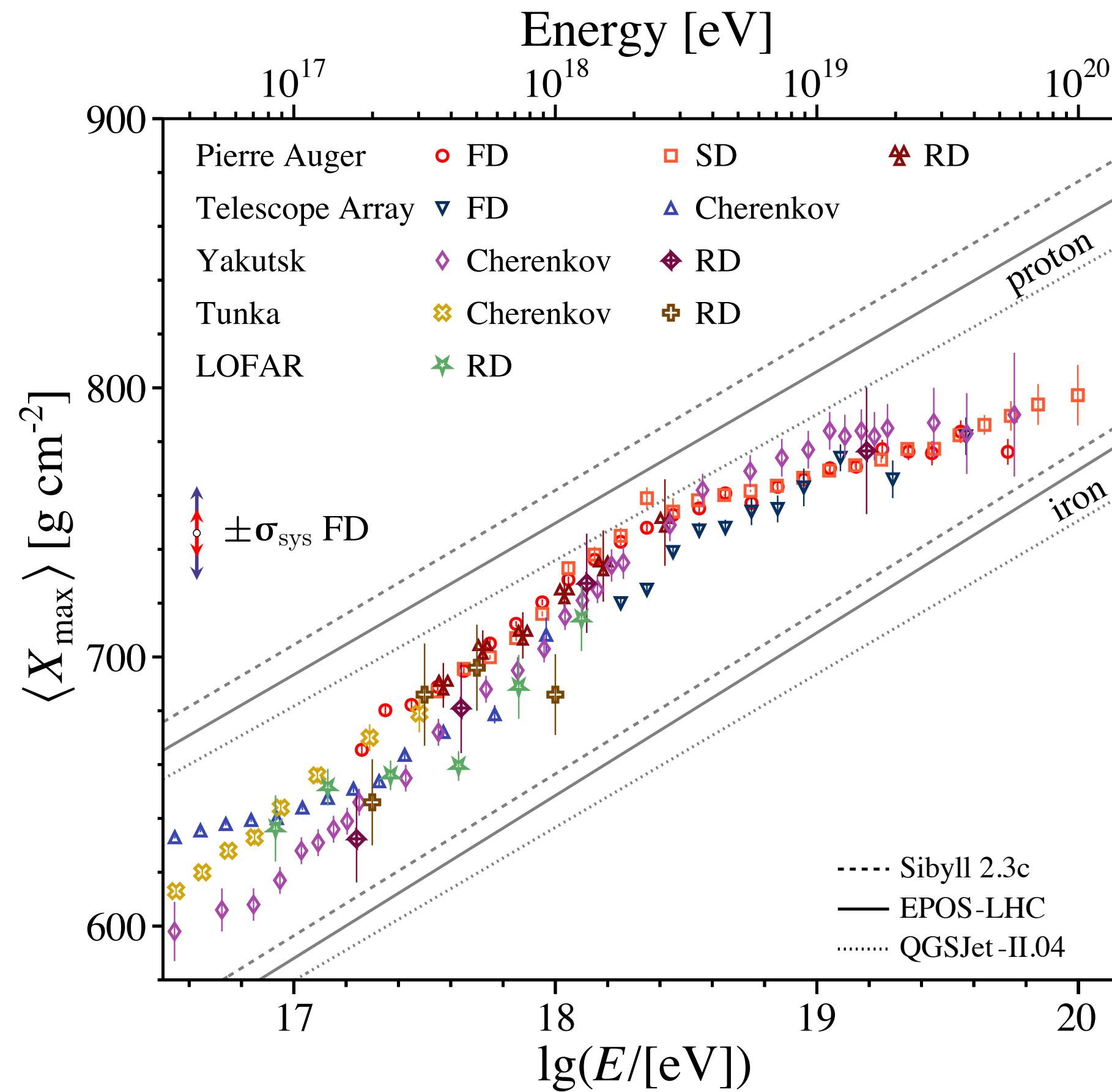


$$\frac{dP}{dX_1} = \frac{1}{\lambda_{\text{int}}} e^{-X_1/\lambda_{\text{int}}}$$

$\sigma_{X_1, \text{p}} \sim 45 - 55 \text{ g}/\text{cm}^2$
 $\sigma_{X_1, \text{Fe}} \sim 10 \text{ g}/\text{cm}^2$

Important: LHC-tuned interaction models used for interpretation

Mass composition results – world data

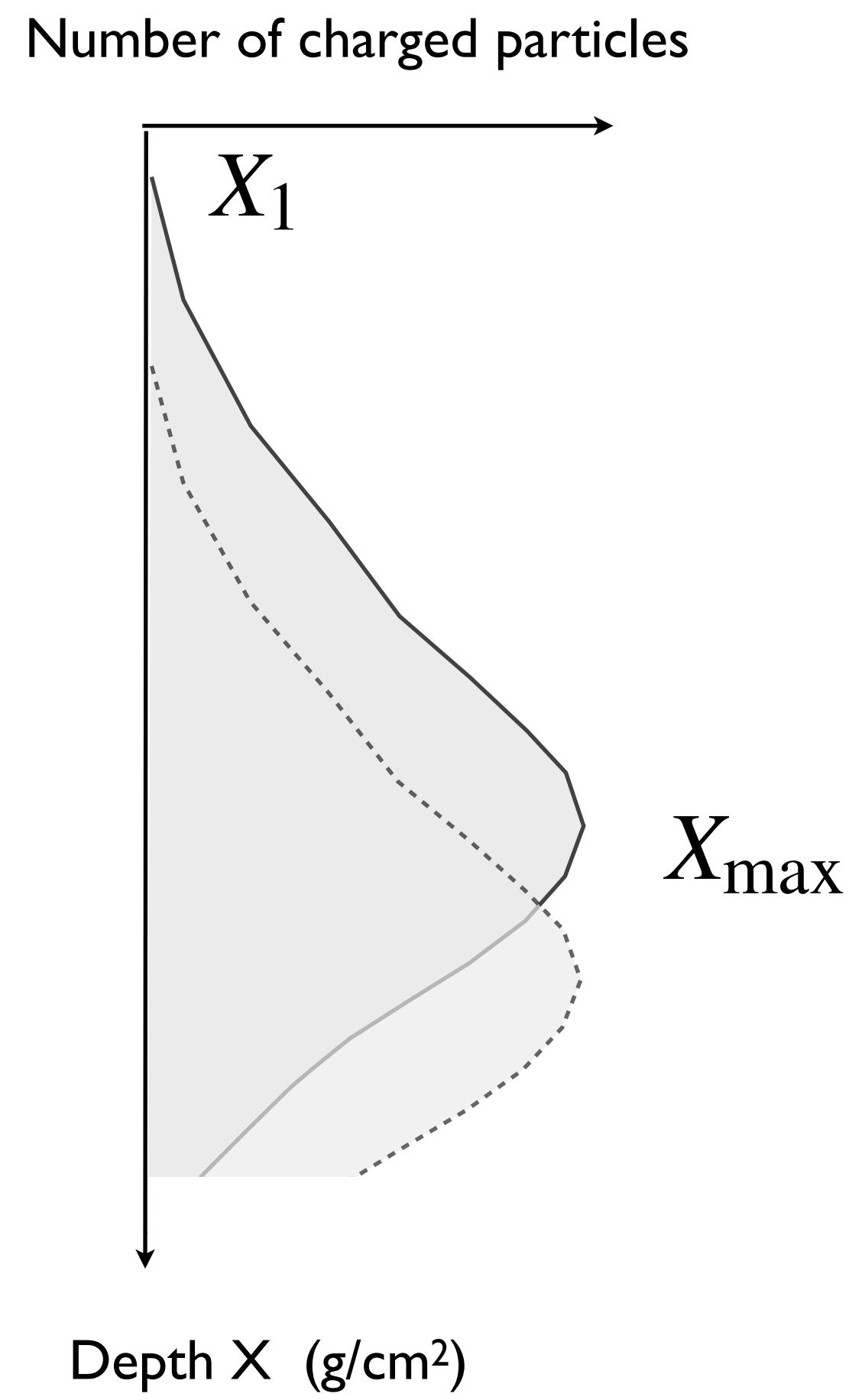
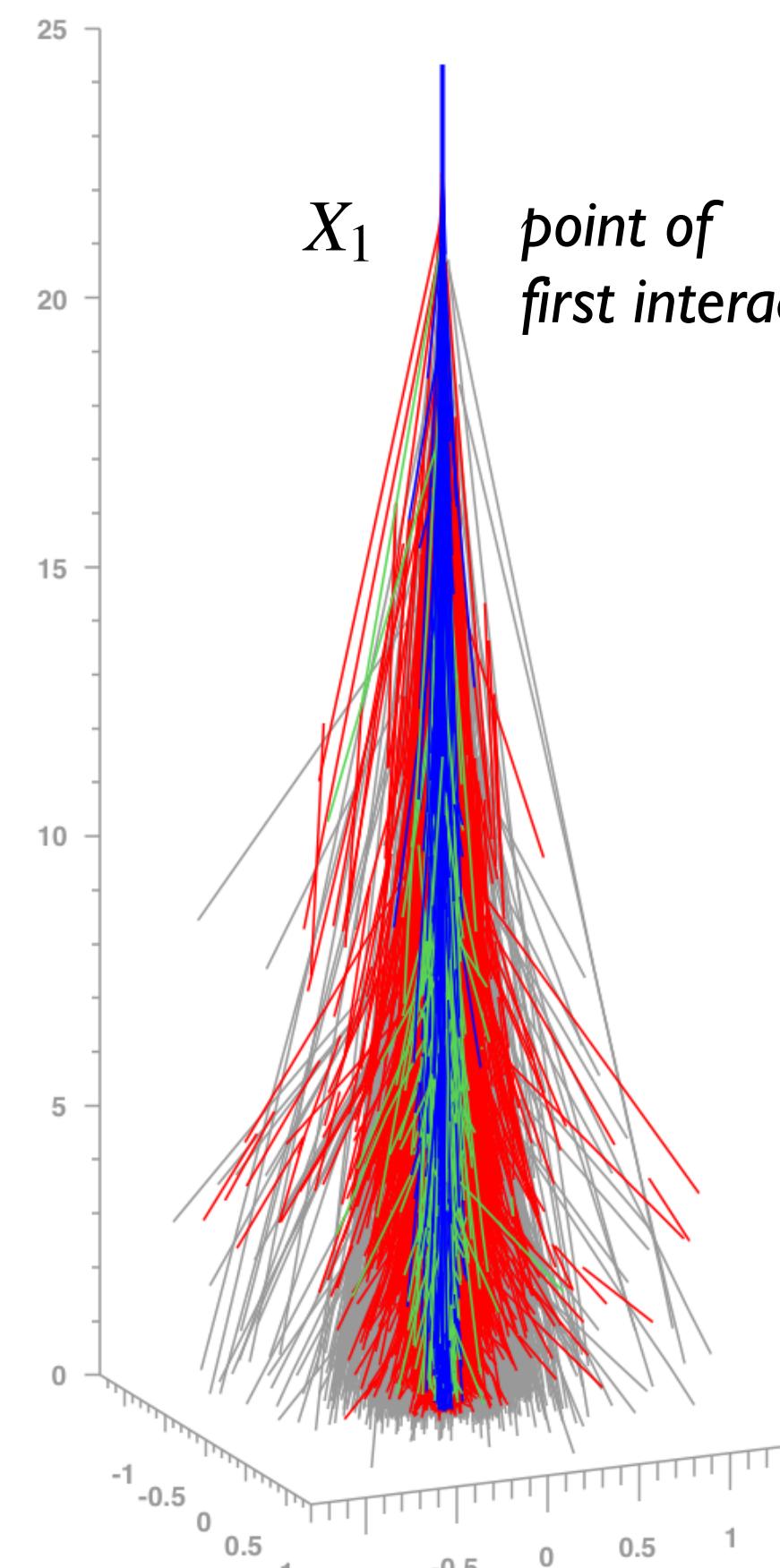


$$\frac{dP}{dX_1} = \frac{1}{\lambda_{\text{int}}} e^{-X_1/\lambda_{\text{int}}}$$

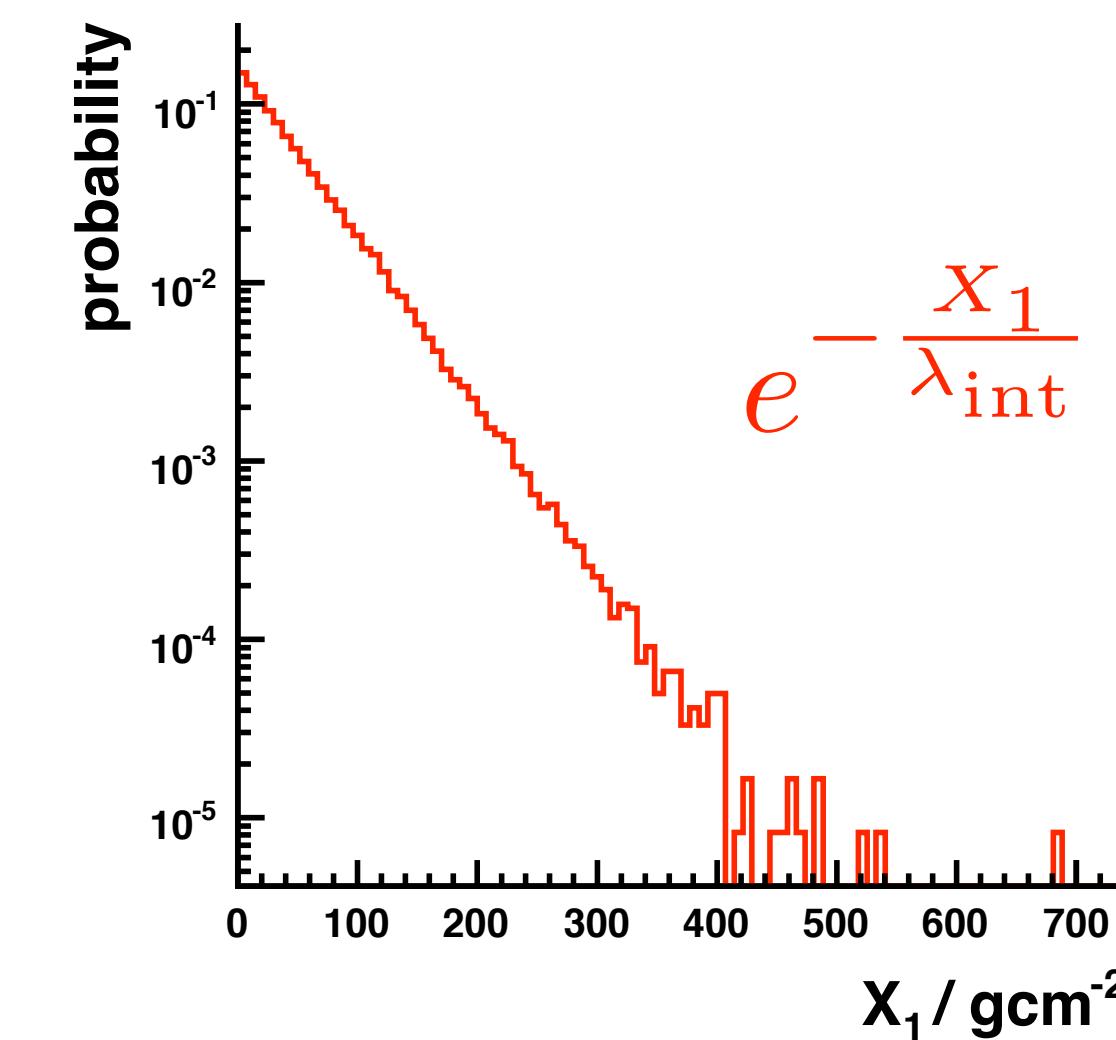
$\sigma_{X_1, \text{p}} \sim 45 - 55 \text{ g/cm}^2$

$\sigma_{X_1, \text{Fe}} \sim 10 \text{ g/cm}^2$

Cross section measurement with air showers



Depth of first interaction

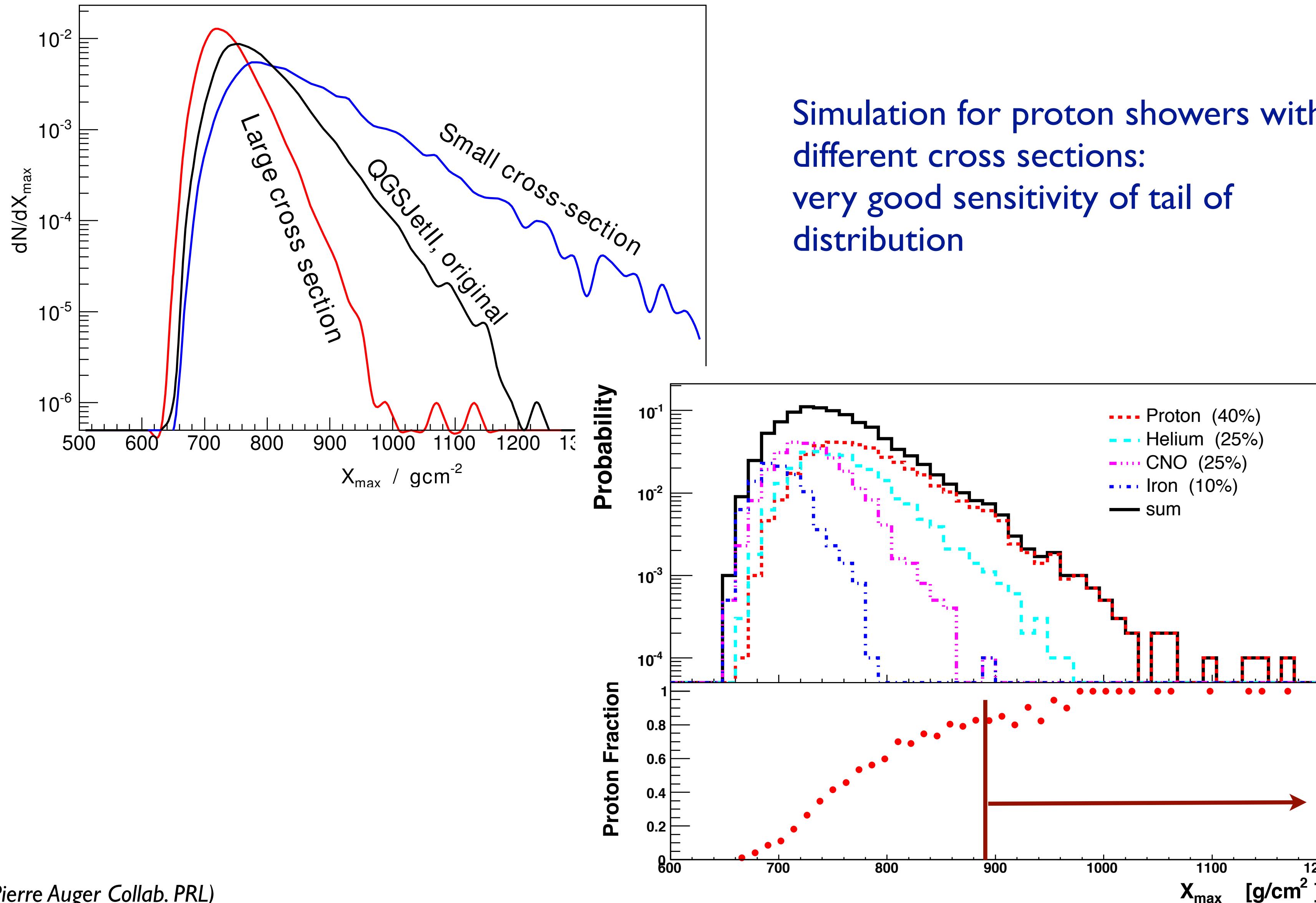


$$\sigma_{\text{prod}} = \frac{\langle m_{\text{air}} \rangle}{\lambda_{\text{int}}}$$

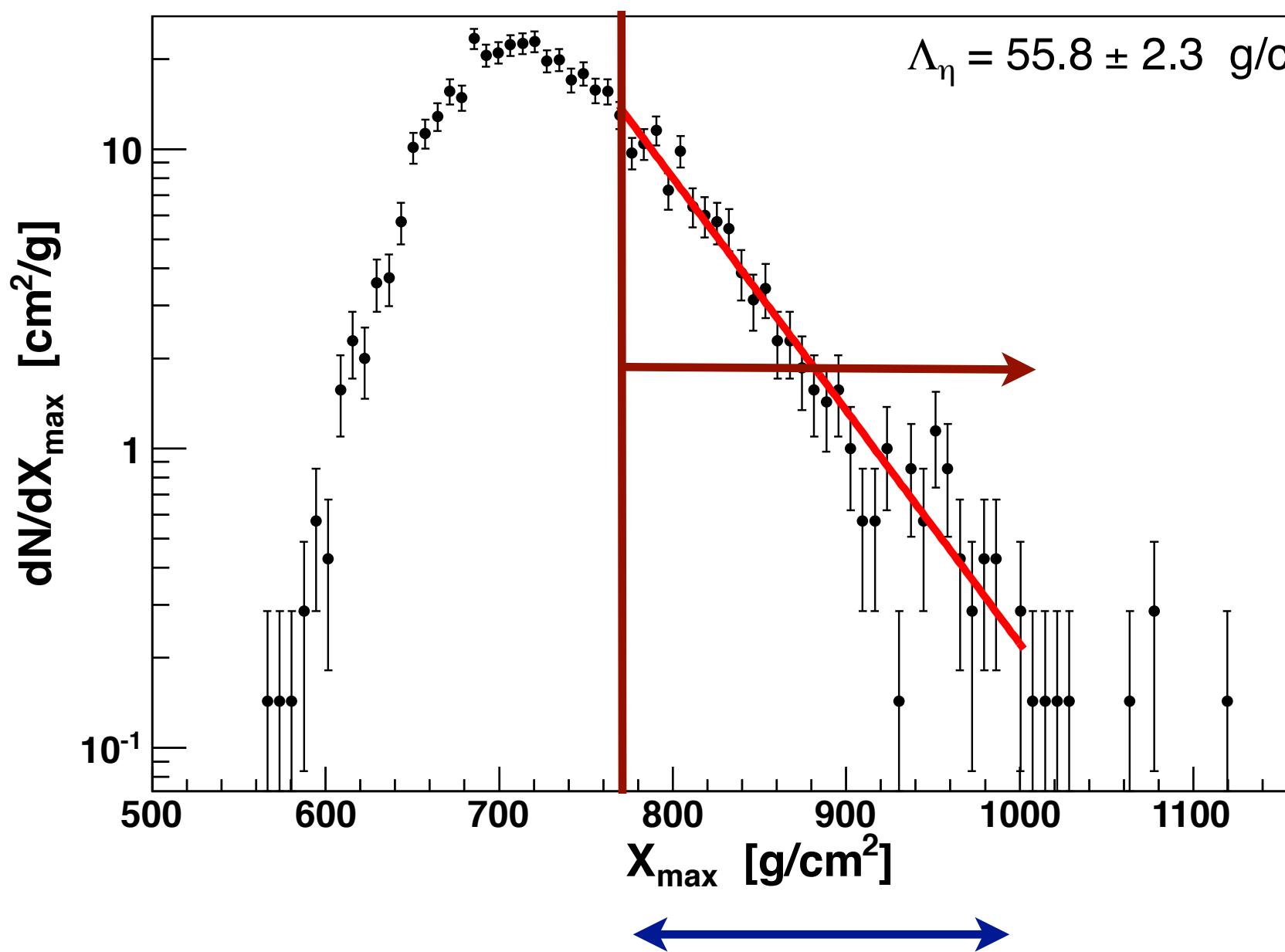
Difficulties

- mass composition (protons?)
- X_1 cannot be measured directly

Cross section measurement: composition bias?

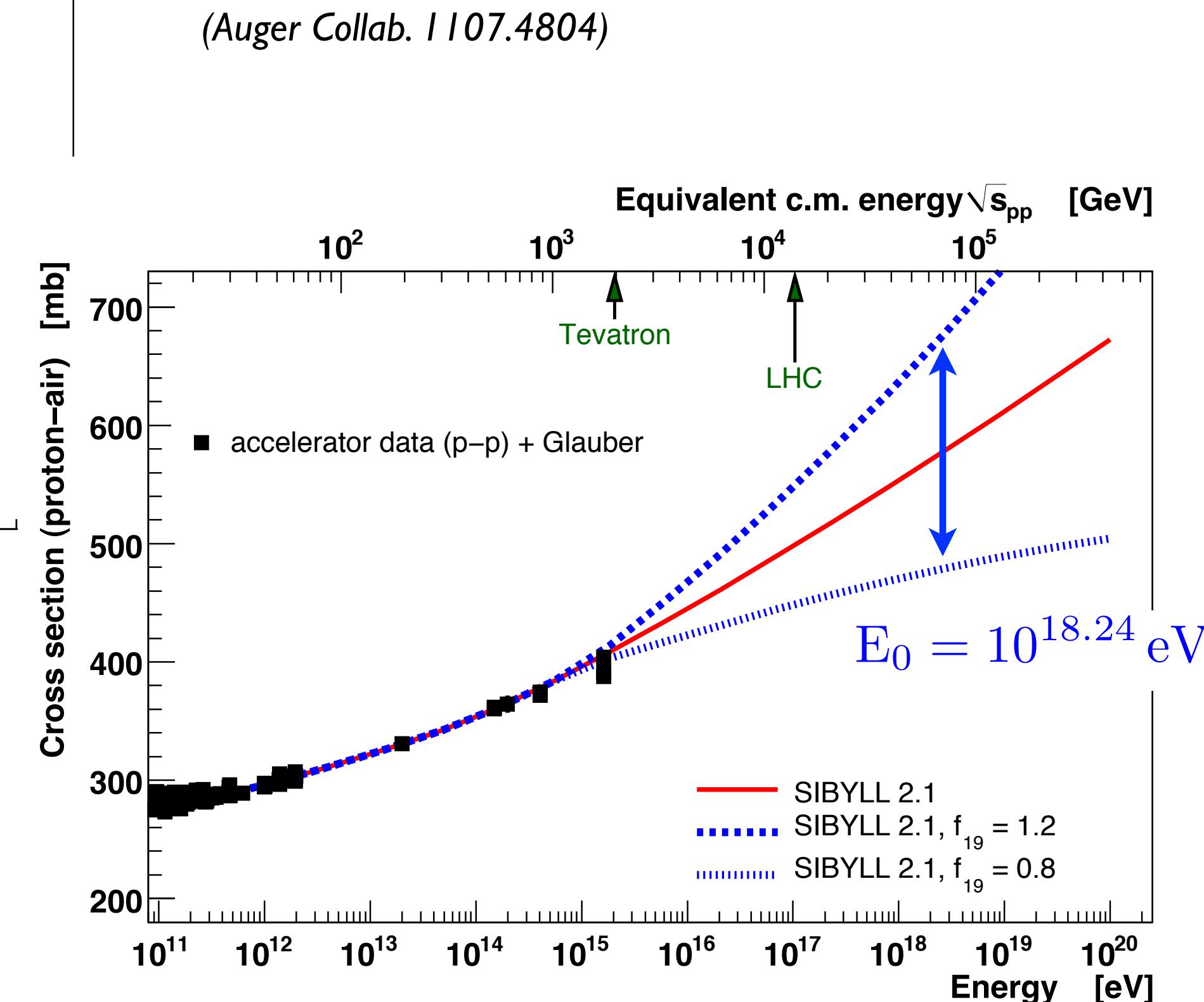


Cross section measurement: self-consistency



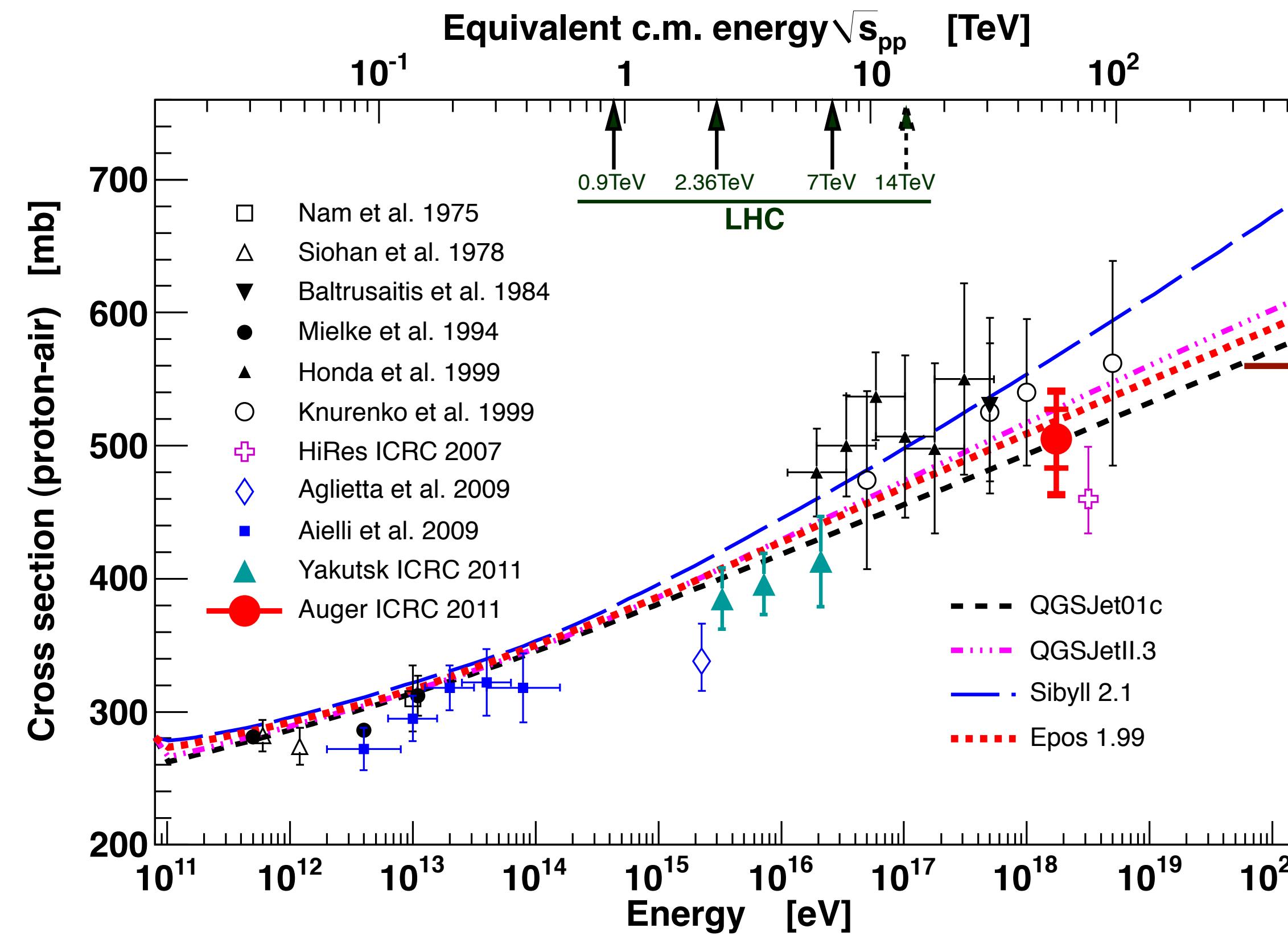
Cross section accepted if simulated slope fits measured slope of X_{\max} distribution

$$\sigma_{p\text{-air}} = (505 \pm 22_{\text{stat}} \quad {}^{+26}_{-34}_{\text{sys}}) \text{ mb}$$



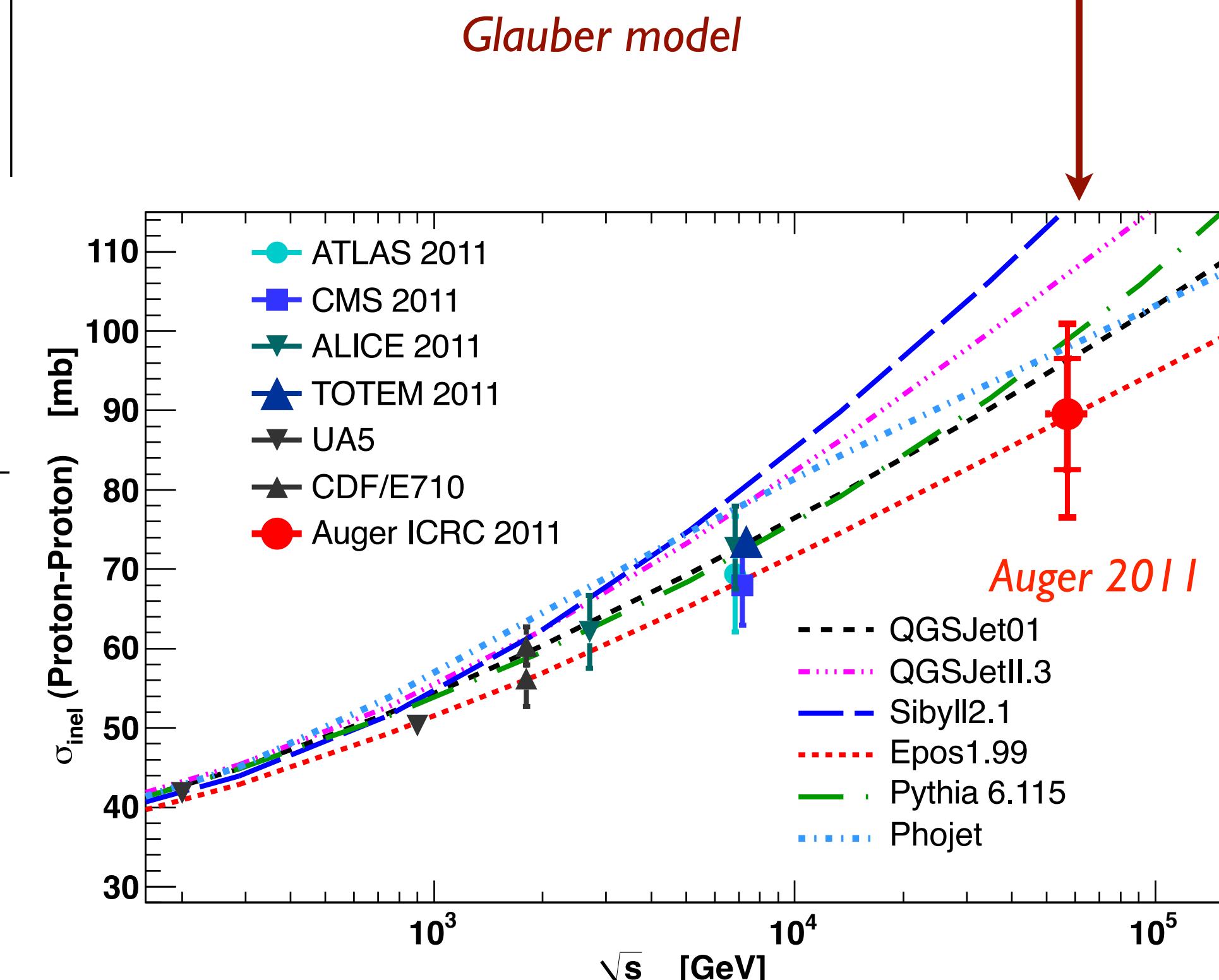
Simulation of data sample with different cross sections, interpolation to measured low-energy values

High-energy frontier: proton-air cross section

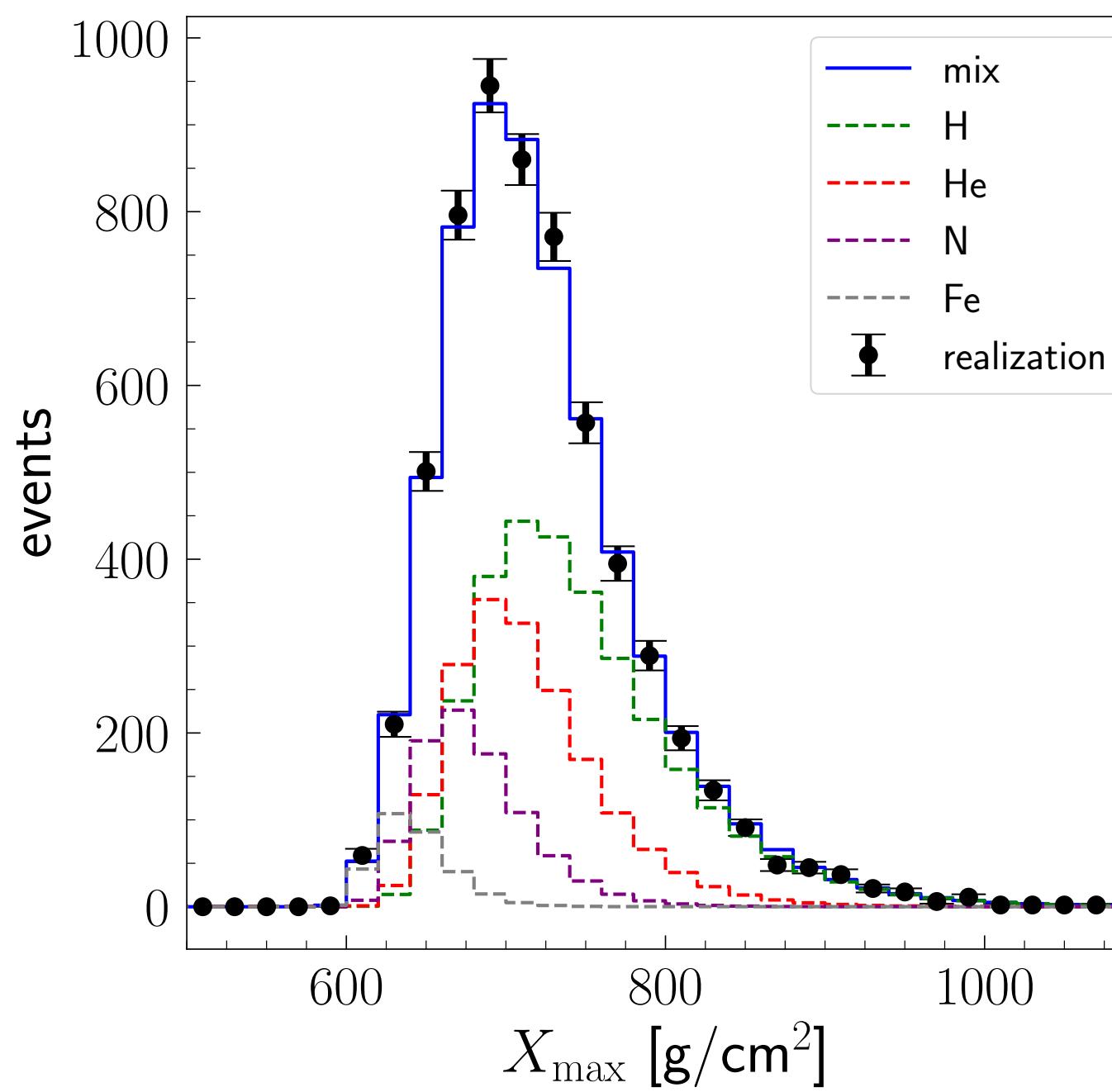


Cross section independent of LHC data,
very good agreement with extrapolated data

Conversion from p-air to p-p cross
section always model-dependent

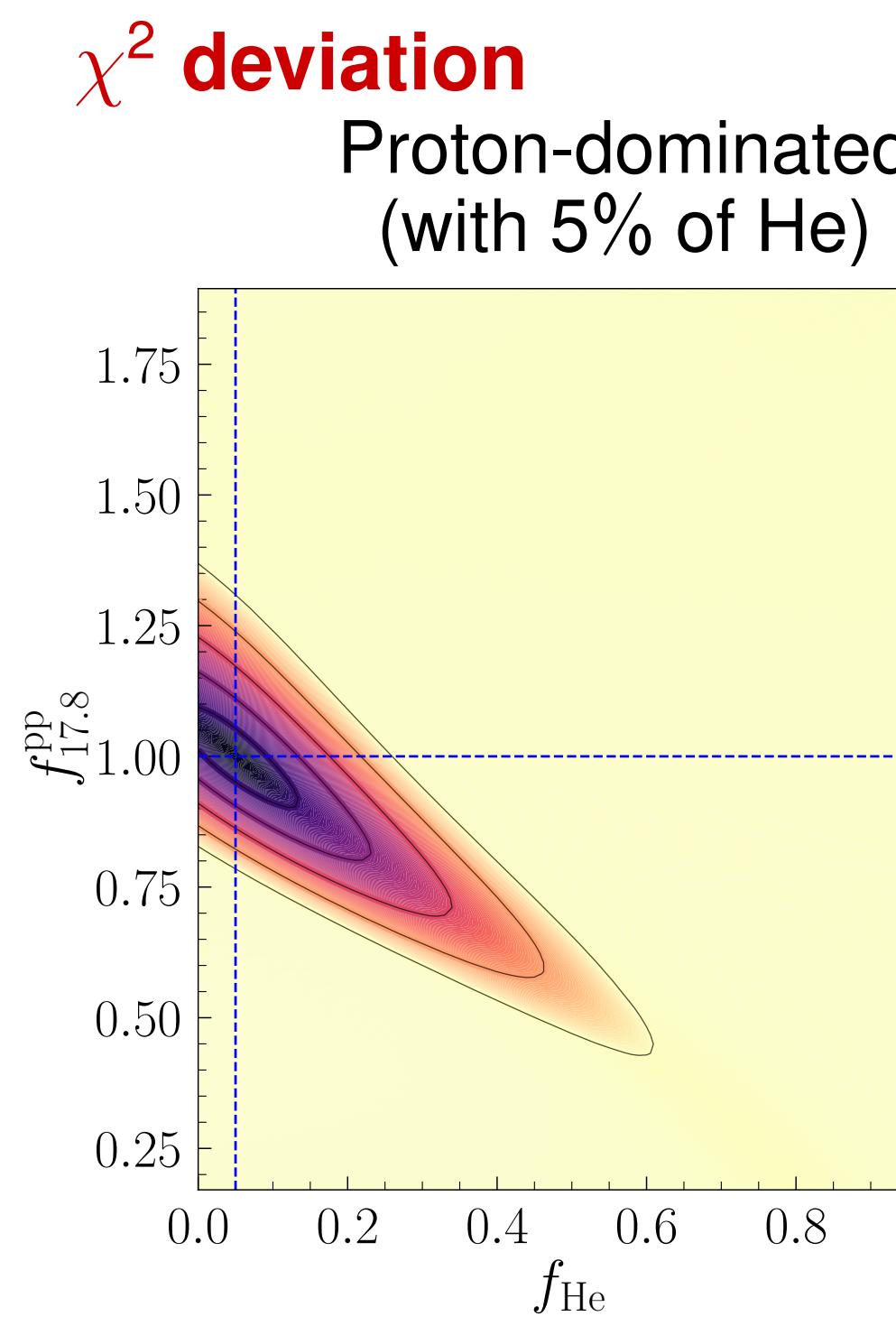


Measurement of composition and cross section?

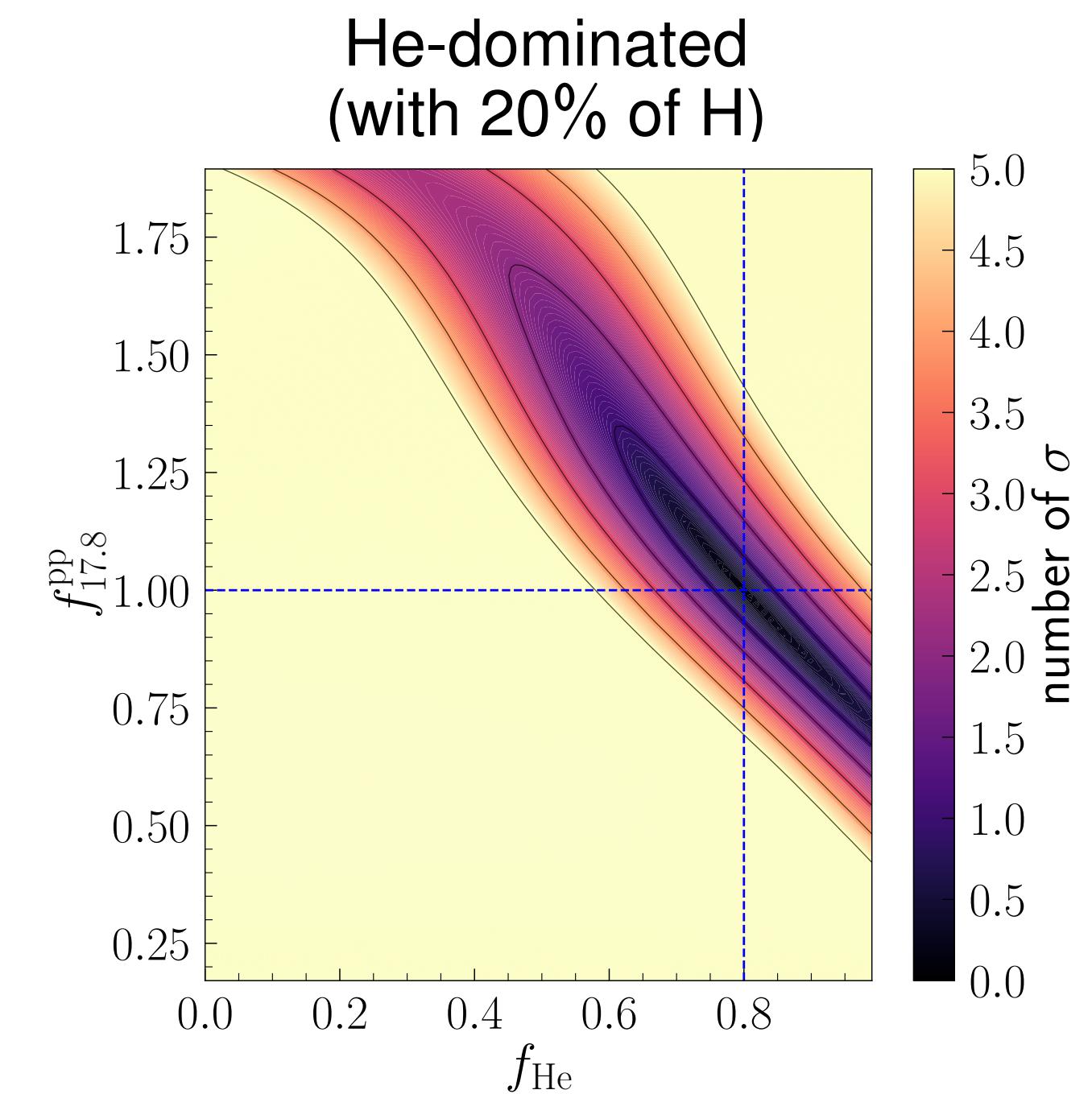
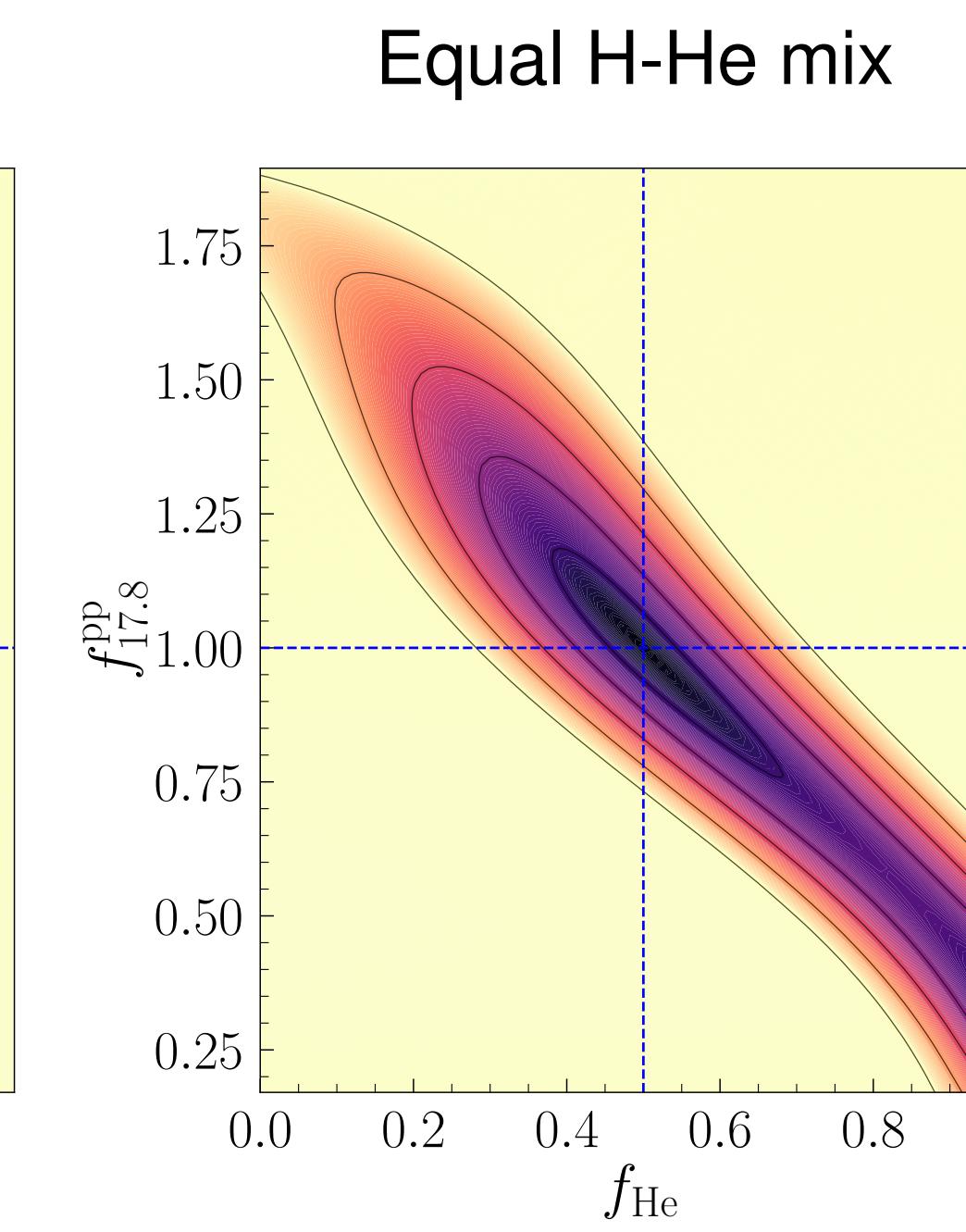


Analysis of Monte Carlo data set

Olena Tkachenko (ICRC 2021)



■ $\lg(E/\text{eV}) = 10^{17.8}-10^{17.9}$ eV

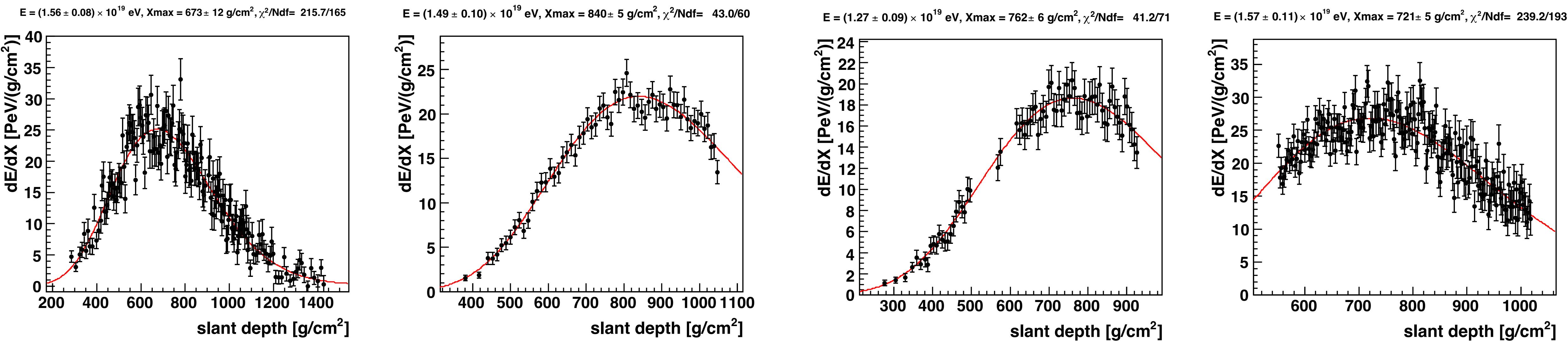


■ Model: Sibyll 2.3d

*blue dashed line shows the simulated He fraction and scaling factor, and colorbar shows the χ^2 deviation in units of sigma.

Consistent analysis, results stable only for large proton fraction

Gaisser-Hillas parametrization of shower profile



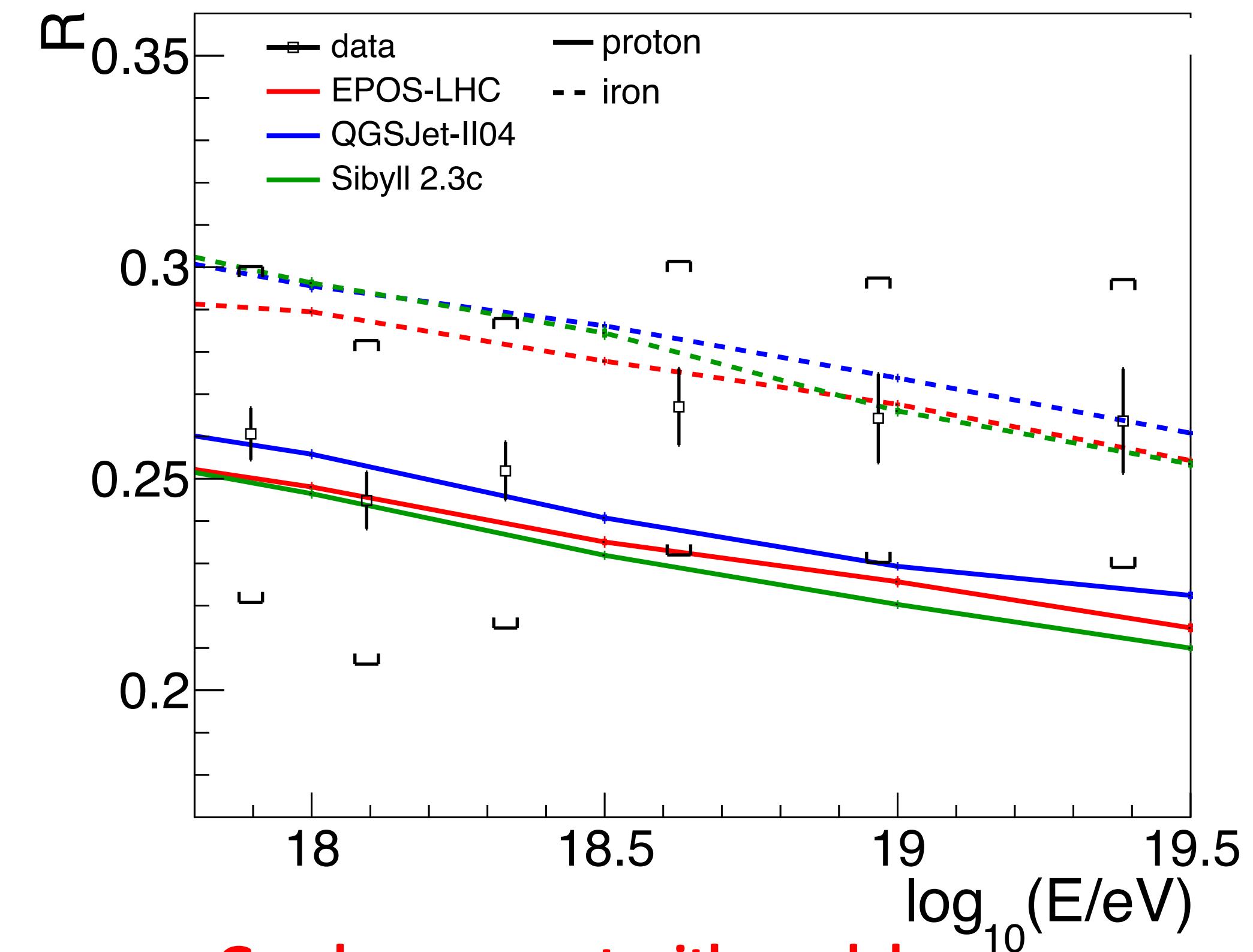
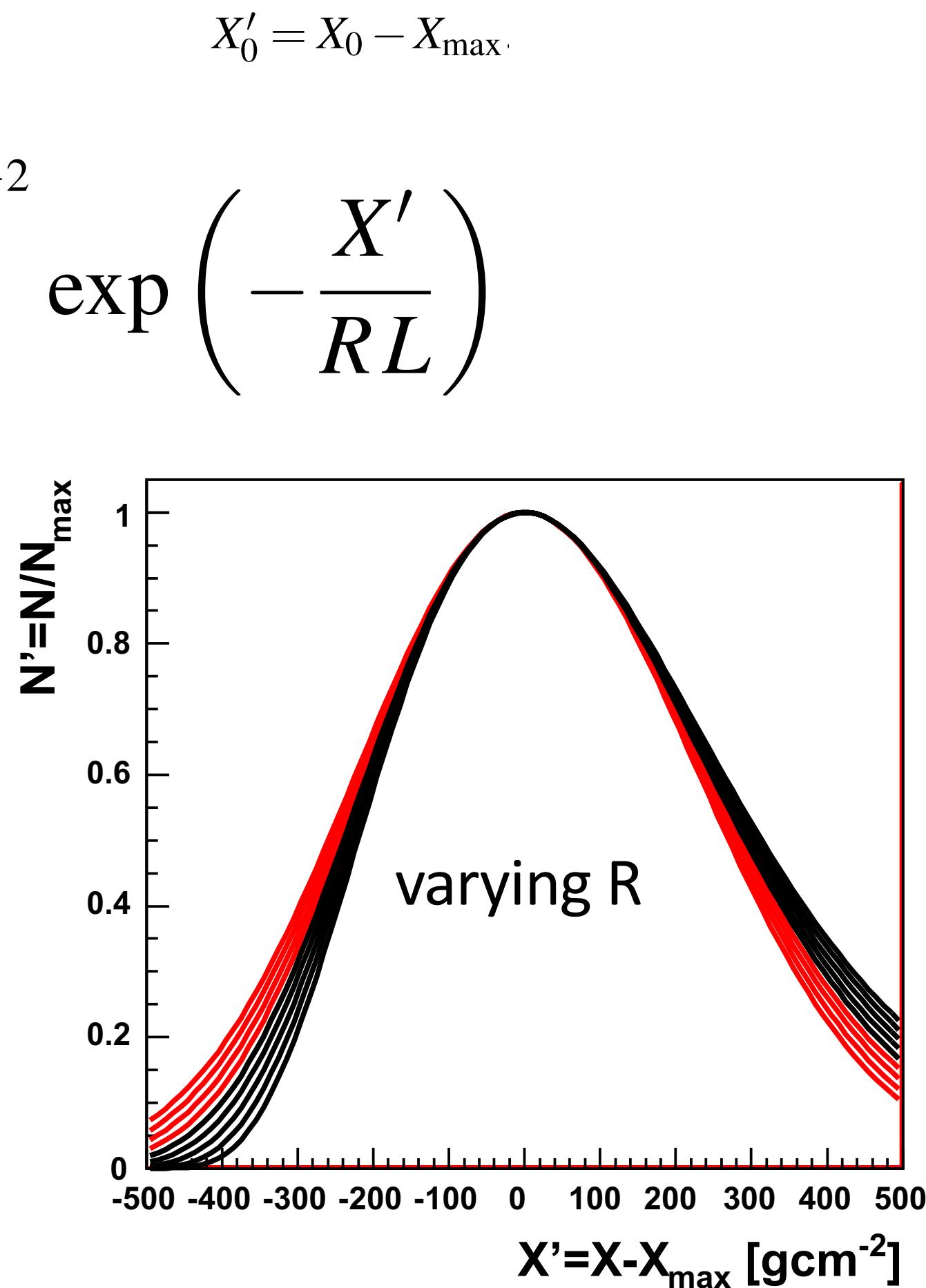
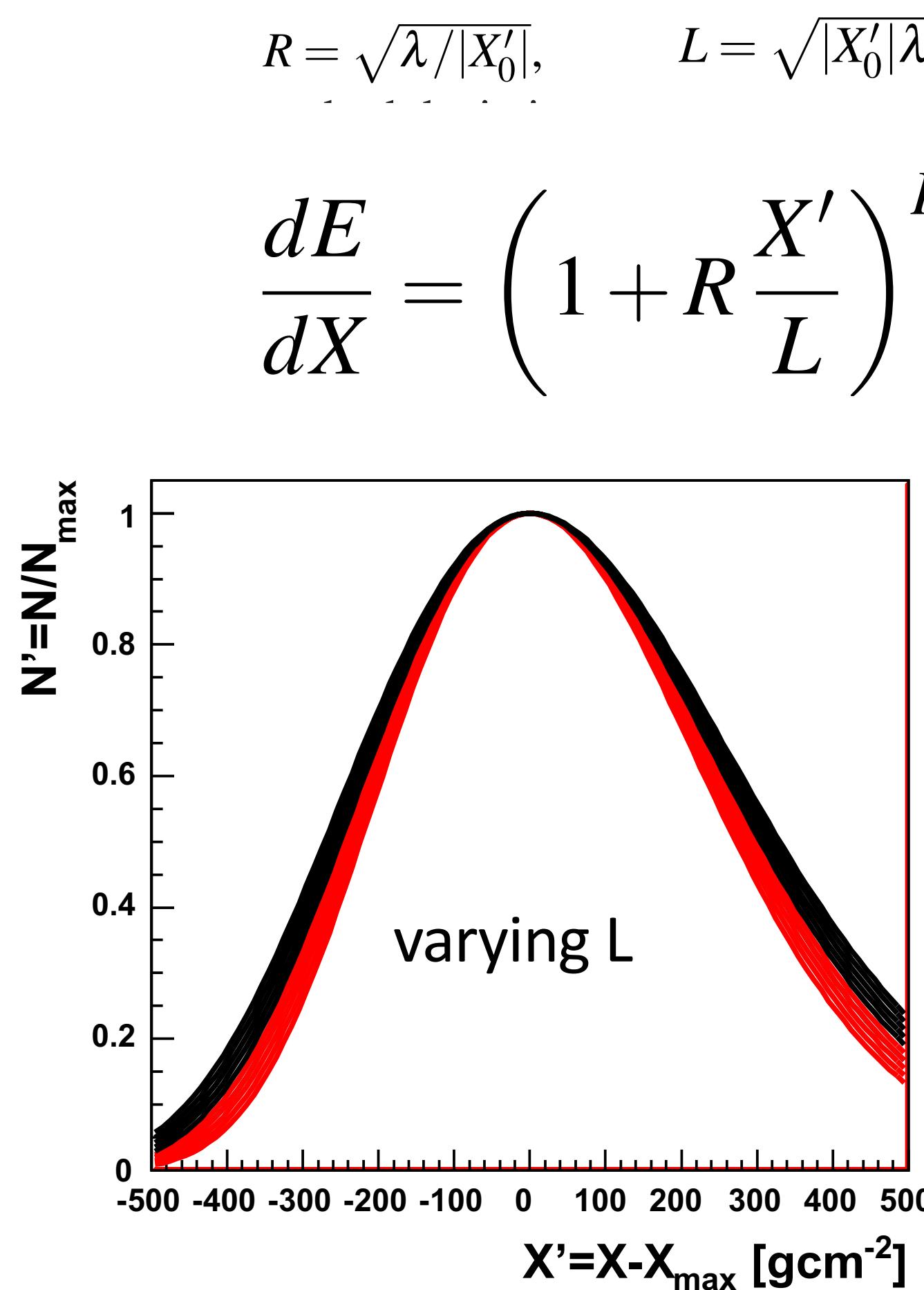
$$N(X) = N_{\max} \left(\frac{X - X_1}{X_{\max} - X_1} \right)^{(X_{\max} - X_1)/\Lambda} \exp \left(-\frac{X - X_{\max}}{\Lambda} \right)$$

Gaisser & Hillas ICRC 1977

Very good description of individual shower profiles and of mean shower profile (effective parameters)

Alternative way of writing GH parametrization

R is sensitive to the injection of **high energy π^0** in the start up of the shower.



Good agreement with models.

Too large systematics for hadronic physics, for the moment.

End of Lecture 1